Dynamic Parameterized Problems

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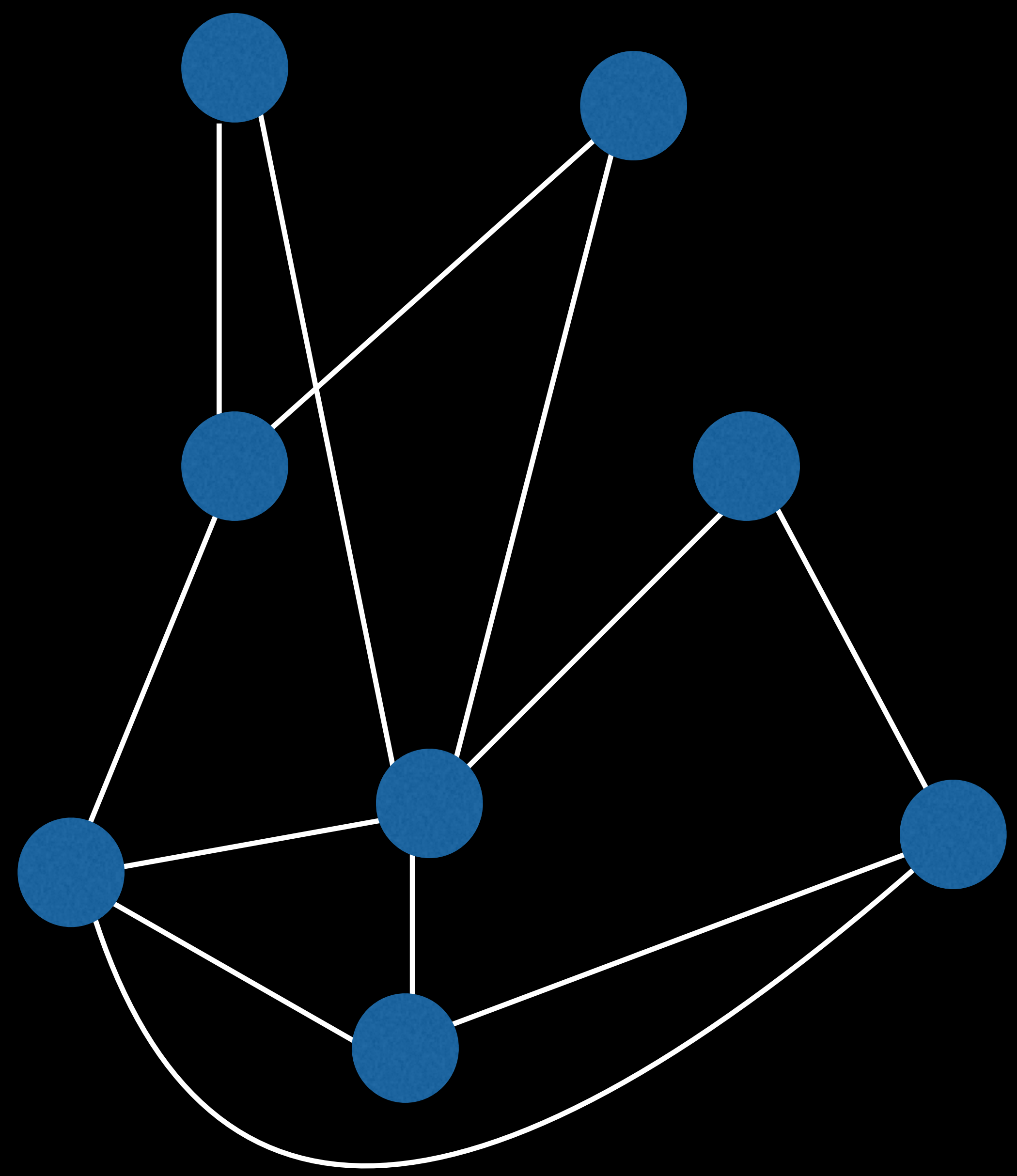
IPEC 2016
Aarhus University, Denmark

Dynamic Graphs



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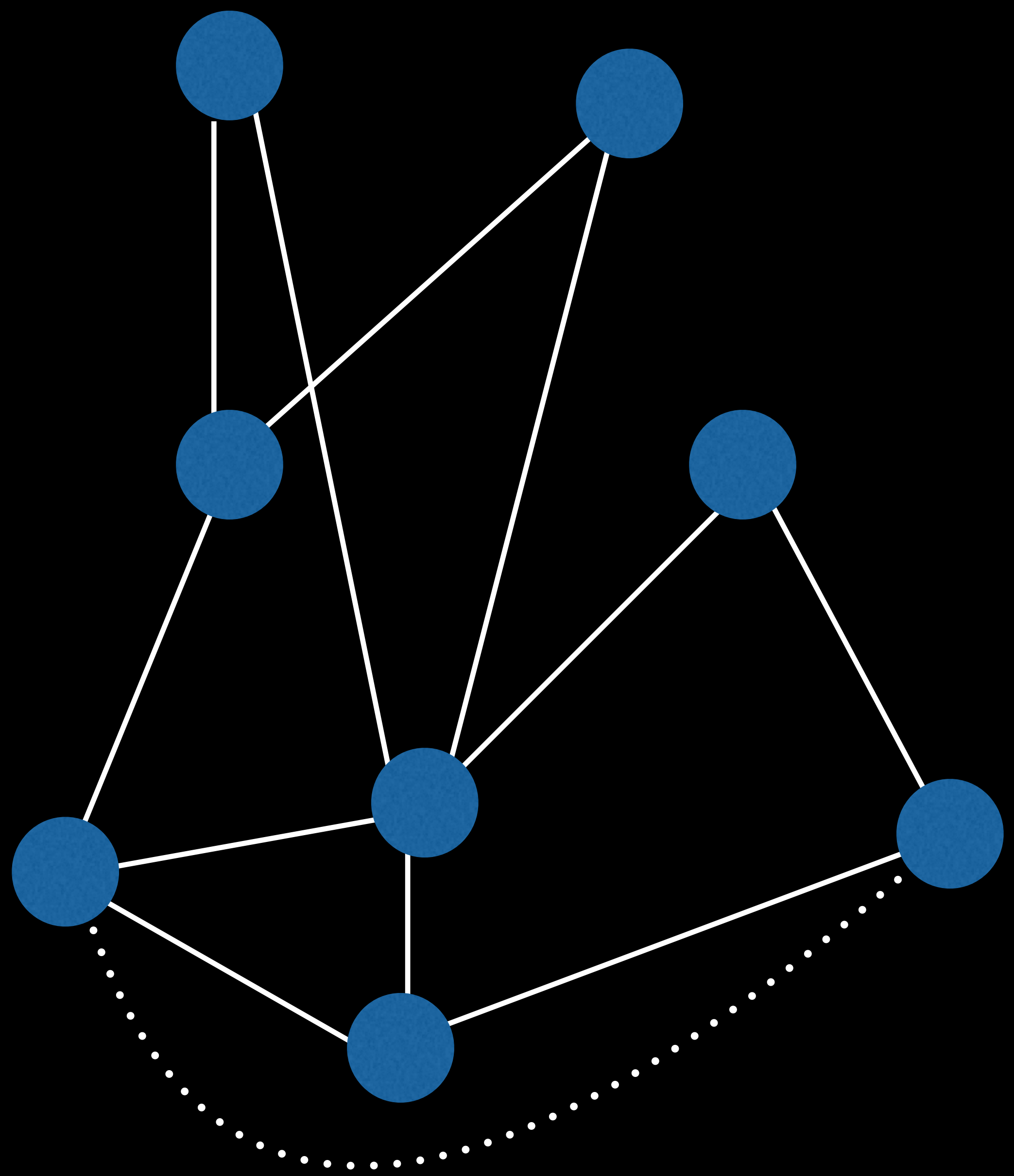

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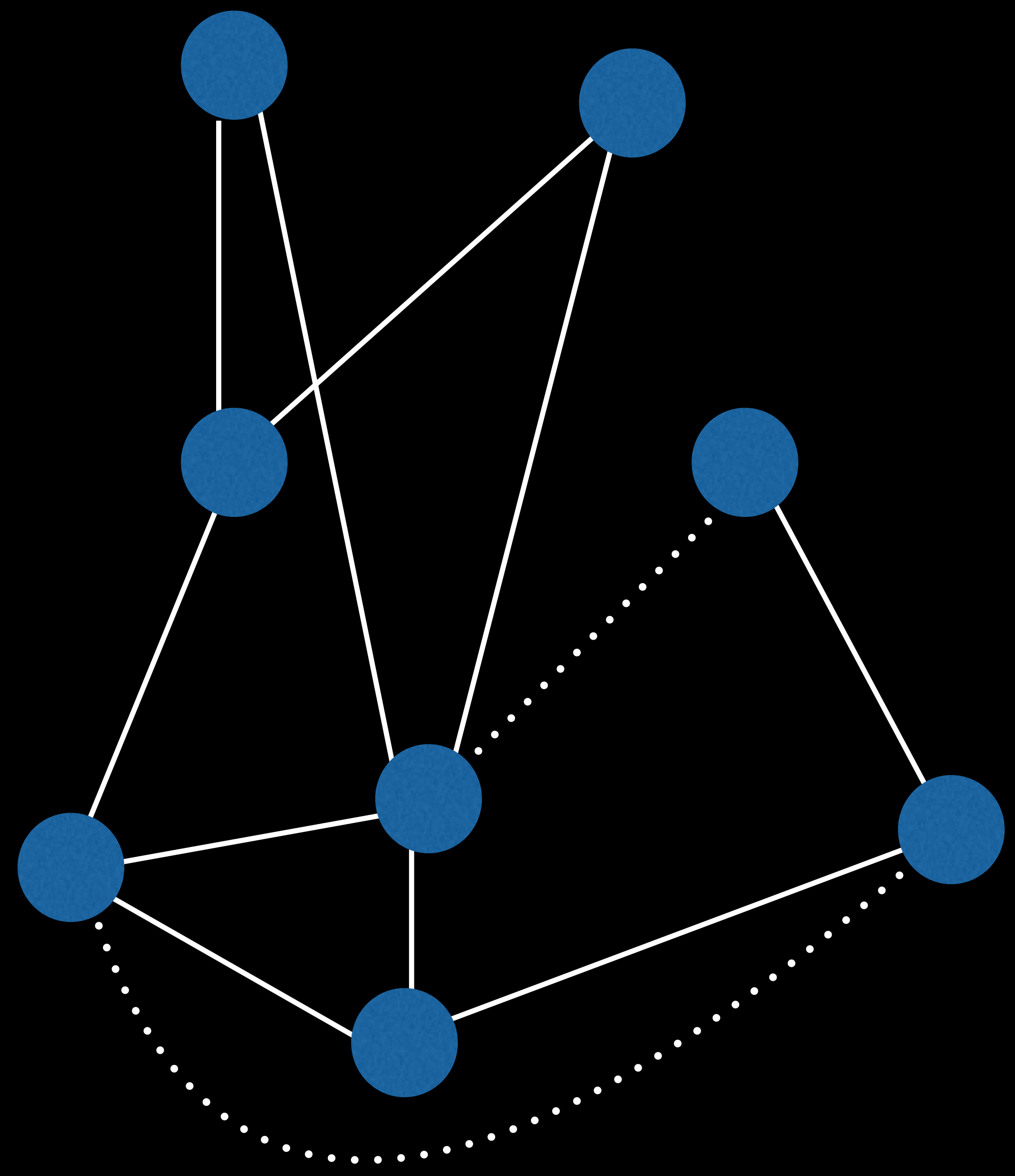

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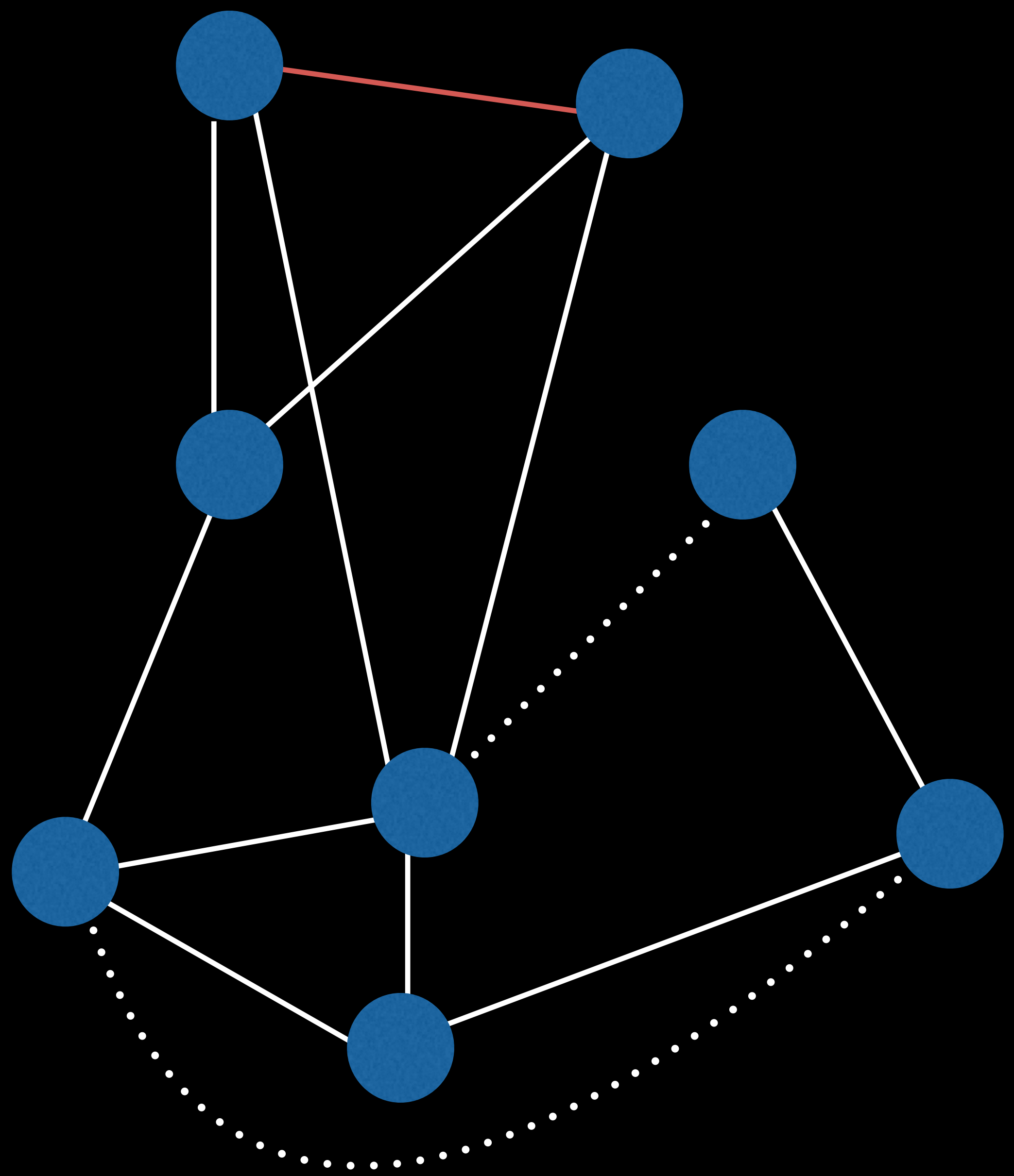

