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Quiz 01 for MT3444: Combinatorial Optimization

Date: 4^{th} Feb 2026 (Wednesday) Time: 12:05pm – 01:05pm (1 hour)
Number of questions: 2; Maximum number of points: 15

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1. (a) Define a *vertex cover* of a graph. Formulate the problem of computing minimum vertex cover of a graph as an instance of ILP. [2]

   Repeat the exercise for *independent set*.

   (b) Let $\mathbf{x}$ be a basic feasible solution of $\mathbf{Ax} = \mathbf{b}$ and $\mathbf{x} \geq \mathbf{0}$. Then, there exists a cost vector $\mathbf{c}$ such that $\mathbf{x}$ is the unique optimal solution for the LP $\min \mathbf{c}^T\mathbf{x}$ subjected to $\mathbf{Ax} = b$ and $\mathbf{x} \geq \mathbf{0}$. [2]

   (c) Demonstrate two steps (i.e. three simplex tabulea) of the Simple method for the following problem. At each step, clearly mention basic and non-basic variables. [3]

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\begin{aligned}
\text{Maximize} \quad & z = 3x_1 + 2x_2 + 4x_3 \\
\text{subject to} \quad & x_1 + x_2 + x_3 \leq 30, \\
& 2x_1 + x_2 + 3x_3 \leq 60, \\
& x_1 + 2x_2 + x_3 \leq 40, \\
& x_1, x_2, x_3 \geq 0.
\end{aligned}
$$

2. (a) Define basic feasible solution for a linear program which is in equational form. Prove that a feasible solution $\mathbf{x}$ of a linear program in equational form is basic if and only if the columns of the matrix A_K are linearly independent, where $K = \{j \in [n] \mid x_j > 0\}$. [4]

   (b) Prove that if P is the set of all feasible solutions of a linear program in equational form then the following two conditions for a point $\mathbf{v} \in P$ are equivalent: [4]

   1. $\mathbf{v}$ is a vertex of the polyhedron P.
   2. $\mathbf{v}$ is a basic feasible solution of the linear program.