Quiz 4: MT3164: Numerical Analysis

10:00 am to 10:50 am on 14^{th} Nov, 2025

The documents contain series of instructions, questions, and skeleton for solution. Do not change the input format.

The commands assumes that your enrollment number is 20301234.
Please change 20301234 to your enrollment number.

To obtain a relevant chapter of the book, please use the following command.
`cp /nfscommon/common/prafullkumar/public/Chapter-6.pdf ./`

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1. (a) Open VS Code (or some other editor) to create a new file `20301234-q4p1.py` and save it on Desktop.

   (b) There is no sample input for this question.

   (c) You don't need to convert any text file for this question.

   (d) Write a NumPy program that computs Chebychev polynomial of first kind of order 10, i.e. $T_{10}(x)$.

   (e) Output the following quantities:

   i. Sum of coefficient in $T_{10}(x)$.

   ii. Value of $T_{10}(1.0045)$.

   iii. Roots of $T_{10}(x)$.

   iv. Consider the following statement: In an interval $[-1, 1]$, we have

   $$T_n(x) = \cos(n \cdot \cos^{-1}(x)).$$

   (f) Check the output of your program using the following command.
   `python3 20301234-q4p1.py`

   (g) Submit the solutions only if you are confident with it. **You are only allowed to submit code once**. Use the following command for submission.
   `/nfscommon/common/prafullkumar/submit 20301234-q4p1.py`

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2. (a) Open VS Code (or some other editor) to create a new file `20301234-q4p2.py` and save it on Desktop.

   (b) To obtain a sample input, run the following command.

   `cp /nfscommon/common/prafullkumar/public/input-q4p2.txt ./`

   (c) Use the following code to convert the above text file into a $n \times 2$ matrix A.

```
import numpy as np

print("20301234 \t Alan Turing")
# Replace 20301234 by your roll number and 'Alan Turing' by your
name.

# Read matrix from input file
with open("input-q4p2.txt", "r") as f:
    lines = f.readlines()

# Convert file contents to numpy array
A = np.array([[float(num) for num in line.split()] for line in
lines])

```

   (d) Let A be an $n \times 2$ matrix such that

$$A = \begin{bmatrix} x_0 & f(x_0) \\ x_1 & f(x_1) \\ \vdots & \vdots \\ x_{n-1} & f(x_{n-1}) \end{bmatrix},$$

   where the first column contains n distinct nodes $x_0, x_1, \ldots, x_{n-1}$ in the interval $[-10, 10]$, and the second column contains the corresponding function values $f(x_i)$. Suppose we want to approximate $f(x_i)$ by polynomial $p(x)$. Compute $p(x)$ by constructing Vandermonde matrix V. You can use `numpy.vander` and `numpy.linalg.solve`.

   (e) After outputting your roll number and name, output the following.

   i. ℓ_2 norm of 3^{rd} column of V.
   ii. Determinant of V.
   iii. Sum of coefficient of $p(x)$.
   iv. $p(5.5)$.

   (f) Check the output of your program using the following command.

   `python3 20301234-q4p2.py`

   (g) Submit the solutions only if you are confident with it. **You are only allowed to submit code once**. Use the following command for submission.

   `/nfscommon/common/prafullkumar/submit 20301234-q4p2.py`

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3. (a) Open VS Code (or some other editor) to create a new file `20301234-q4p3.py` and save it on Desktop.

   (b) To obtain a sample input, run the following command.

   ```
   cp /nfscommon/common/prafullkumar/public/input-q4p3.txt ./
   ```

   (c) Using the commands similar to the previous questions, import the above files to create $n \times 2$ matrix A.

   (d) Our objective is to compute cubic splines to approximate the function. Towards that, consider the values specified on Page Nr 319 of the book (Page Nr 42 of the pdf). Compute all the quantities mentioned there, and suppose the system of linear equation is written as $Dz = v$.

   (e) Print your enrollment number and name in the specified format. In the next lines, print the following quantities.

   i. Determinant of D.
   ii. $\sum_{i=1}^{n-2} i \cdot h_i$.
   iii. $\sum_{i=1}^{n-1} i \cdot u_i$.
   iv. $\sum_{i=0}^{n-1} i \cdot b_i$.
   v. $\sum_{i=1}^{n-1} i \cdot v_i$.
   vi. $\sum_{i=1}^{n-1} i \cdot z_i$.

   (f) Check the output of your program using the following command.

   ```
   python3 20301234-q1p3.py
   ```

   (g) Submit the solutions only if you are confident with it. **You are only allowed to submit code once**. Use the following command for submission.

   ```
   /nfscommon/common/prafullkumar/submit 20301234-q4p3.py
   ```
