
Mid-semester Examination for MT3164: Numerical Analysis

Date: 29th Sept. 2025 (Monday) Time: 10:00am – noon (2 hours)

Number of questions: 5; Maximum number of points: 30

1. (a) Prove that A is positive definite and B is non-singular if and only if BAB^T is positive definite. 3
- (b) Prove the following facts. 3
 1. If U is upper triangular and invertible, then U^{-1} is upper triangular.
 2. The inverse of a unit lower triangular matrix is unit lower triangular.
 3. The product of two upper triangular matrices is upper triangular.
2. (a) Define a *vector norm* and state the four conditions it must satisfy. Define a *matrix norm* subordinate to this vector norm. 3
- (b) Define $\|A\| = \sum_{i=1}^n \sum_{j=1}^n |a_{ij}|$. Show that this is a matrix norm. 3
3. (a) Define the order of convergence of a sequence. 2
- (b) In the Secant method, prove that the order of convergence of the error is approximately 1.62. 4
4. (a) Prove that if A is symmetric and positive definite, then solving $Ax = b$ is equivalent to minimizing the quadratic form 4

$$q(x) = \langle x, Ax \rangle - 2\langle x, b \rangle.$$

- (b) Define the *spectral radius* of a matrix G . Prove that if the spectral radius of G is less than 1, then the iterative scheme 4

$$x^{(k)} = Gx^{(k-1)} + c$$

converges to $(I - G)^{-1}c$ for any initial vector $x^{(0)}$.

5. (a) Let $I \subseteq \mathbb{R}$ be an interval and let $f : I \rightarrow \mathbb{R}$ be differentiable on I . Prove that f is Lipschitz continuous on I if and only if f' is bounded on I . 4