INDIAN INSTITUTE OF SCIENCE EDUCATION AND RESEARCH PUNE

Mid-Semester Examination, 2023 August Semester

Course Name: **Mathematics of Network Algorithms**	Course Code: **DS4114/DS6144 and MT4154/MT6174**
Date of Exam: **25 September 2023**	Duration: **2 hours**
Instructor: **Prafullkumar Tale**	Total Score: **30**

Instructions: All 5 questions are compulsory. No queries will be entertained during the mid-term exam. Use your judgment about any possible doubts.

Questions:

Q.I. (a) (2 pts) Suppose u_1 and u_2 are eigenvectors of A with eigenvalues λ_1 and λ_2, respectively. Prove that if $\lambda_1 = \lambda_2$, then $u = \alpha \cdot u_1 + \beta \cdot u_2$ is also an eigenvector with the same eigenvalue. Here, α, β are some scalars.

(b) (4 pts) Define probability space and illustrate it with an example where we toss an unfair coin (which turns head with probability p) n-times. Define an appropriate random variable and write an expression to compute its values if we are interested in the number of heads.

Q.II. (a) (2 pts) Define *learning* in the context of Machine Learning and describe *supervised learning* and *unsupervised learning*.

(b) (4 pts) Write short description on four types of *tasks* (in the context of Machine Learning).

Q.III. (a) (2 pts) Prove the following:

$$\partial(\mathbf{x}^\top \mathbf{x})/\partial \mathbf{x} = 2\mathbf{x}^\top, \ \partial(\mathbf{x}^\top \mathbf{a})/\partial \mathbf{x} = \mathbf{a}^\top, \ \partial(\mathbf{a}^\top \mathbf{x})/\partial \mathbf{x} = \mathbf{a}^\top$$

(b) (4 pts) Prove that a function $f(\mathbf{x}) : \mathbb{R}^n \mapsto \mathbb{R}$ decreases fastest in the direction opposite to its gradient (assuming gradient exists everywhere).

Q.IV. (6 pts) Consider a set of samples $\{x^{(1)}, \ldots, x^{(m)}\}$ that are independently and identically distributed according to a Bernoulli distribution with mean θ. Consider the following estimator $\hat{\theta}_m = \frac{1}{m}\sum_{i=1}^{m} x^{(i)}$. Compute `bias` and `variance` of the estimator.

Q.V. (a) (2 pts) Construct a neural network for logical AND, logical OR, and logical NOT function.

(b) (4 pts) Construct a neural network that fits *all* the following points, i.e., given $[x, y]$ as input, it should output the corresponding color.

$$\langle [0,0]; red \rangle, \langle [0.5, 0.5]; red \rangle, \langle [1,0]; blue \rangle, \langle [0,1]; blue \rangle, \langle [1,1]; red \rangle$$