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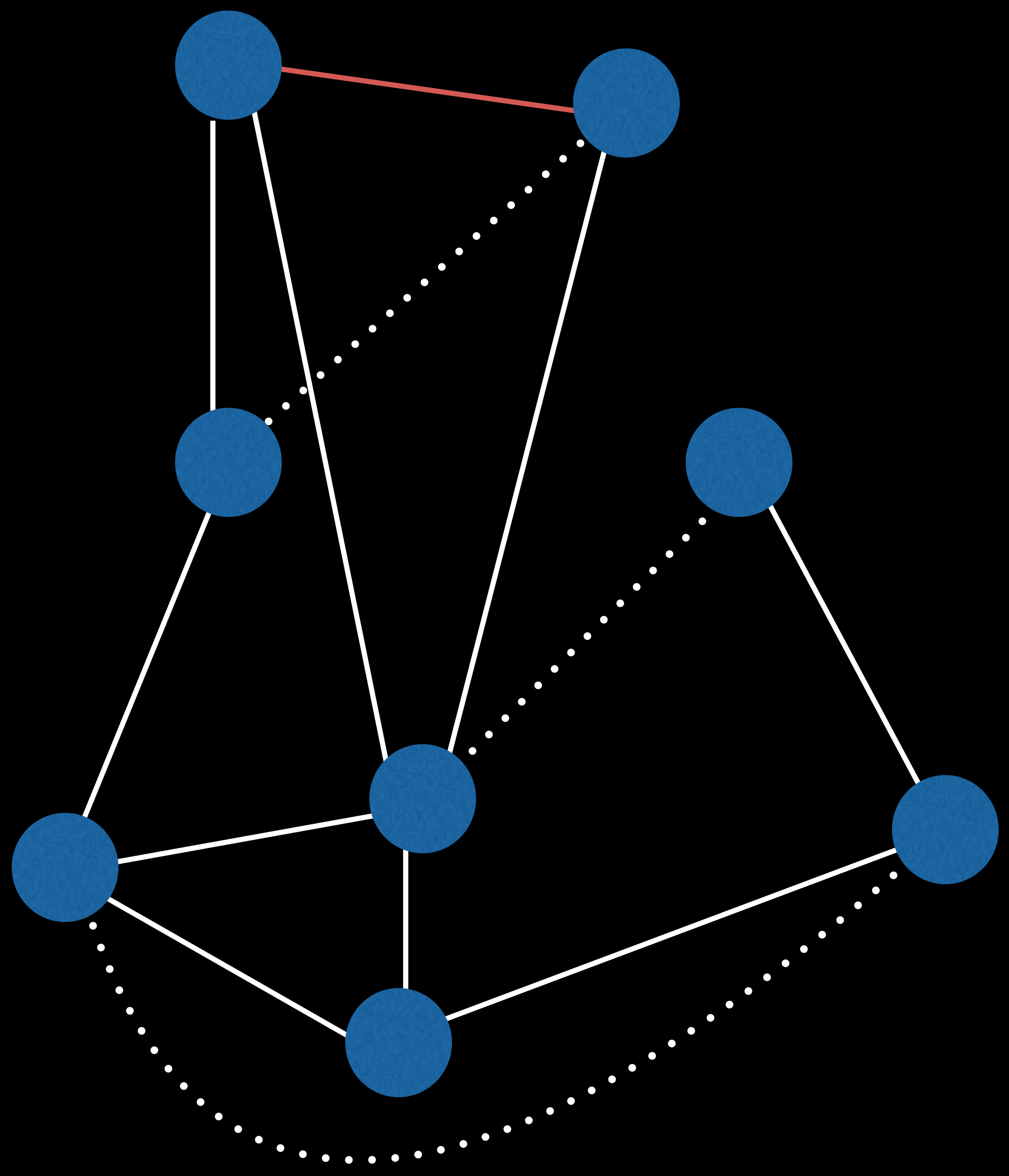

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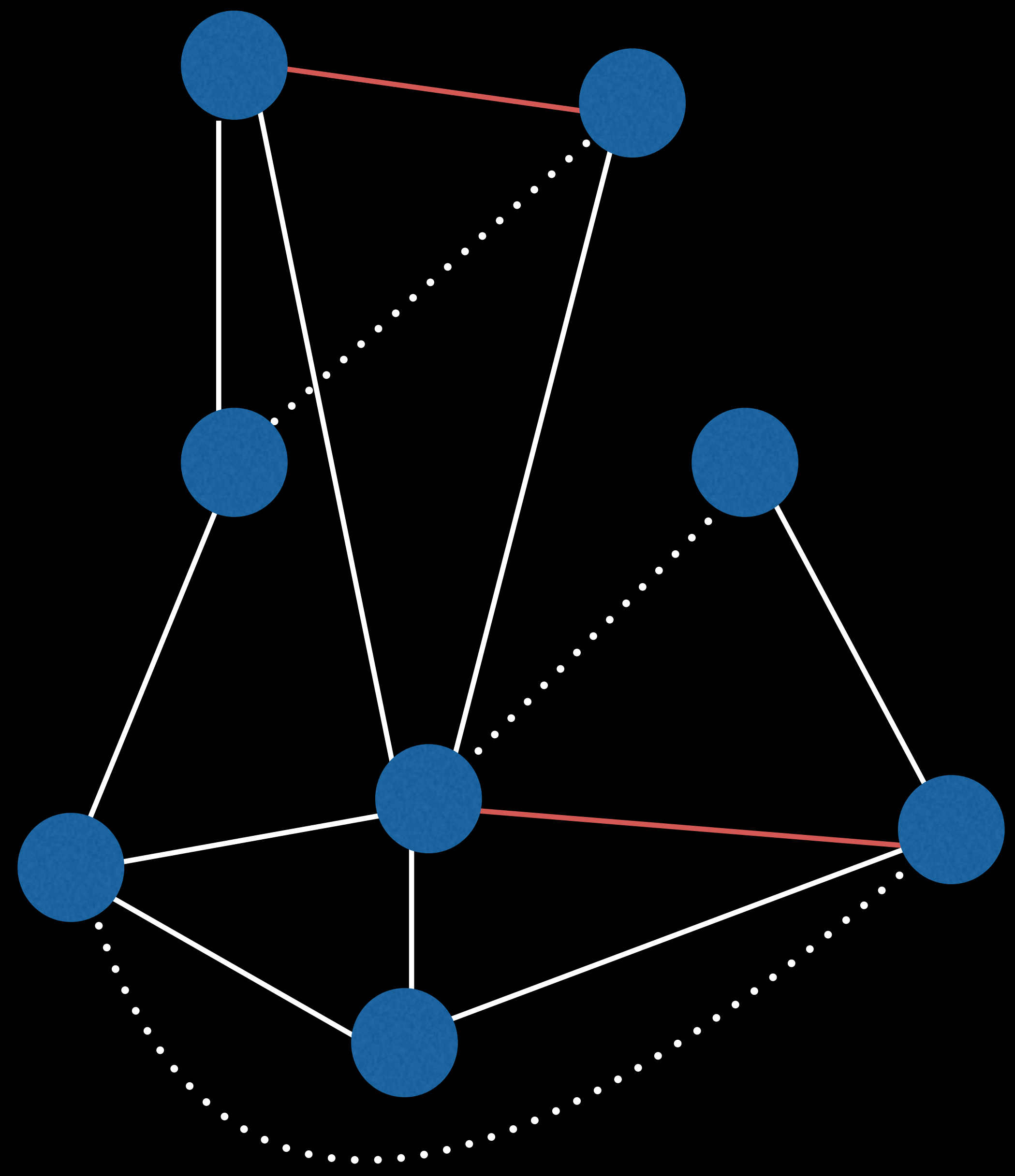

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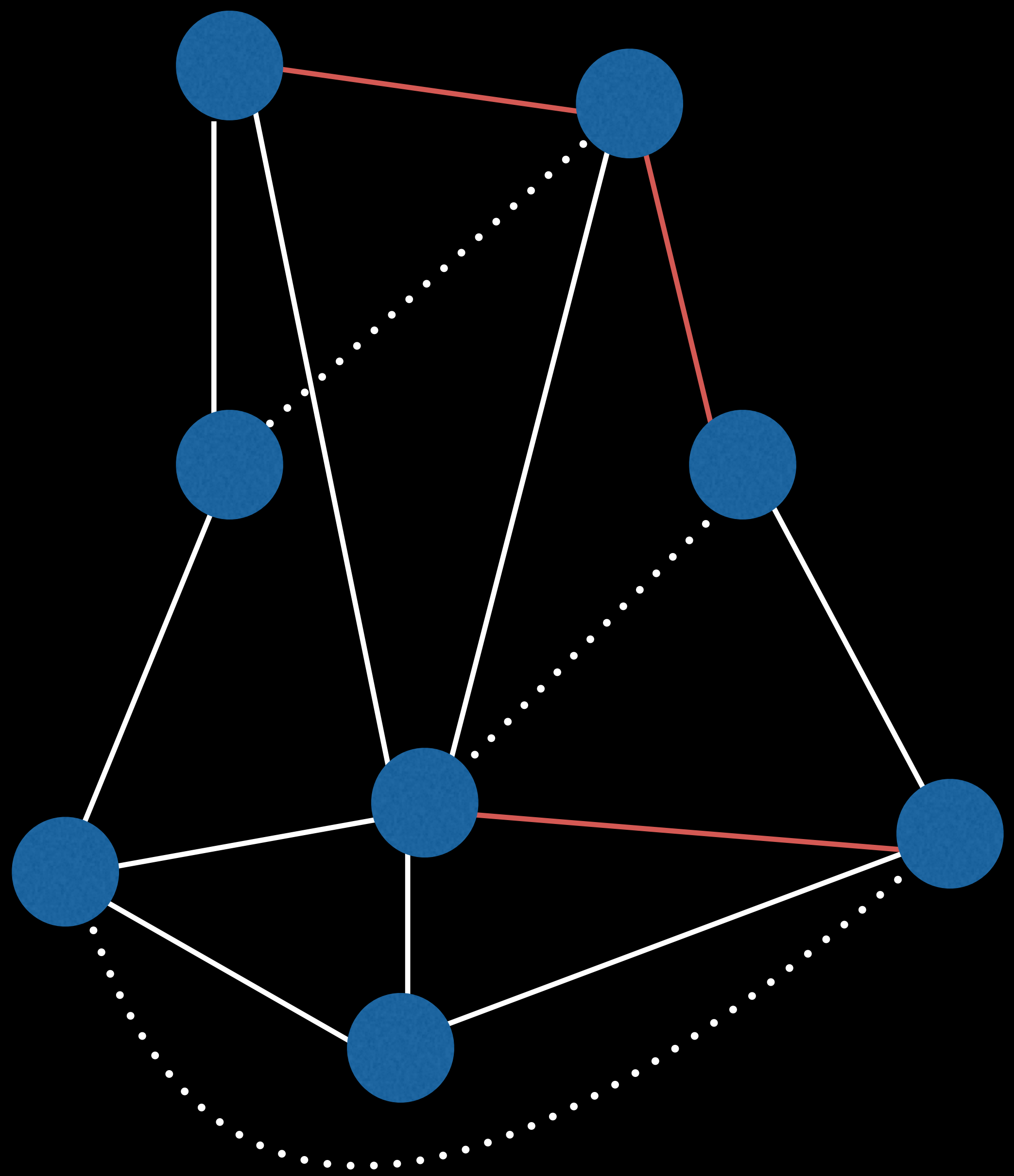

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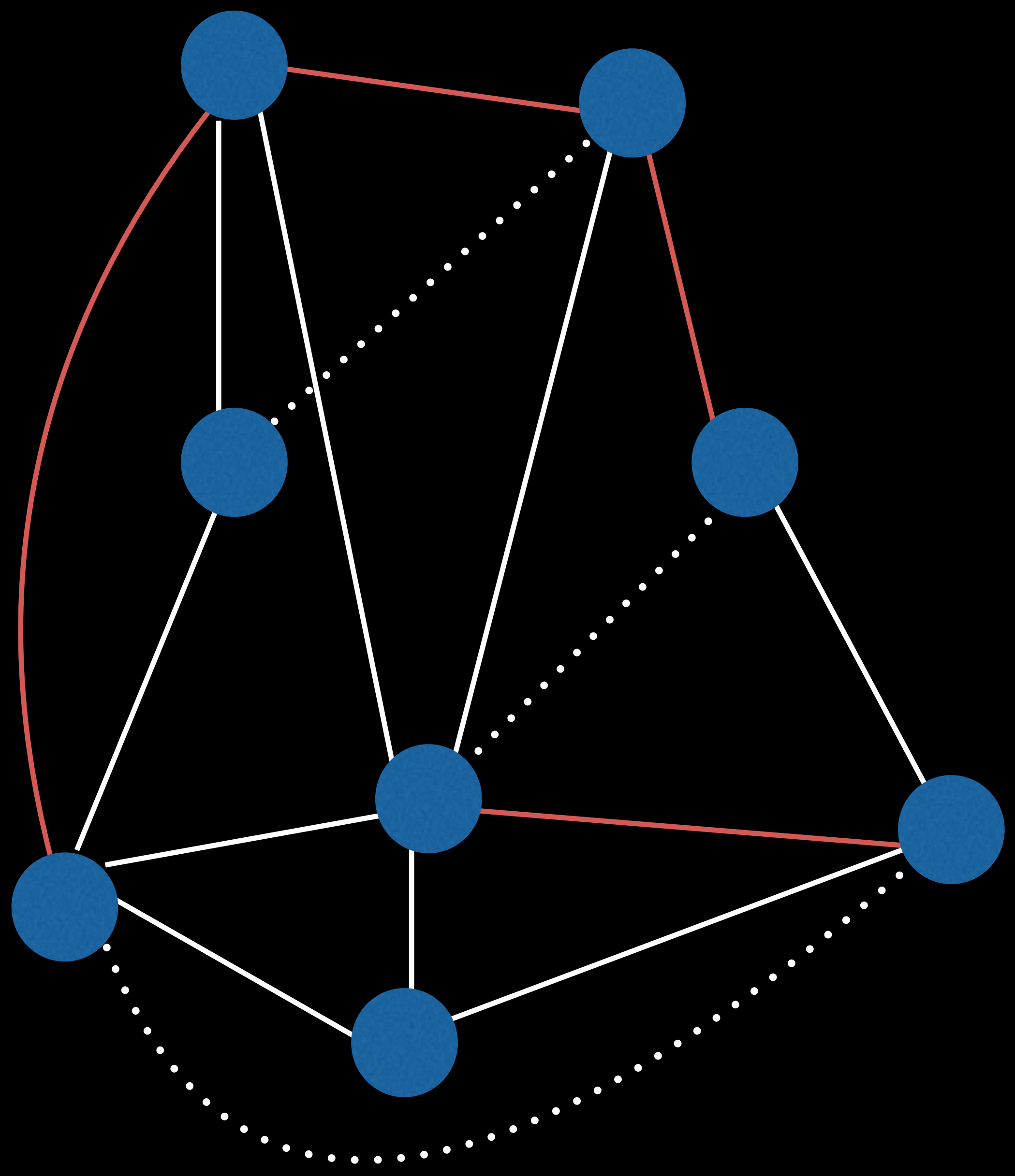

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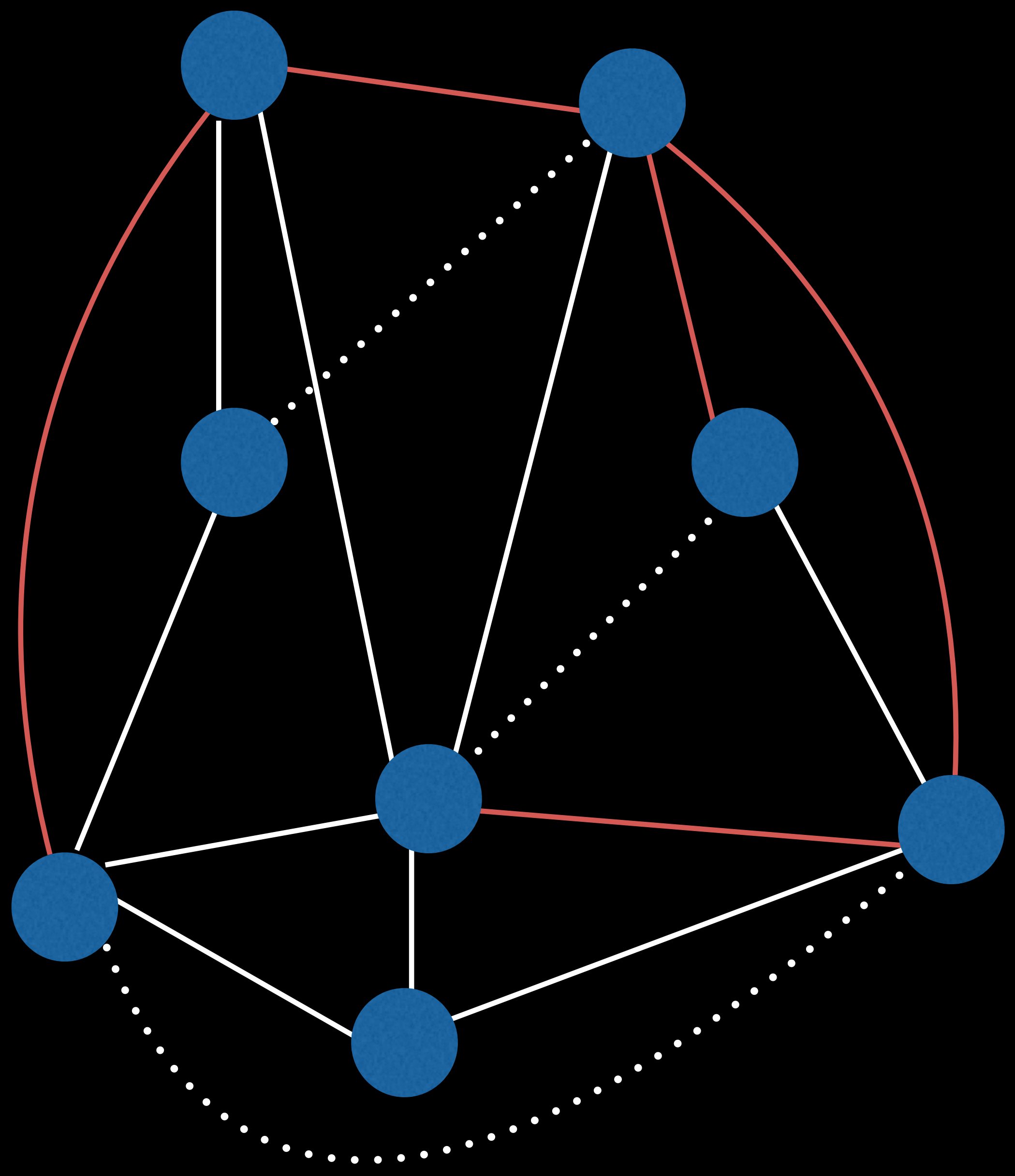

