End-Term Exam: ECS 342/442/642 Competitive Programming

9:30 am to 12:10 pm on 21^{st} April, 2025

Instructions

- You can upload your files only once. Please follow the instructions carefully.
- Suppose your roll number is `20001`.
  - Open Linux and create folder `end-term-20001`.
  - The folder should contain files `et-01-20001.cpp`, ..., `et-08-20001.cpp` corresponding to the following **eight questions**.
  - Zip the folder and upload it at http://172.28.153.65:5000
- All the problems have time limit of 2 sec.
- Your output should use the following line of code.

```
int main()
{
   int final_output; // or other relevant declaration
   cout << ``20001\t'' << ``Donald Knuth\t'' << final_output << endl;
   //Replace '20001' by your roll number and 'Donald Knuth' by your name.
 }
```

Questions

1. (4 pts) Given a string, your task is to reorder its letters in such a way that it becomes a palindrome (i.e., it reads the same forwards and backwards).

   Input: The only input line has a string consisting of characters A–Z.

   Output: Print a palindrome consisting of the characters of the original string. You may print any valid solution. If there are no solutions, print "NO SOLUTION".

2. (4 pts) You have n coins with positive integer values. What is the smallest sum you cannot create using a subset of the coins? You can use any coin at most once.

   Input: The first line has an integer n: the number of coins. The second line has n integers $x_1, x_2, \ldots, x_n$: the value of each coin.

   Output: Print one integer: the smallest coin sum.

3. (4 pts) Consider a money system consisting of n coins. Each coin has a positive integer value. Your task is to calculate the number of distinct ordered ways you can produce a money sum x using the available coins. For example, if the coins are $\{2, 3, 5\}$ and the desired sum is 9, there are 3 ways: (i) $2+2+5$, (ii) $3+3+3$, and (iii) $2+2+2+3$.

   Input: The first input line has two integers n and x: the number of coins and the desired sum of money. The second line has n distinct integers $c_1, c_2, \ldots, c_n$: the value of each coin.

   Output: Print one integer: the number of ways modulo $10^9 + 7$.

   Constraints $1 \leq n \leq 100$, $1 \leq x \leq 10^6$, and $1 \leq c_i \leq 10^6$.

4. (4 pts) You are given an array of n positive integers. Your task is to find two integers such that their greatest common divisor is as large as possible.

   Input: The first input line has an integer n: the size of the array. The second line has n integers $x_1, x_2, \ldots, x_n$: the contents of the array.

   Output: Print the maximum greatest common divisor.

5. (6 pts) Your task is to count the number of ways numbers $1, 2, \ldots, n$ can be divided into two sets of equal sum. For example, if $n = 7$, there are four solutions: (i) $\{1, 3, 4, 6\}$ and $\{2, 5, 7\}$, (ii) $\{1, 2, 5, 6\}$ and $\{3, 4, 7\}$, (iii) $\{1, 2, 4, 7\}$ and $\{3, 5, 6\}$, and (iv) $\{1, 6, 7\}$ and $\{2, 3, 4, 5\}$.

   Input: The only input line contains a positive integer n which is at most 500.

   Output: Print the answer modulo $10^9 + 7$.

6. (6 pts) Syrjala's network has n computers and m connections. Your task is to find out if Uolevi can send a message to Maija, and if it is possible, what is the minimum number of computers on such a route.

   Input: The first input line has two integers n and m: the number of computers and connections. The computers are numbered $1, 2, \ldots, n$. Uolevi's computer is 1 and Maija's computer is n. Then, there are m lines describing the connections. Each line has two integers a and b: there is a connection between those computers. Every connection is between two different computers, and there is at most one connection between any two computers.

   Output: If it is possible to send a message, first print k: the minimum number of computers on a valid route. After this, print an example of such a route. You can print any valid solution. If there are no routes, print "IMPOSSIBLE".

7. (6 pts) You are given a tree consisting of n nodes, and m paths in the tree. Your task is to calculate for each node the number of paths containing that node.

   Input: The first input line contains integers n and m: the number of nodes and paths. The nodes are numbered $1, 2, \ldots, n$. Then there are $n-1$ lines describing the edges. Each line contains two integers a and b: there is an edge between nodes a and b. Finally, there are m lines describing the paths. Each line contains two integers a and b: there is a path between nodes a and b.

   Output: Print n integers: for each node $1, 2, \ldots, n$, the number of paths containing that node.

8. (6 pts) Given an array of n integers, your task is to process q queries of the following types:

   (a) increase each value in range $[a, b]$ by u

   (b) what is the value at position k?

   Input: The first input line has two integers n and q: the number of values and queries. The second line has n integers $x_1, x_2, \ldots, x_n$: the array values. Finally, there are q lines describing the queries. Each line has either "1 a b u" or "2 k".

   Output: Print one integer which is the sum of the results of queries of type 2.