Quiz 3: MT3164: Numerical Analysis

10:00 am to 10:50 am on 17^{th} October, 2025

The documents contain series of instructions, questions, and skeleton for solution. Do not change the input format.

The commands assumes that your enrollment number is 20301234.
Please change 20301234 to your enrollment number.

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1. (a) Open VS Code (or some other editor) to create a new file `20301234-q3p1.py` and save it on Desktop.

   (b) To obtain a sample input, run the following command.

   ```
   cp /nfscommon/common/prafullkumar/public/input-q3p1.txt ./
   ```

   (c) Use the following code to convert the above text file into matrix A.

   ```
   import numpy as np

   print("20301234 \t Alan Turing")
   # Replace 20301234 by your roll number and 'Alan Turing' by your name.

   # Read matrix from input file
   with open("input-q3p1.txt", "r") as f:
       lines = f.readlines()

   # Convert file contents to numpy array
   A = np.array([[float(num) for num in line.split()] for line in lines])

   ```

   (d) Write a NumPy program that implements the Power Method to estimate the dominant eigenvalue and eigenvector of a given square matrix A.

   i. Implement two versions of the Power Method: (a) Normalizing $x_{k+1} = \frac{Ax_k}{\|Ax_k\|_2}$, (b) Normalizing by maximum absolute value, $x_{k+1} = \frac{Ax_k}{\|Ax_k\|_\infty}$.

   ii. Modify your implementation so that the iteration stops when for both methods $\frac{\|Ax_k - \lambda_k x_k\|_2}{\|x_k\|_2} < 10^{-6}$.

   (e) For each iteration, output the number of iterations and the corresponding approximation for the largest eigenvalue for both methods.

   (f) Check the output of your program using the following command.

   ```
   python3 20301234-q3p1.py
   ```

   (g) Show your working code to the instructor.

   (h) Submit the solutions only if you are confident with it. **You are only allowed to submit code once**. Use the following command for submission.

   ```
   /nfscommon/common/prafullkumar/submit 20301234-q3p1.py
   ```

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2. (a) Open VS Code (or some other editor) to create a new file `20301234-q3p2.py` and save it on Desktop.

   (b) To obtain a sample input, run the following command.

   ```
   cp /nfscommon/common/prafullkumar/public/input-q3p2.txt ./
   ```

   (c) Use the following code to convert the above text file into a $m \times n$ matrix A.

```python
import numpy as np

print("20301234 \t Alan Turing")
# Replace 20301234 by your roll number and 'Alan Turing' by your name.

# Read matrix from input file
with open("input-q3p2.txt", "r") as f:
    lines = f.readlines()

# Convert file contents to numpy array
A = np.array([[float(num) for num in line.split()] for line in lines])

```

   (d) Compute the Singular Value Decomposition of A using $A = U\Sigma V^T$ where $U \in \mathbb{R}^{m \times m}$, $V \in \mathbb{R}^{n \times n}$, and $\Sigma \in \mathbb{R}^{m \times n}$.

   (e) After you have printed your roll number and name, print the following quantities, one in each line.

   i. ℓ_2 norm of 3^{rd} column of U.
   ii. ℓ_1 norm of 5^{th} column of V.
   iii. Leagest singular value of A.
   iv. Sum of all singular values of A.
   v. Trace of U plus trace of V.
   vi. Square root of the sum of squared singular values of A.
   vii. Construct the best rank-4 approximation of A by keeping only the top 4 singular values: $A_4 = U_4\Sigma_4 V_4^T$. Print Frobenius norm of $A - A_4$.
   viii. Repeat the above exercise for rank-10 approximation of A.

   (f) Check the output of your program using the following command.

   ```
   python3 20301234-q3p2.py
   ```

   (g) Show your working code to the instructor.

   (h) Submit the solutions only if you are confident with it. **You are only allowed to submit code once**. Use the following command for submission.

   ```
   /nfscommon/common/prafullkumar/submit 20301234-q3p2.py
   ```

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3. (a) Open VS Code (or some other editor) to create a new file `20301234-q3p3.py` and save it on Desktop.

   (b) To obtain a sample input, run the following command.

   ```
   cp /nfscommon/common/prafullkumar/public/input-q3p3-A.txt ./
   cp /nfscommon/common/prafullkumar/public/input-q3p3-b.txt ./
   ```

   (c) Using the commands similar to the previous questions, import the above files to create 30×20 matrix A and 30×1 vector b.

   (d) Solve the Least Squares problem for $Ax = b$ using the following two methods.

   i. Using QR-decomposition of A.
   ii. Using pseudo-inverse A^+ of A.

   Suppose x_1 and x_2, respectively, be the answer obtained using these two processes.

   (e) Print your enrollment number and name in the specified format. In the next lines, print the following quantities.

   i. ℓ_2 norm of 2^{nd} column of Q.
   ii. ℓ_1 norm of 4^{th} column of R.
   iii. ℓ_∞ norm of 3^{rd} column of A^+.
   iv. ℓ_2-norm of residual vector $r_1 = b - Ax_1$.
   v. ℓ_2-norm of residual vector $r_2 = b - Ax_2$.

   (f) Check the output of your program using the following command.

   ```
   python3 20301234-q1p3.py
   ```

   (g) Show your working code to the instructor.

   (h) Submit the solutions only if you are confident with it. **You are only allowed to submit code once**. Use the following command for submission.

   ```
   /nfscommon/common/prafullkumar/submit 20301234-q3p3.py
   ```