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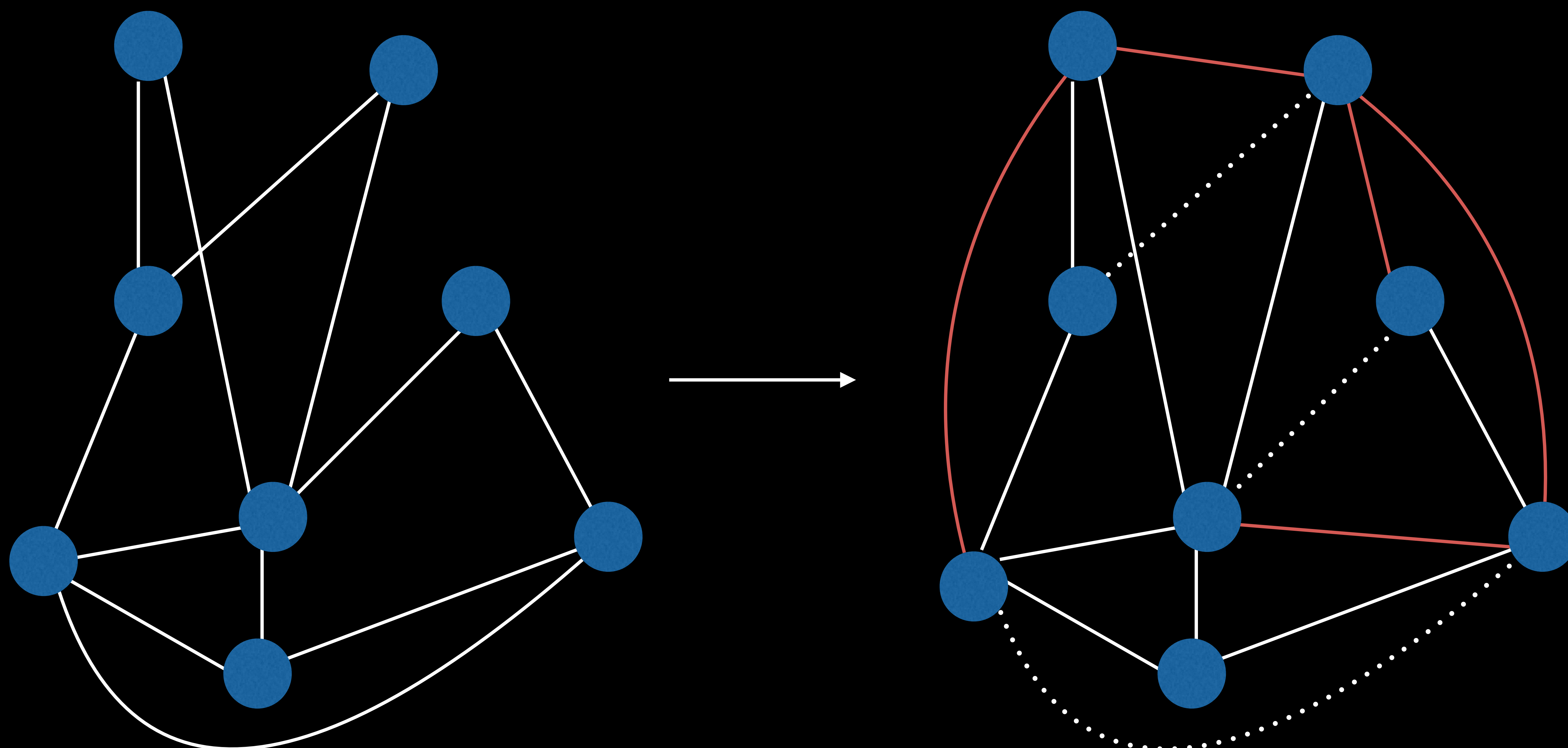

Current graph is at a distance 8 from the initial graph

Dynamic Graphs

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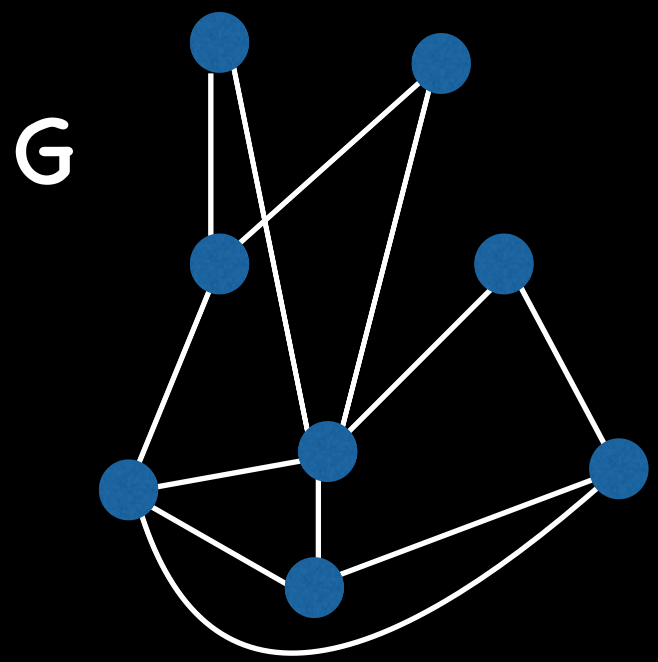
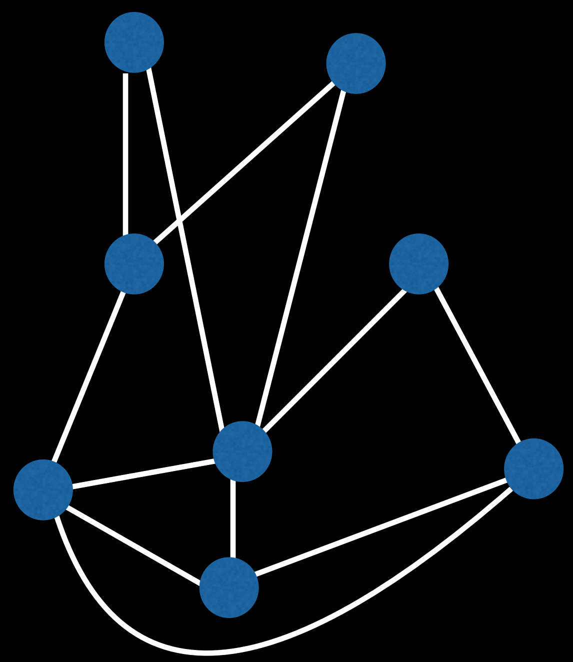
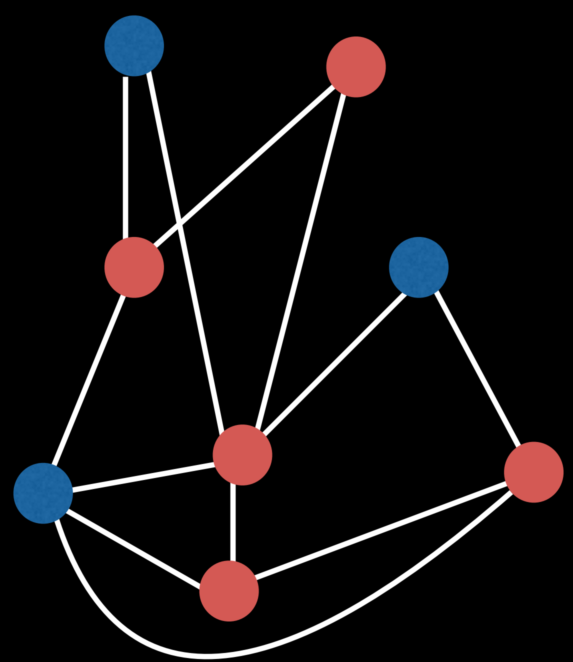


Dynamic Graphs

G





vertex cover of G

Dynamic Graphs

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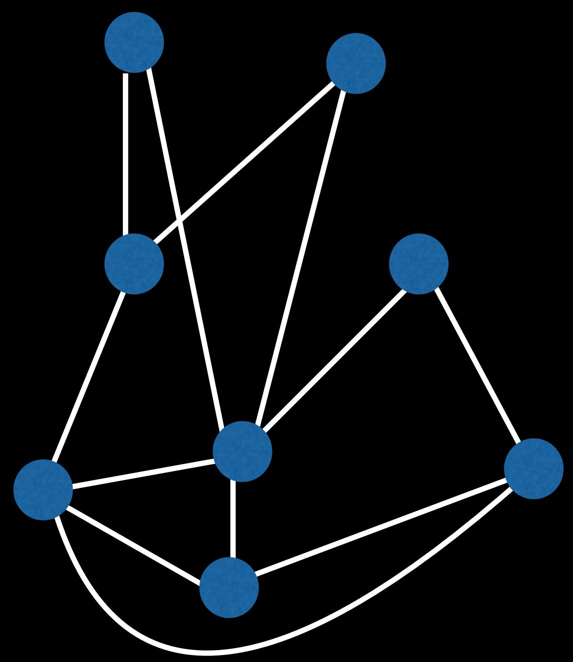

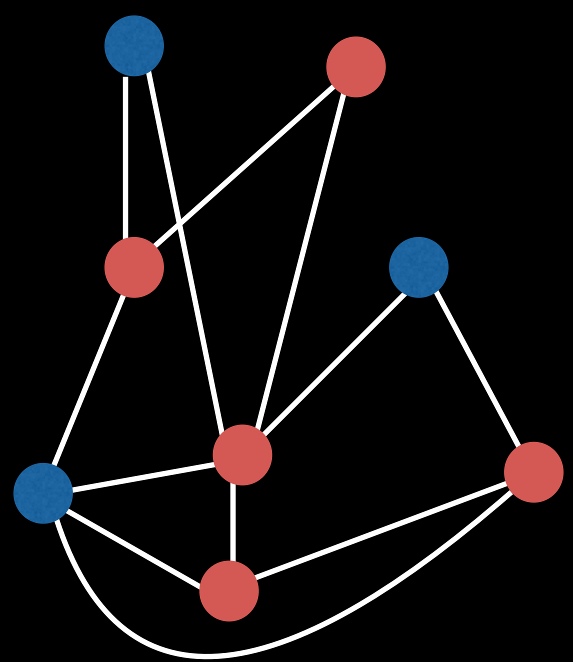

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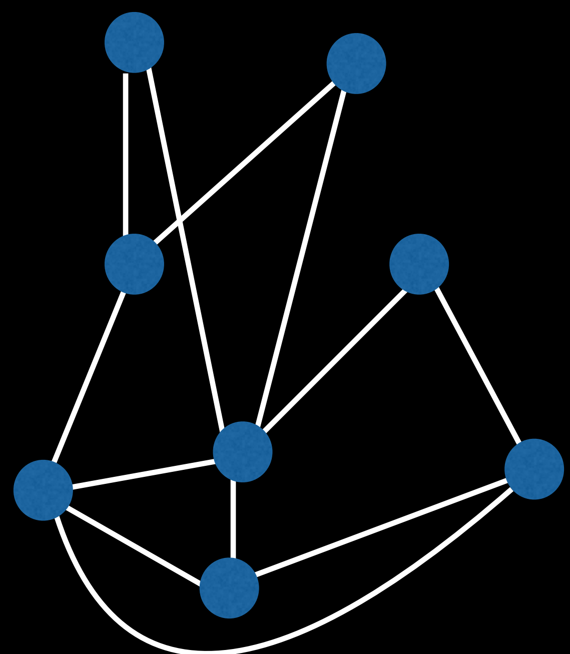
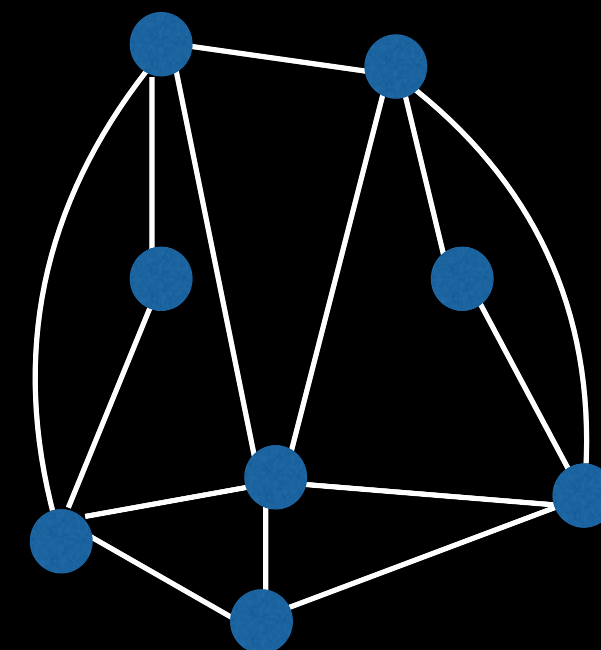
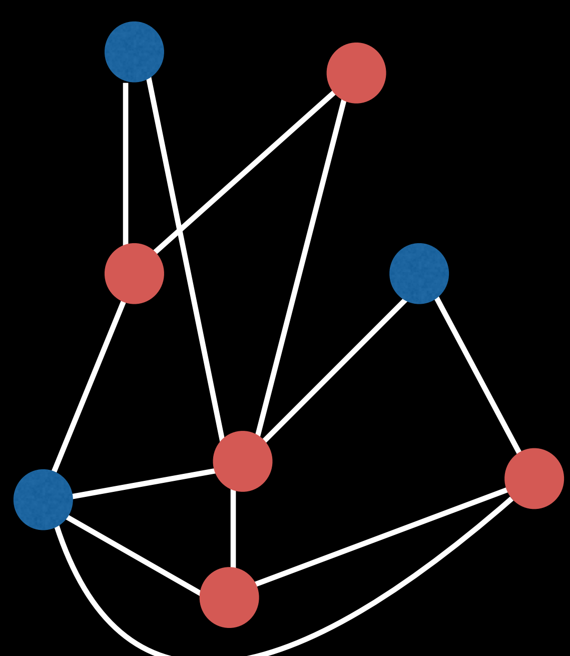




