# IISER Pune

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## End-Term Exam for MT3444: Combinatorial Optimization

Date: $23^{rd}$ April 2026 (Thursday) Time: 3:00pm – 05:00pm (2 hours)
Number of questions: 4; Maximum number of points: 40

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1. (a) Write formal definitions of the following classes: (*i*) NP, (*ii*) P, (*iii*) NP-hard, and (*iv*) NP-Complete, [4]

   (b) (2 pts) Write primal and dual the complementary slackness conditions for an LP. (4 pts) Design an approximation algorithm for Set Cover using the Primal-Dual schema. [6]

2. (a) (1 pt) Define a totally unimodular matrix. (4 pts) Prove that the incident matrix of a bipartite graph is totally unimodular. [5]

   (b) Suppose an algorithm $\mathcal{A}$ is polynomial time $c$-factor approximation algorithm for Minimum MultiCut on trees. [5]

   1. (2pts) Design a polynomial-time $c$-factor approximation algorithm for Vertex Cover on general graphs.
   2. (3pts) Prove the correctness of your algorithm.

3. (a) Define the following terms: (*i*) hyperplanes, (*ii*) closed half-spaces, (*iii*) convex polyhedra, (*iv*) vertex of a polyhedra. [4]

   (b) Given a set $J$ of jobs, a set $M$ of machines, and for each $j \in J$ and $i \in M$, $p_{ij} \in \mathbb{Z}$, the time taken to process job $j$ on machine $i$, the objective is to schedule the jobs on the machines so as to minimize the makespan. [6]

   1. (2pts) Write an ILP for the problem and prove that the integrality gap is not bounded by any constant.
   2. (2pts) Explicitly write an LP, denoted by $LP(T)$, obtained by *parametric pruning* of the LP-relaxation of the ILP.
   3. (2pts) Prove that any extreme point solution to $LP(T)$ has at most $|J|+|M|$ nonzero variables.

4. (a) Prove that the integrality gap for Set Cover is at least $\ln n$. [2]

   (b) Let $A(n, d)$ denote the maximum possible size of a code $C \subseteq \{0,1\}^n$ with distance $d$. Obtain an upper bound on $A(n, d)$ using the *volume* argument. [2]

   (c) Consider a two-player zero-sum game defined by a payoff matrix $A$ of size $m \times n$. [6]

   1. (1pt) Define a mixed Nash equilibrium.
   2. (5pts) State and prove the Min-Max theorem for zero-sum games.