H



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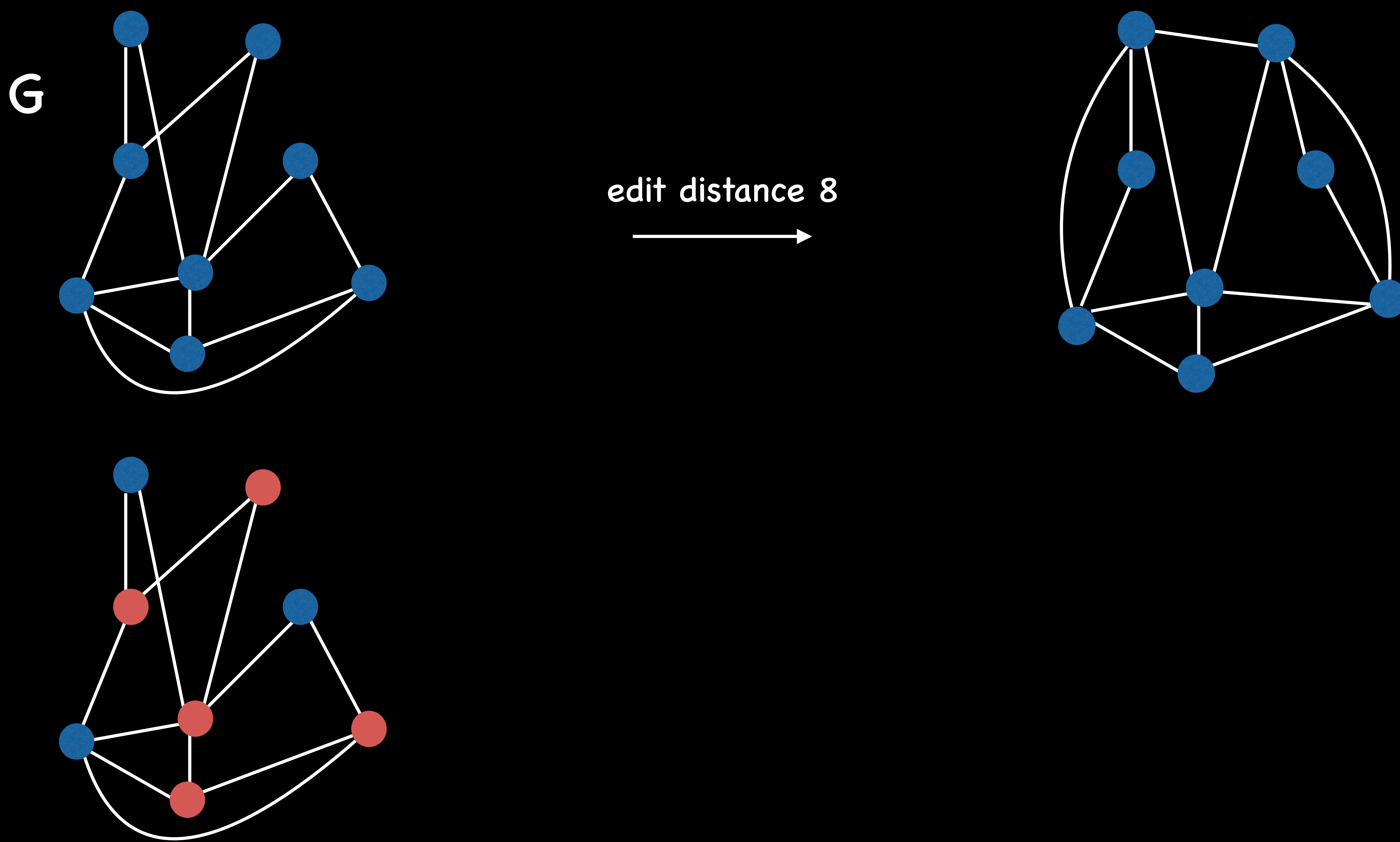

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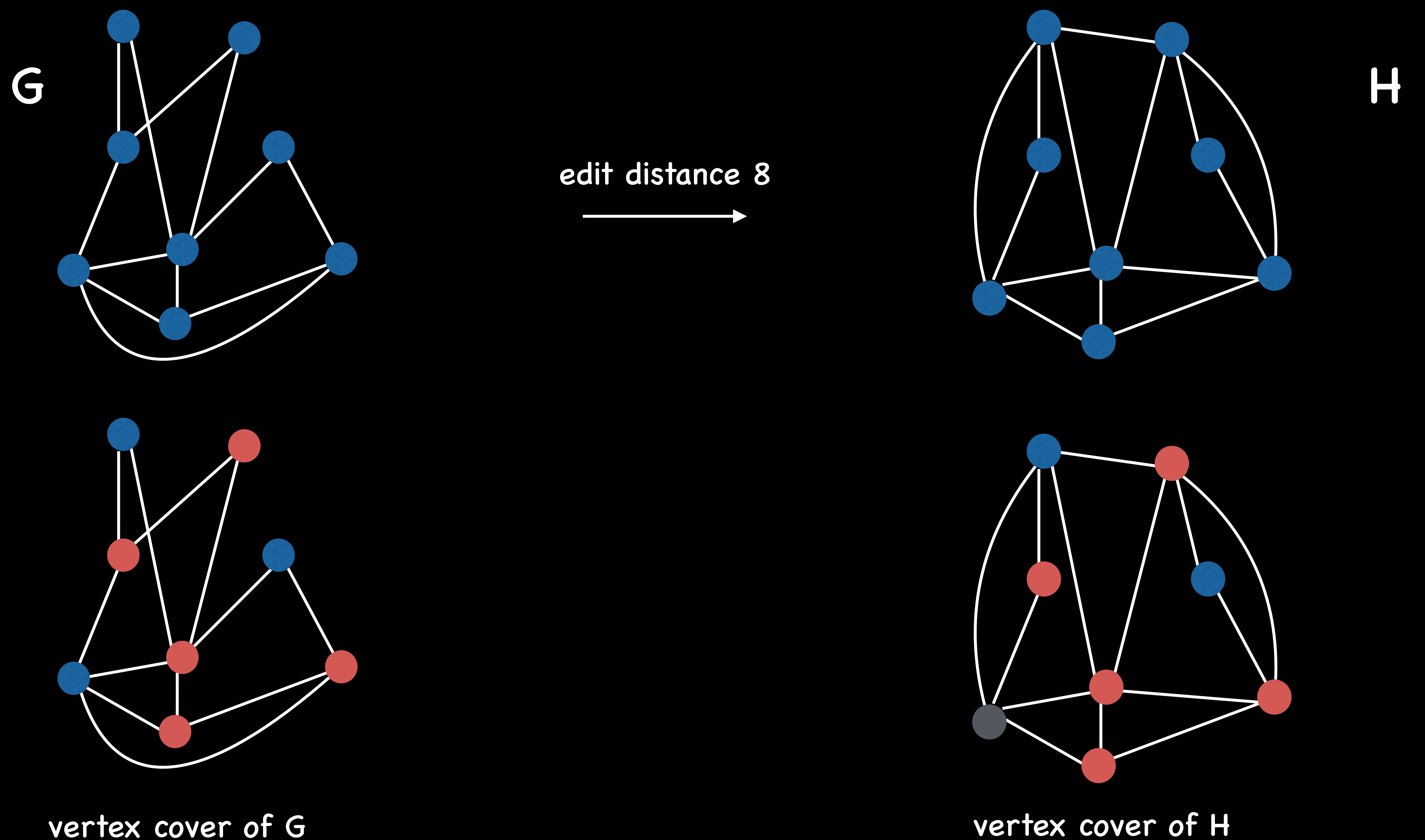

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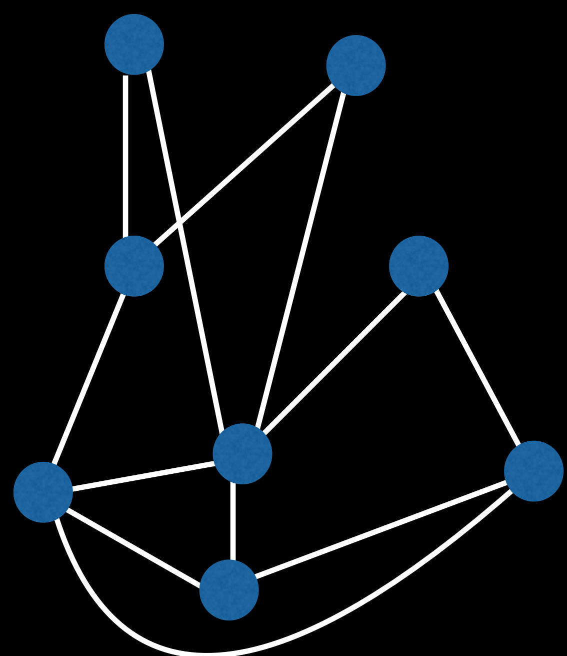
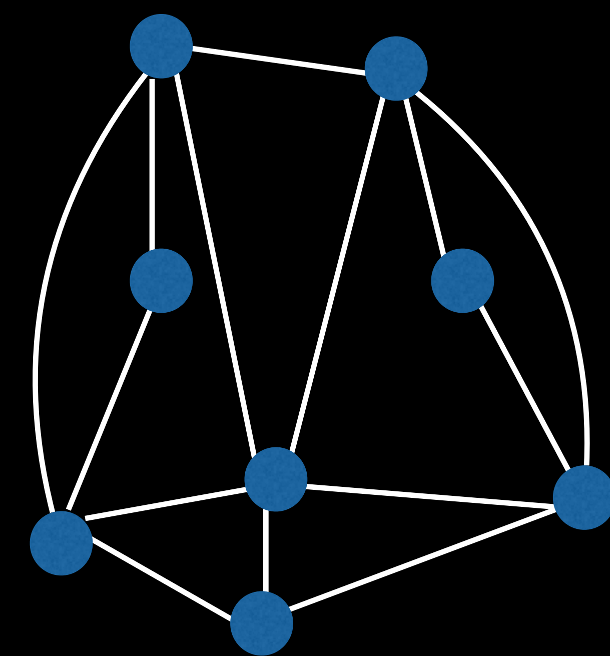


edit distance 8



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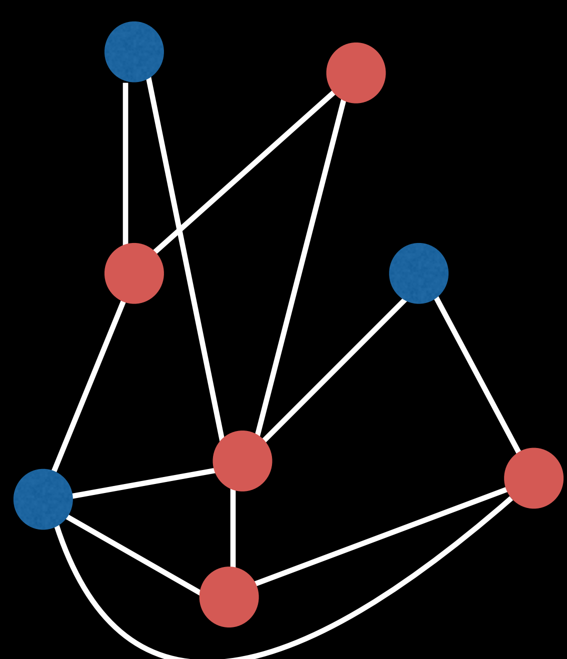
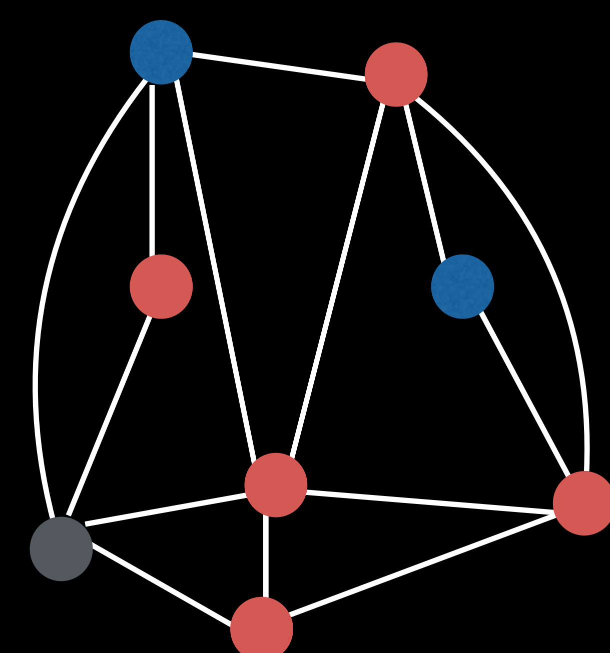


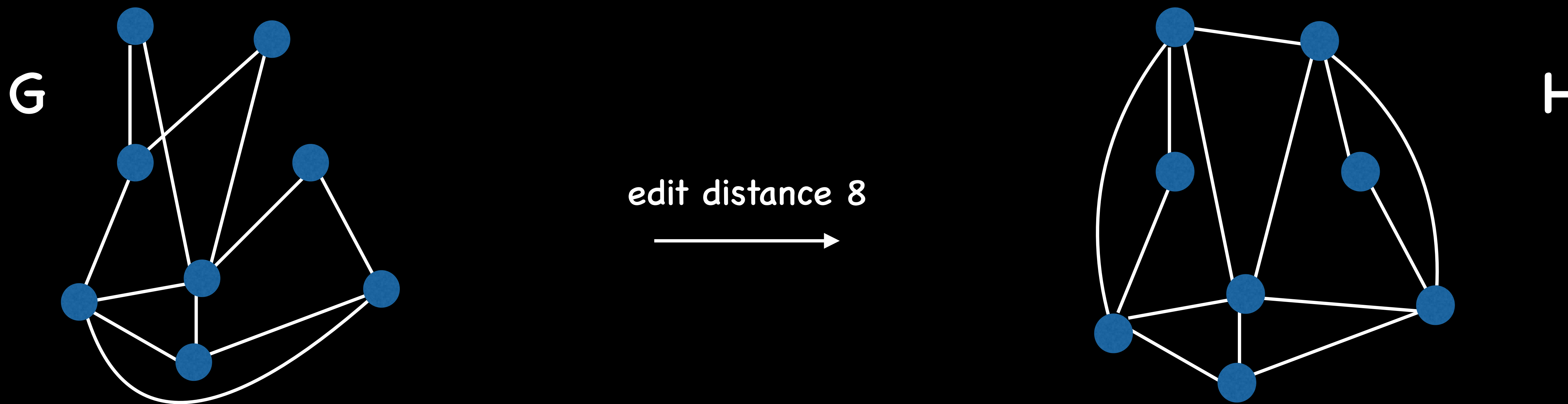
Hamming distance 1



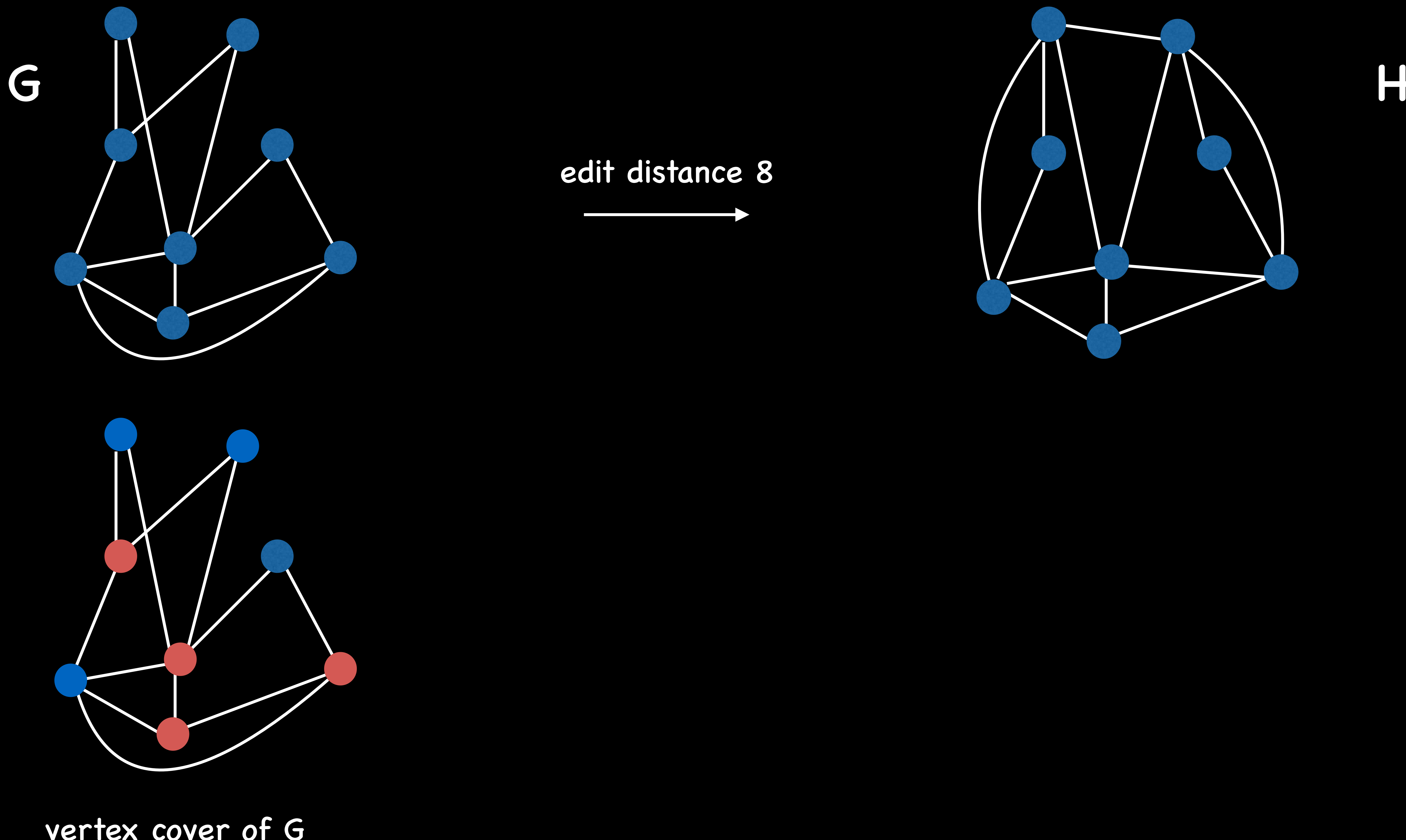
vertex cover of G

vertex cover of H

Dynamic Graphs



Dynamic Graphs



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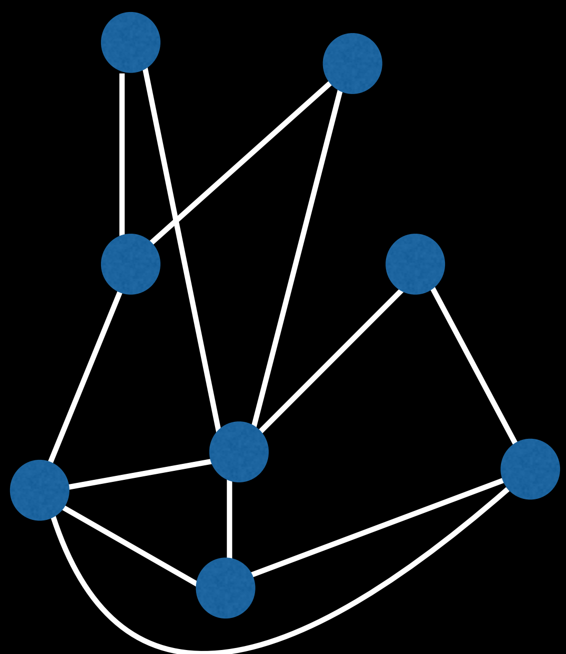
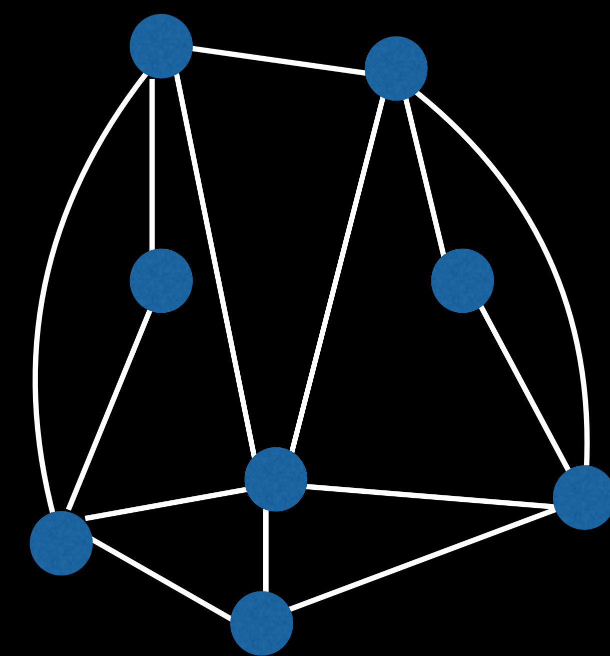
G



edit distance 8



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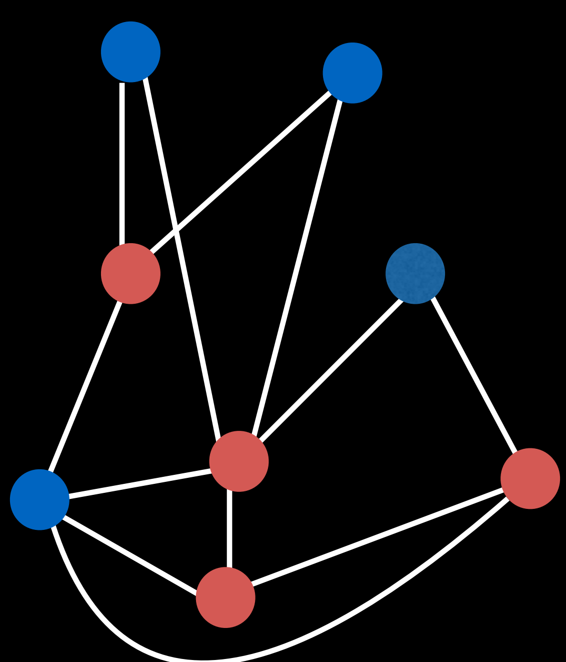
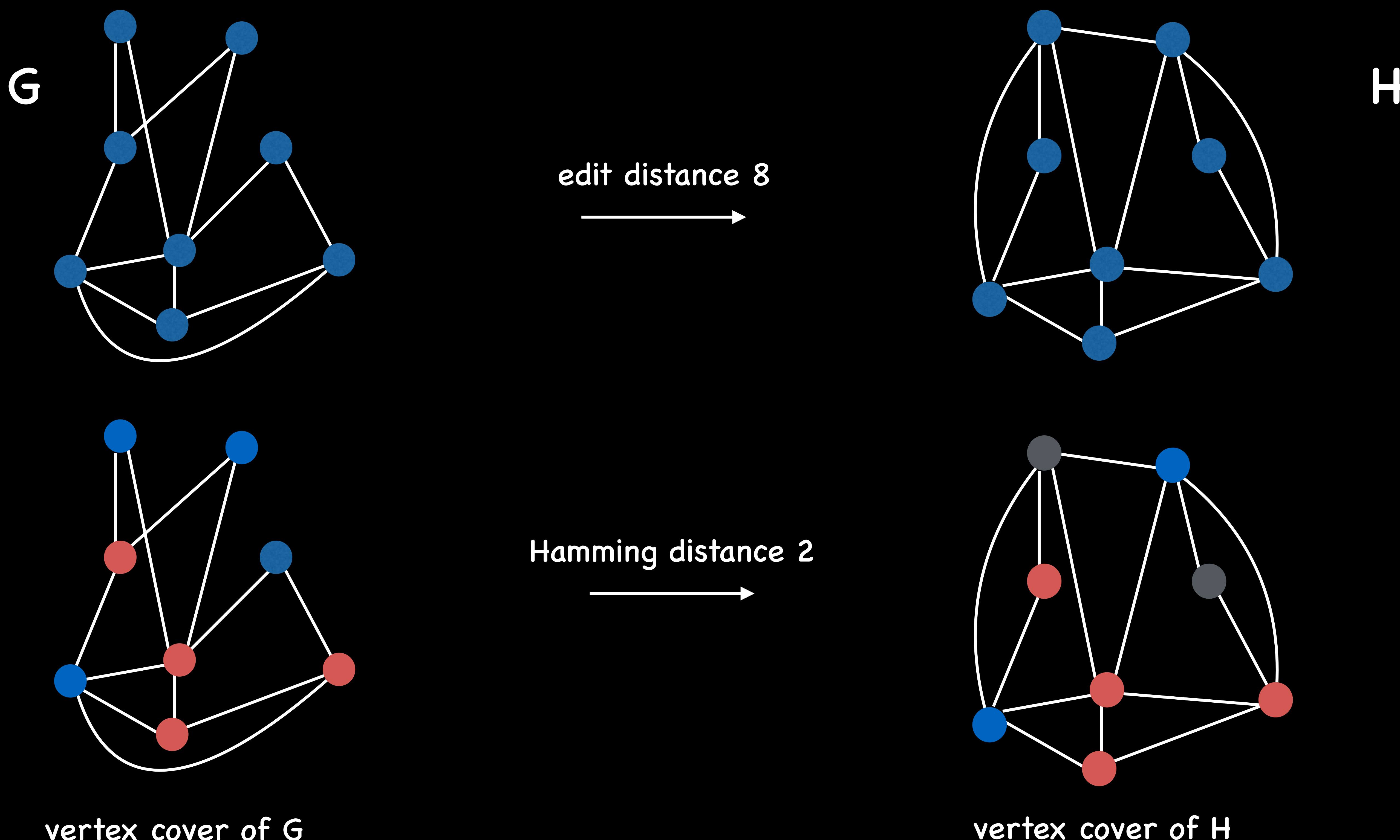


Hamming distance 2

vertex cover of G

Dynamic Graphs



Dynamic Problem Template

Dynamic Problem Template

Instance:

- Graphs G, H on same vertex set s.t $d_e(G,H) \leq k$
- A solution S of G
- An integer r

Question: Does H have a solution T s.t $d_v(S,T) \leq r$?

Parameter(s): k, r

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Parameter(s): k, r

k- edit parameter

r- distance parameter

Dynamic Π-Deletion

Instance:

- Graphs G, H on same vertex set s.t $d_e(G,H) \leq k$
- A set S s.t $G-S \in \Pi$
- An integer r

Question: Does H have a set T s.t $d_v(S,T) \leq r$ and $H-T \in \Pi$?

Parameter(s): k, r

k- edit parameter

r- distance parameter

Dynamic Problem	Parameterized Complexity	
	k	r
Dominating Set	2^{k^2} [DEFRS14], 2^k (tight)	W[2]-hard [DEFRS14]
Connected Dominating Set	4^k [AEFRS15], 2^k (tight)	W[2]-hard [AEFRS15]
Vertex Cover	1.174^k, 1.1277^k (expo space), $O(k)$ kernel	1.2738^r, $O(r^2)$ kernel
Connected Vertex Cover	4^k [AEFRS15], 2^k	W[2]-hard [AEFRS15]
Feedback Vertex Set	1.6667^k (randomized), $O(k)$ kernel	3.592^r, $O(r^2)$ kernel
Π-Deletion	Fixed-parameter (in)tractability related to that of non-dynamic version	

Dynamic Π-Deletion

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Π is hereditary (w.r.t induced subgraphs) $\Rightarrow$ Existence of Incremental Solution

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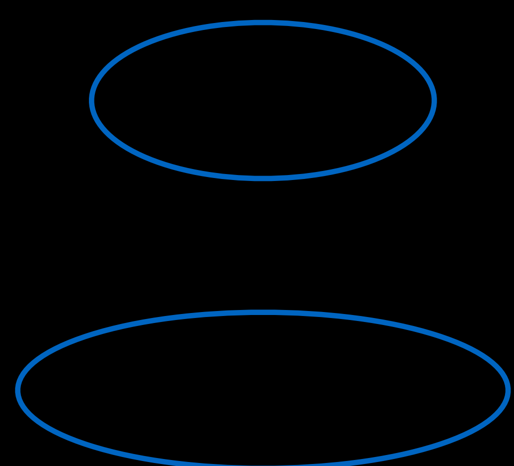
S



G-S $\in$ Π

Dynamic Π-Deletion

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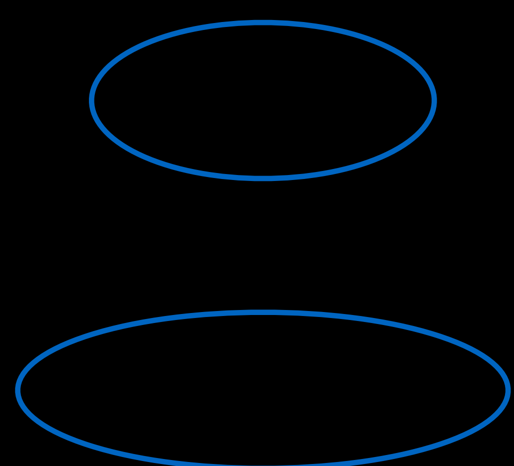
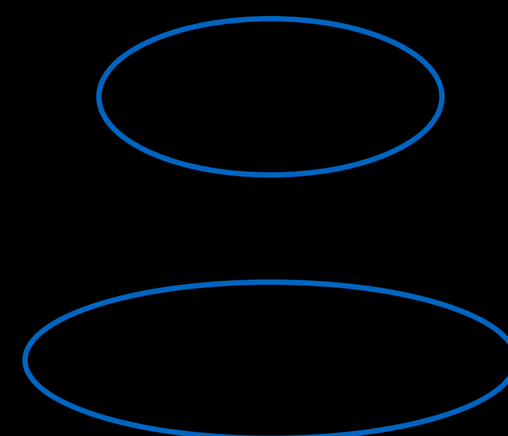


G-S $\in$ Π

T



H-T $\in$ Π

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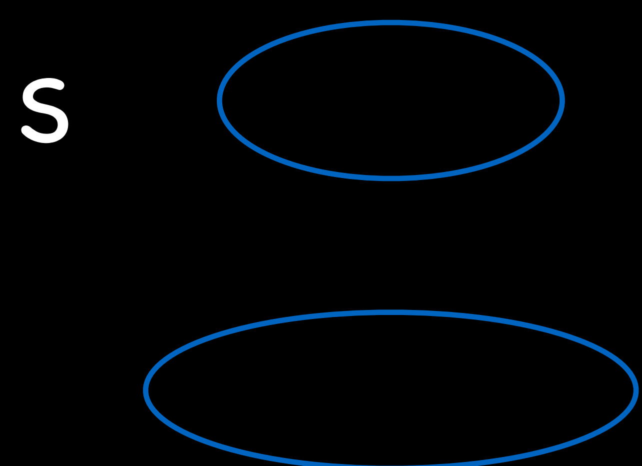

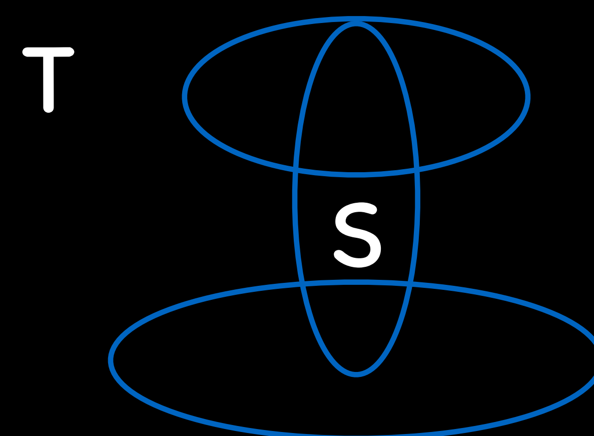

$G - S \in \Pi$

$H - T \in \Pi$

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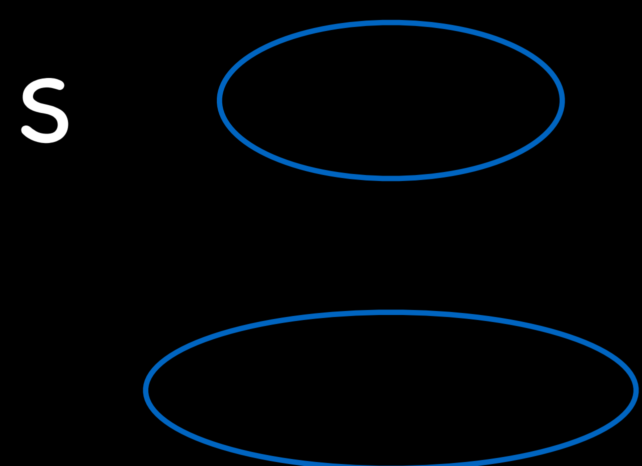

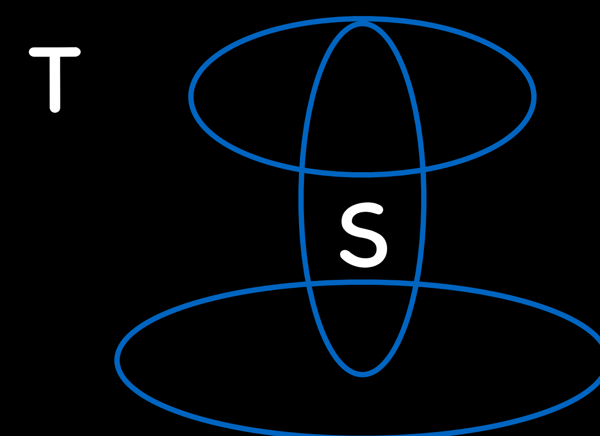

$G-S \in \Pi$

$H-T \in \Pi \quad \Rightarrow H-(S \cup T \in \Pi)$

Dynamic Π-Deletion

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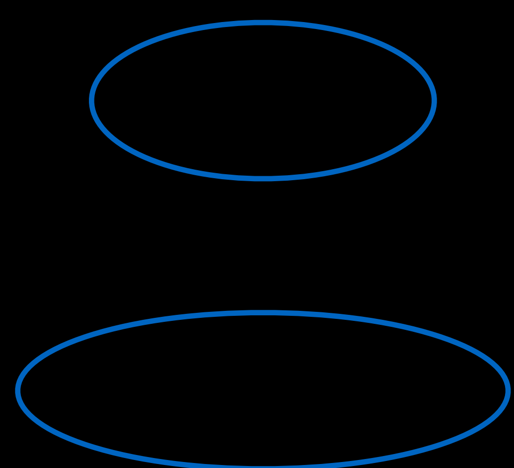
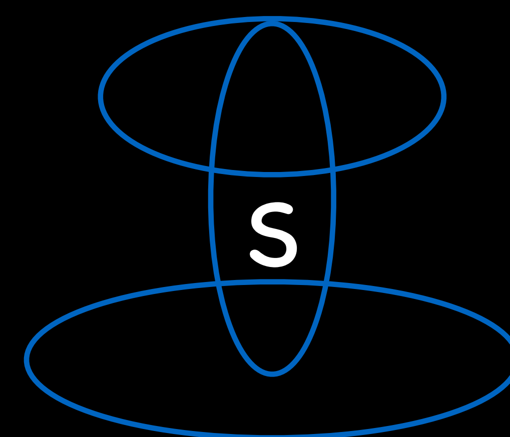


T



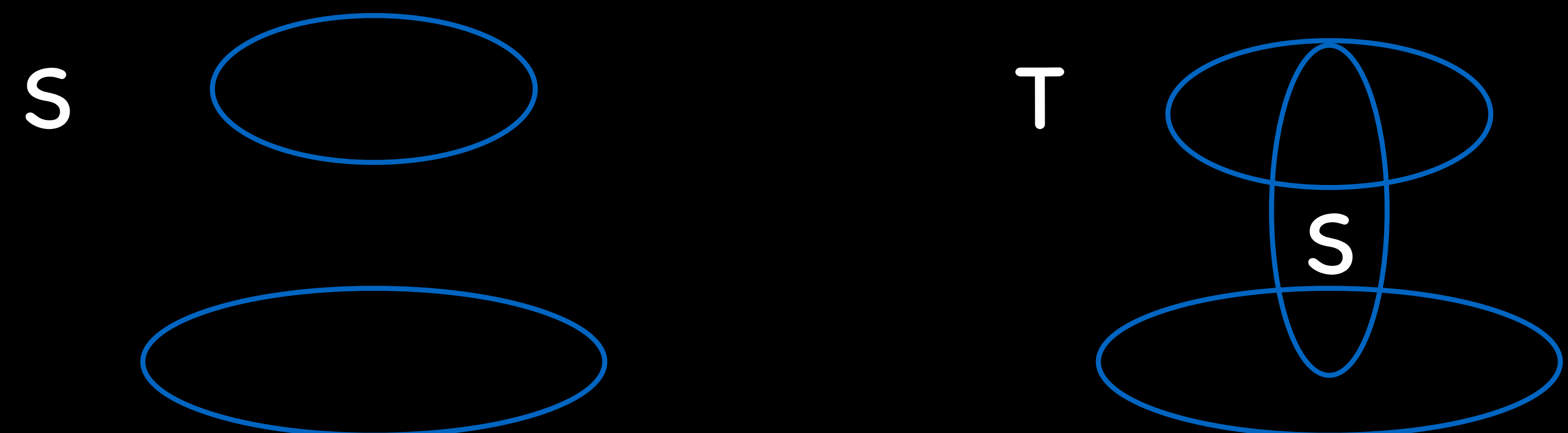
$G - S \in \Pi$

$H - T \in \Pi \quad \Rightarrow H - (S \cup T \in \Pi)$

$$d(S, S \cup T) = |T - S| \leq |T - S| + |S - T| = d(S, T)$$

Dynamic Π-Deletion

Π is hereditary (w.r.t induced subgraphs) ⇒ Existence of Incremental Solution



$G-S \in \Pi$ $\qquad$ $H-T \in \Pi \Rightarrow H-(S \cup T \in \Pi)$

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VC, FVS

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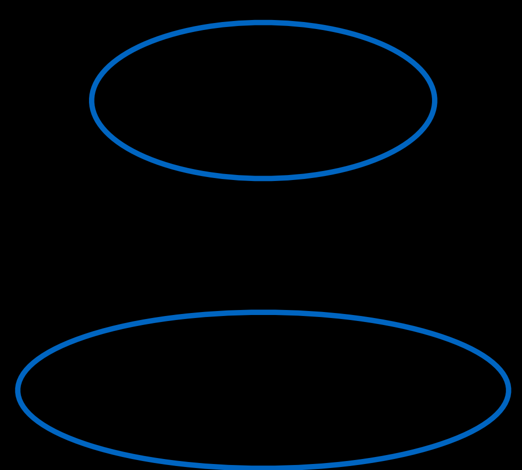

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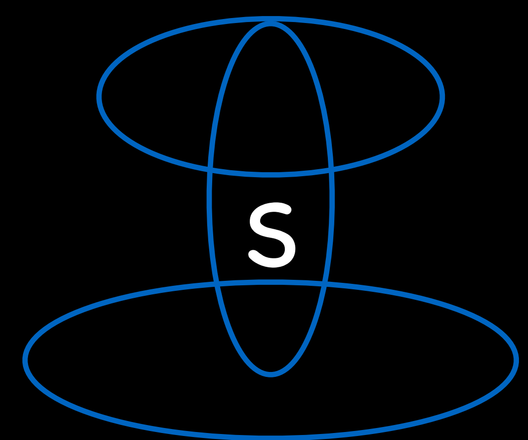

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VC, FVS

CVC, DS, CDS

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Π is hereditary (w.r.t induced subgraphs) and includes all independent sets

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Π-Deletion reduces in poly time to Dynamic Π-Deletion

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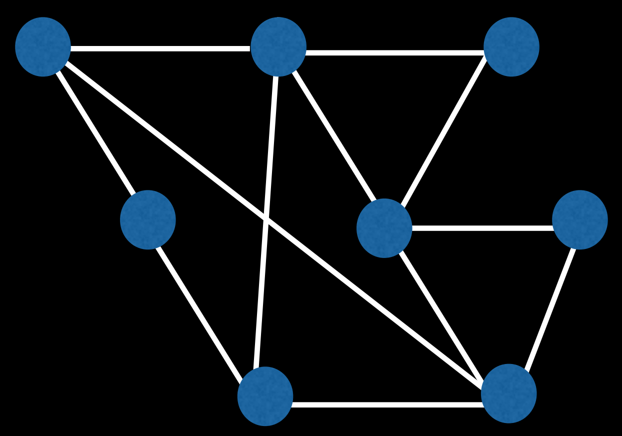

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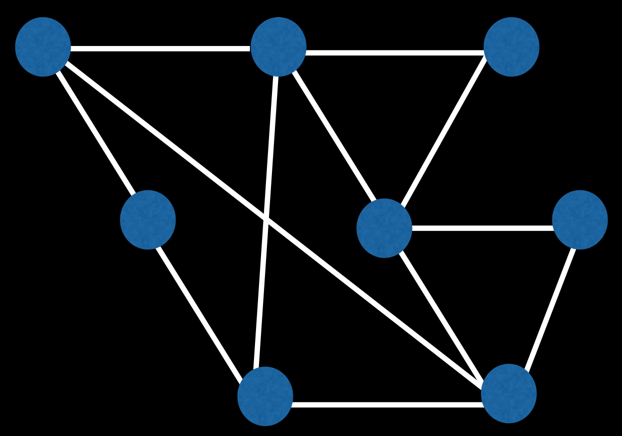

H   l

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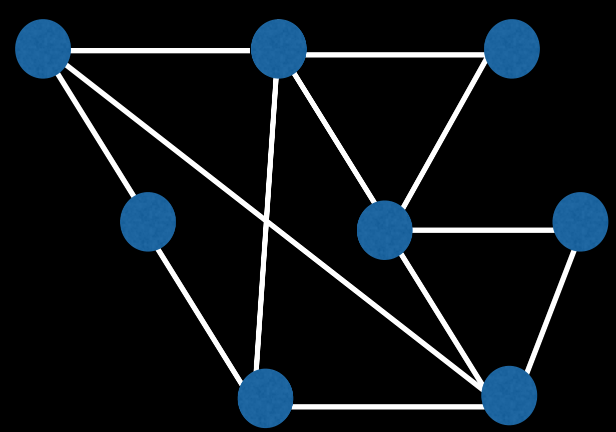

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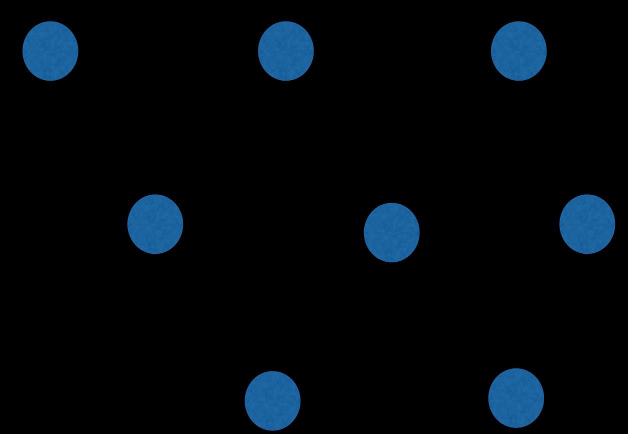

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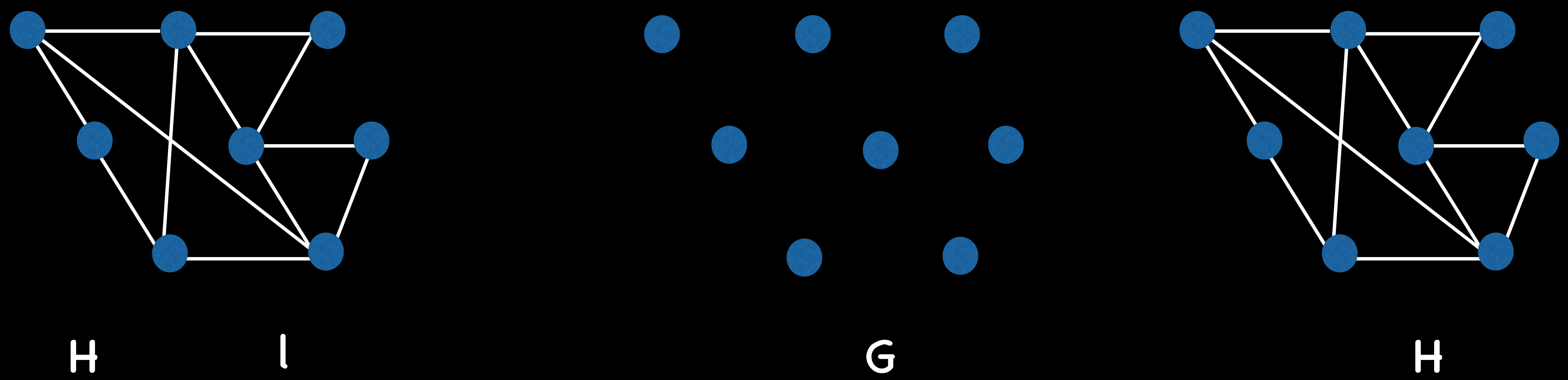

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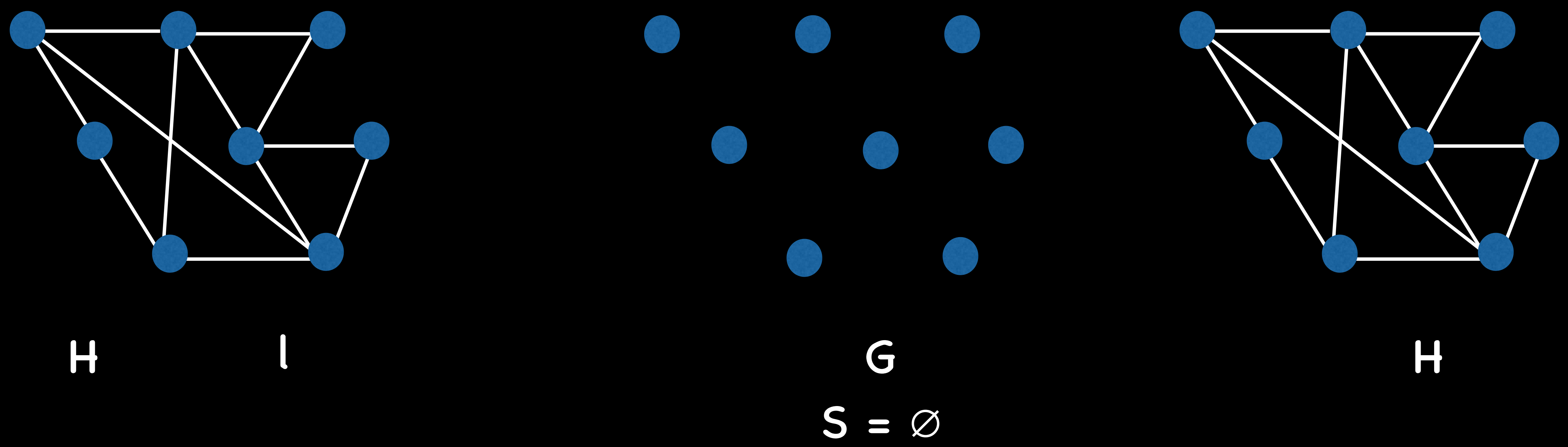

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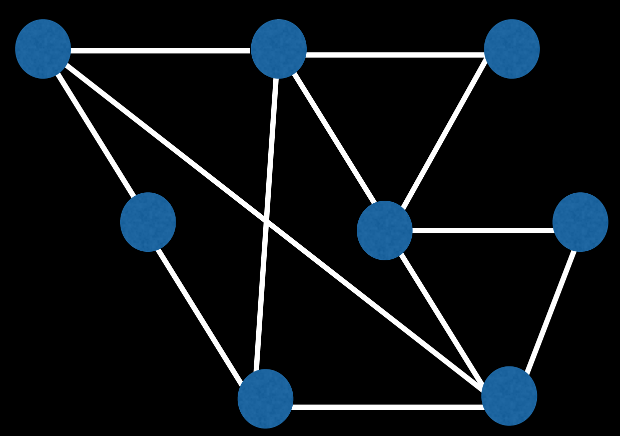

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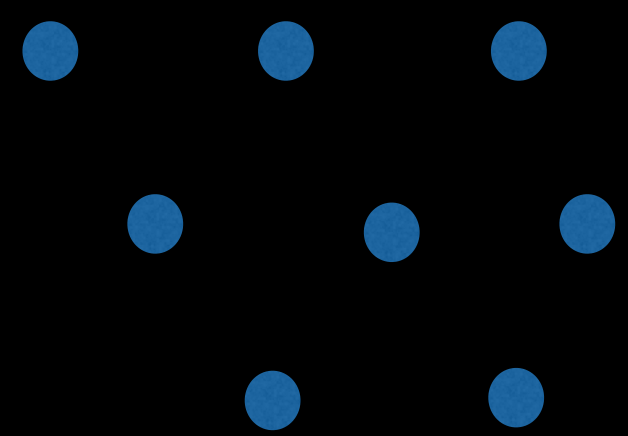
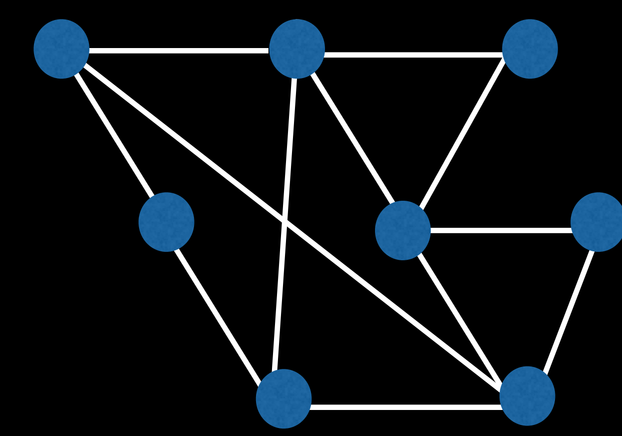


G

$S = \emptyset$

$k=|E(H)|$



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$r=l$

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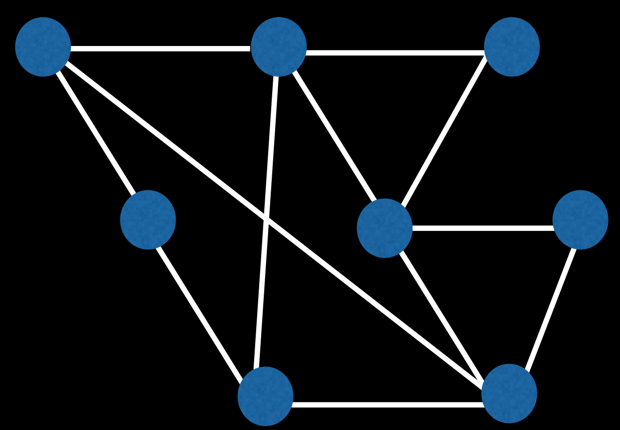

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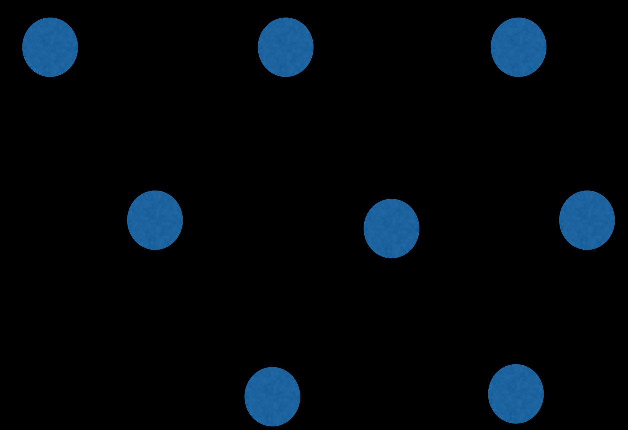

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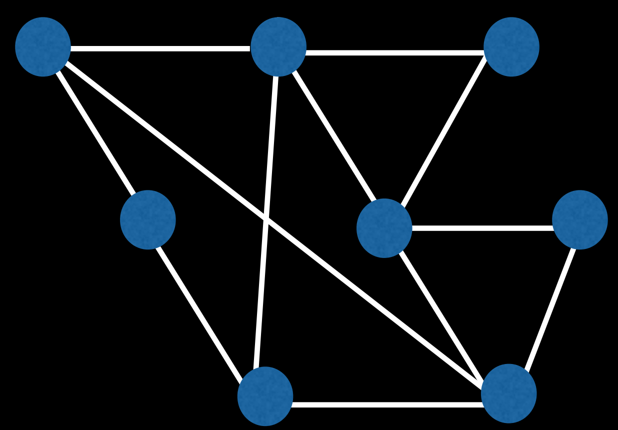

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NP-hard

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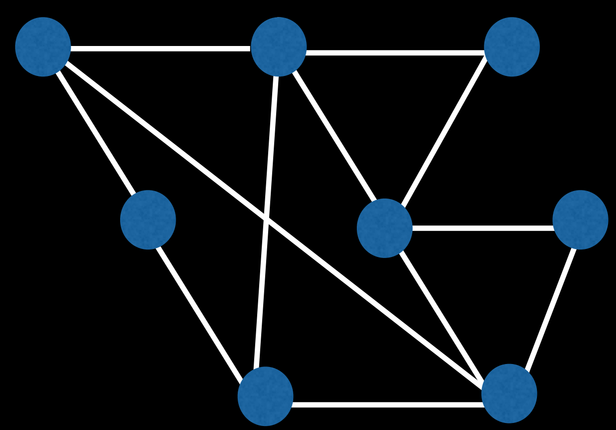

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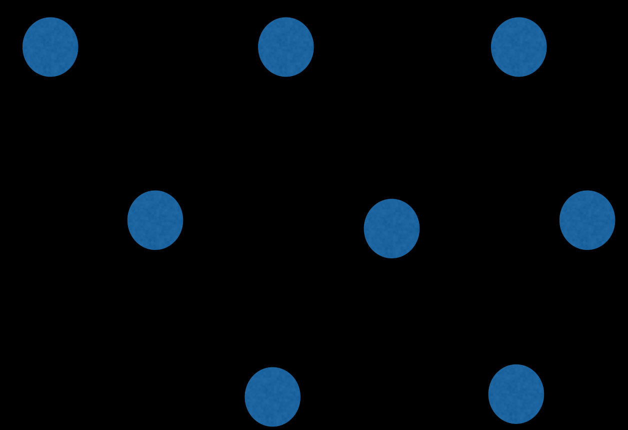

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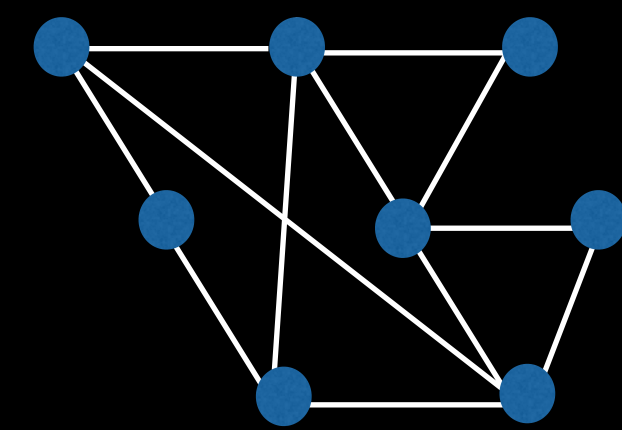

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$\Longrightarrow$

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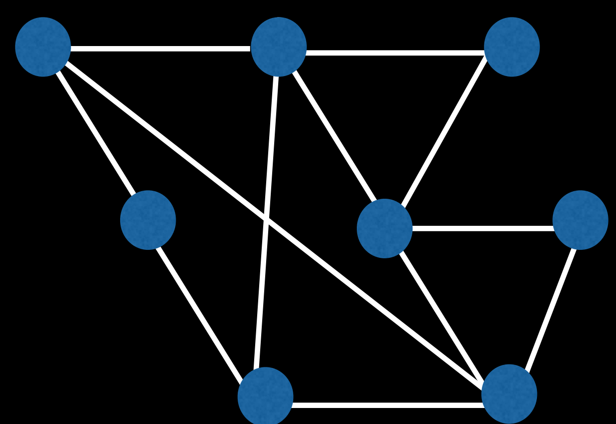
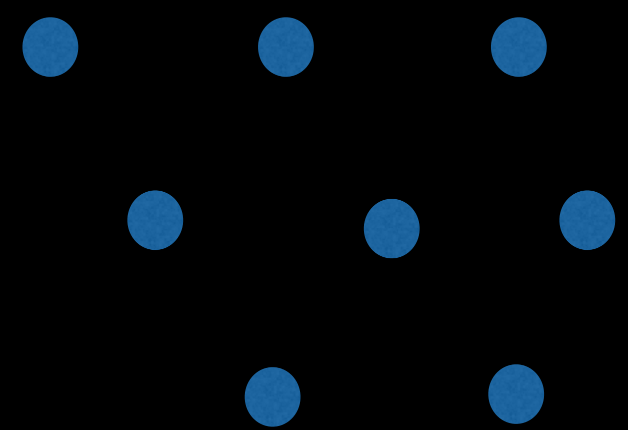


H   l

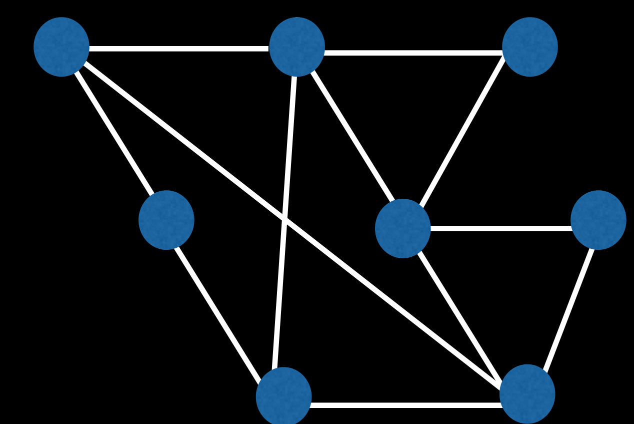
NP-hard



G

$S = \emptyset$

$k=|E(H)|$



H

$r=l$

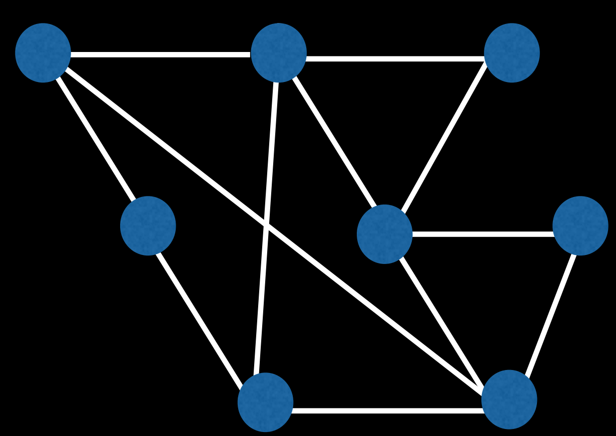
NP-hard

$\Longrightarrow$

Dynamic Π-Deletion

Π is hereditary (w.r.t induced subgraphs) and includes all independent sets

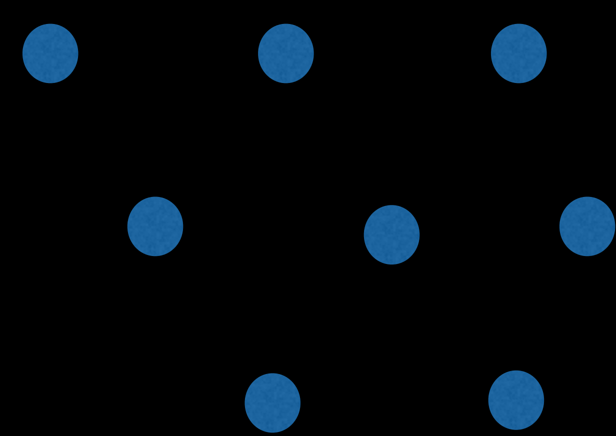
Π-Deletion reduces in poly time to Dynamic Π-Deletion



H l

NP-hard

Not FPT w.r.t sol size l

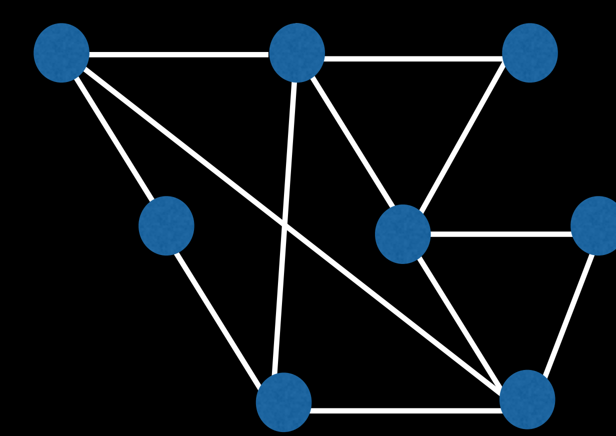


G

S = ∅

k=|E(H)|

⟹



H

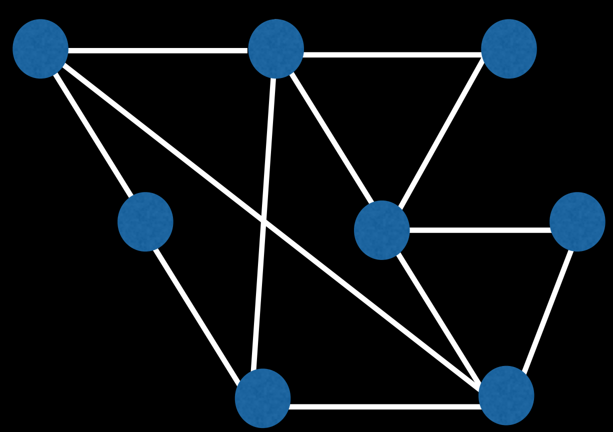
r=l

NP-hard

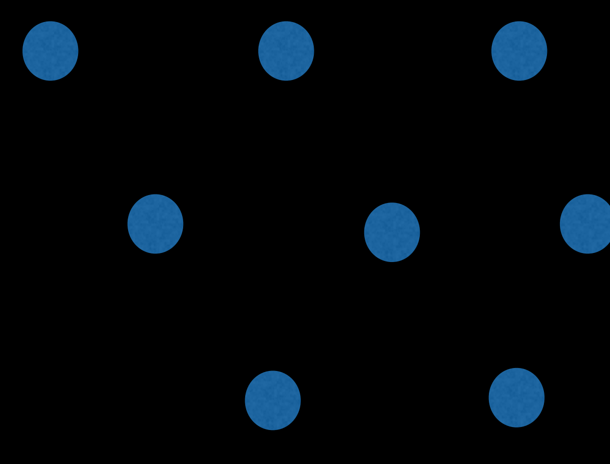
Dynamic Π-Deletion

Π is hereditary (w.r.t induced subgraphs) and includes all independent sets

Π-Deletion reduces in poly time to Dynamic Π-Deletion



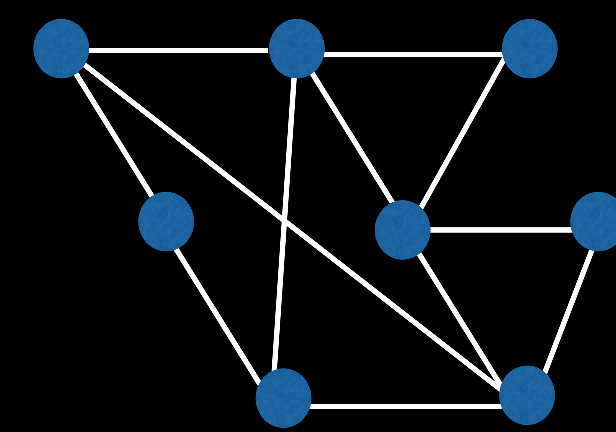
H l



G

$S = \emptyset$

$k=|E(H)|$



H

$r=l$

NP-hard

Not FPT w.r.t sol size l

$\Rightarrow$

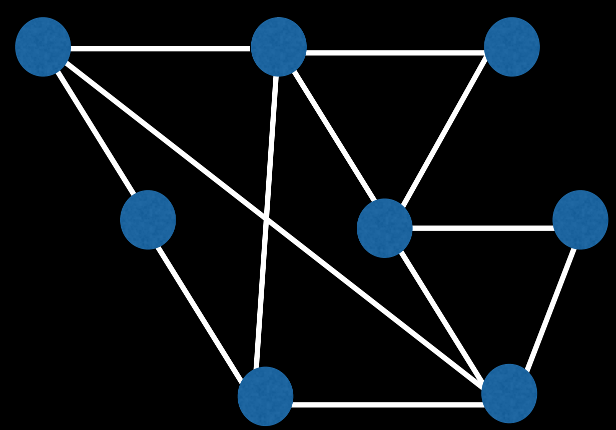
NP-hard

Not FPT w.r.t r

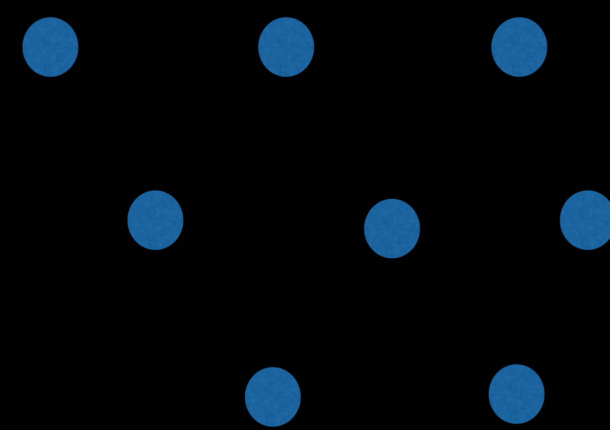
Dynamic Π-Deletion

Π is hereditary (w.r.t induced subgraphs) and includes all independent sets

Π-Deletion reduces in poly time to Dynamic Π-Deletion



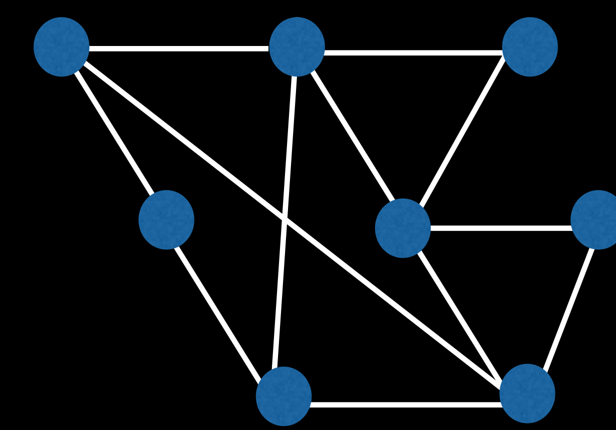
H l



G

$S = \emptyset$

$k = |E(H)|$



H

$r = l$

NP-hard

Not FPT w.r.t sol size l

No poly kernel w.r.t sol size l

$\Rightarrow$

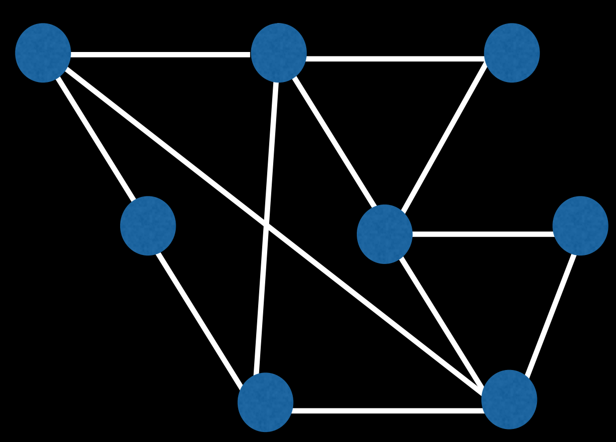
NP-hard

Not FPT w.r.t r

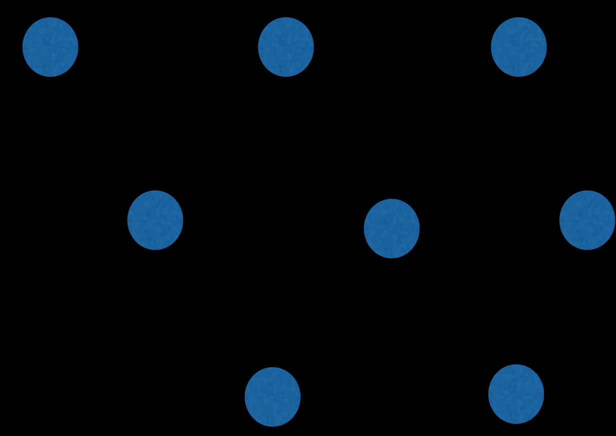
Dynamic Π-Deletion

Π is hereditary (w.r.t induced subgraphs) and includes all independent sets

Π-Deletion reduces in poly time to Dynamic Π-Deletion



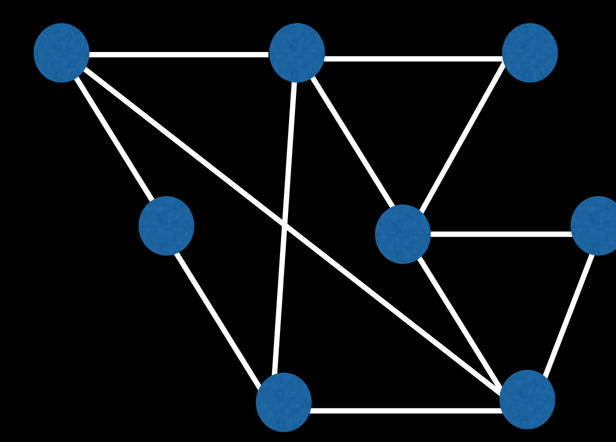
H l



G

$S = \emptyset$

$k=|E(H)|$



H

$r=l$

NP-hard

Not FPT w.r.t sol size l

No poly kernel w.r.t sol size l

$\Longrightarrow$

NP-hard

Not FPT w.r.t r

No poly kernel w.r.t r

Dynamic Π-Deletion

Dynamic Π-Deletion

Π is hereditary (w.r.t induced subgraphs) and includes all cliques

Dynamic Π-Deletion

Π is hereditary (w.r.t induced subgraphs) and includes all cliques

Π-Deletion reduces in poly time to Dynamic Π-Deletion

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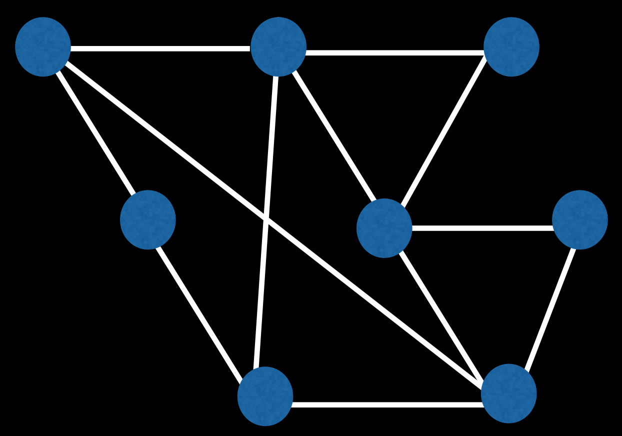
Π is hereditary (w.r.t induced subgraphs) and includes all cliques

Π-Deletion reduces in poly time to Dynamic Π-Deletion

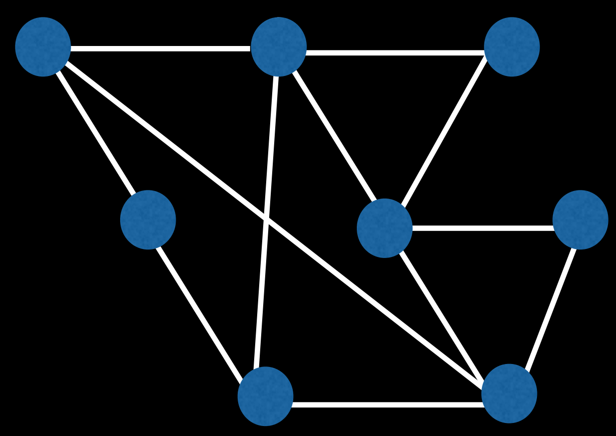


H

Dynamic Π-Deletion

Π is hereditary (w.r.t induced subgraphs) and includes all cliques

Π-Deletion reduces in poly time to Dynamic Π-Deletion

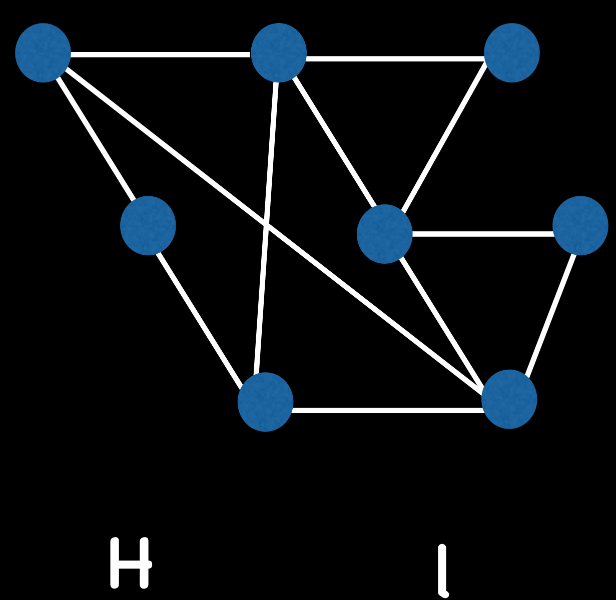


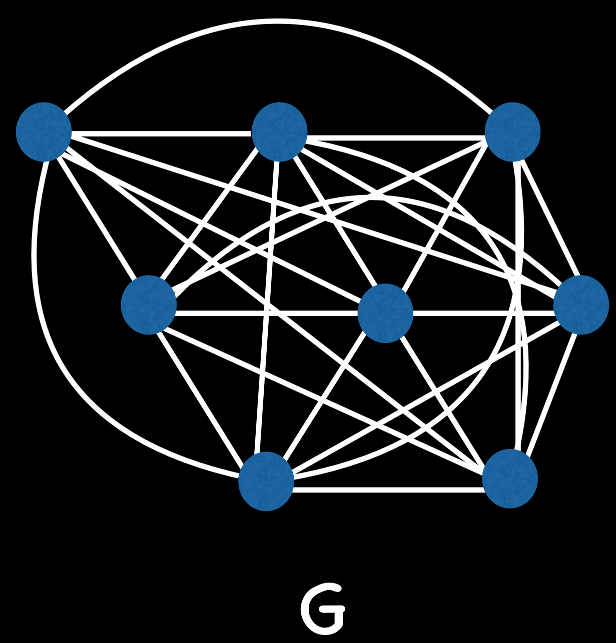
H I

Dynamic Π-Deletion

Π is hereditary (w.r.t induced subgraphs) and includes all cliques

Π-Deletion reduces in poly time to Dynamic Π-Deletion

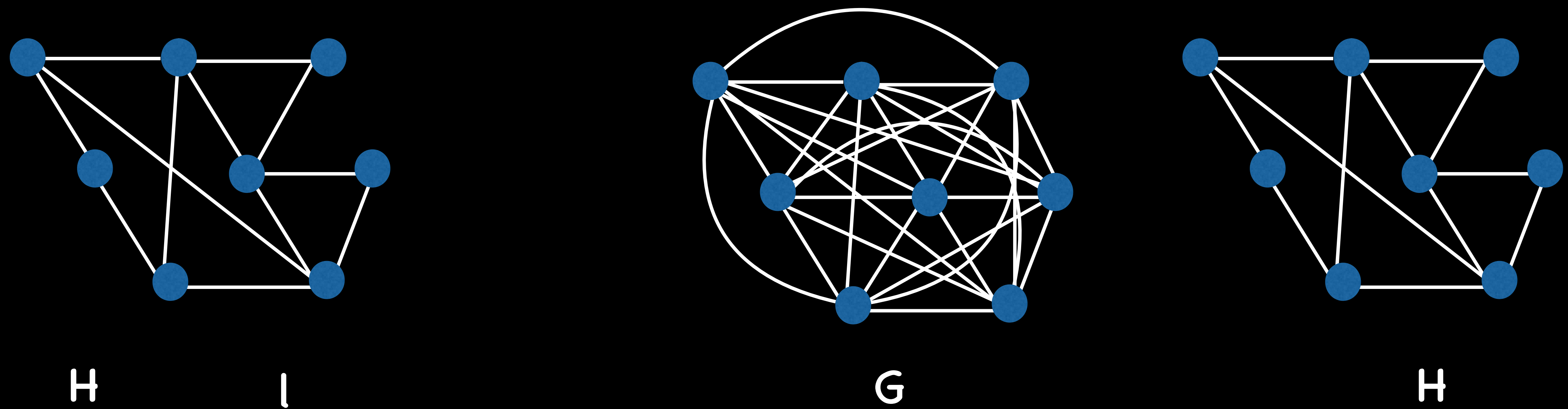




Dynamic Π-Deletion

Π is hereditary (w.r.t induced subgraphs) and includes all cliques

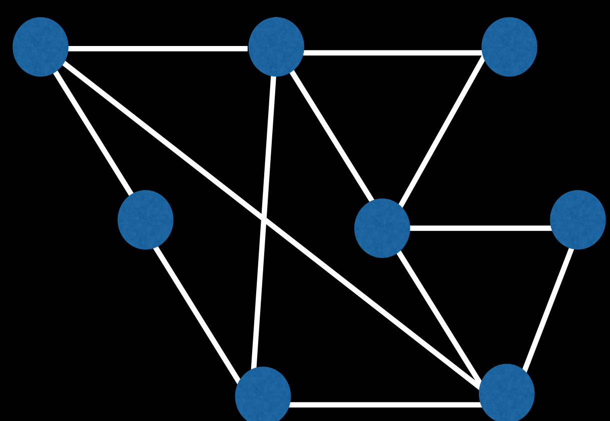
Π-Deletion reduces in poly time to Dynamic Π-Deletion



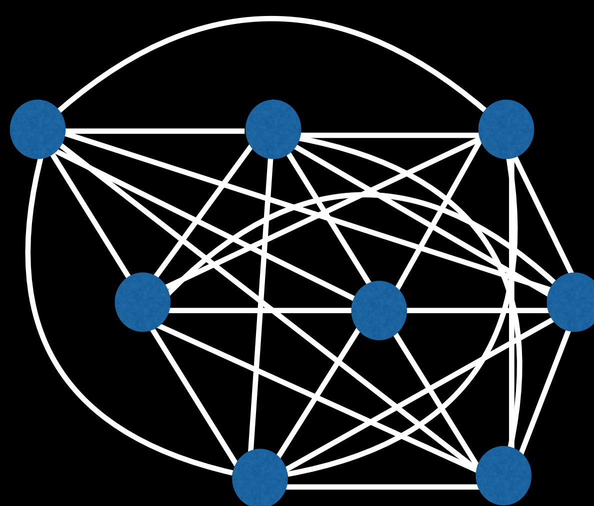
Dynamic Π-Deletion

Π is hereditary (w.r.t induced subgraphs) and includes all cliques

Π-Deletion reduces in poly time to Dynamic Π-Deletion

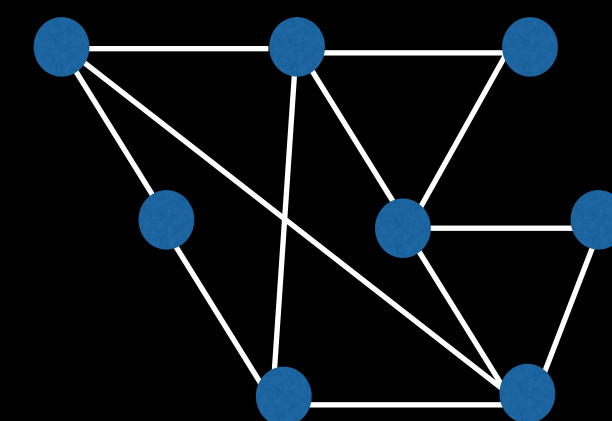


H I



G

S = ∅

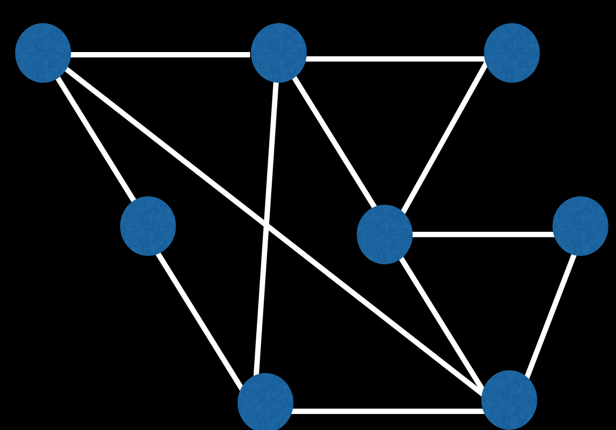


H

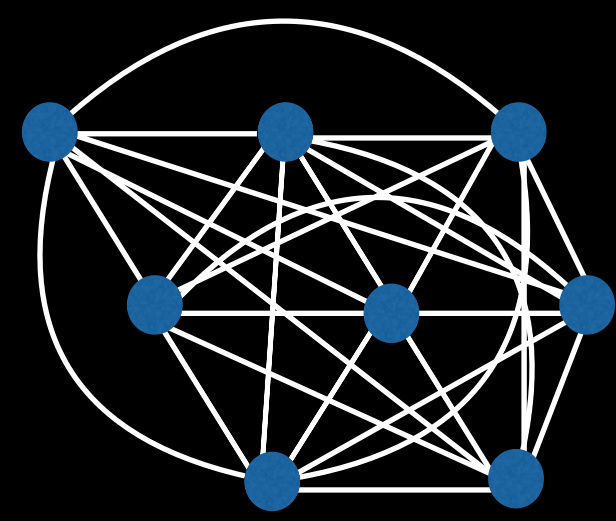
Dynamic Π-Deletion

Π is hereditary (w.r.t induced subgraphs) and includes all cliques

Π-Deletion reduces in poly time to Dynamic Π-Deletion



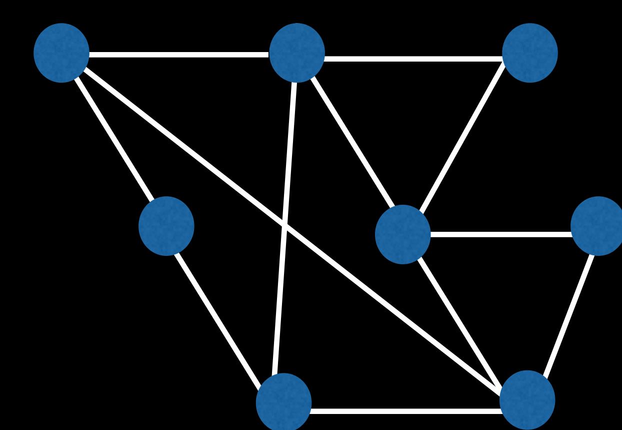
H l



G

$S = \emptyset$

$k = {}^nC_2 - |E(H)|$



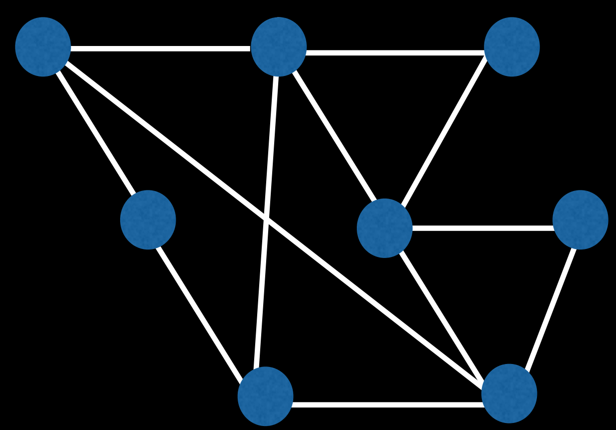
H

$r = l$

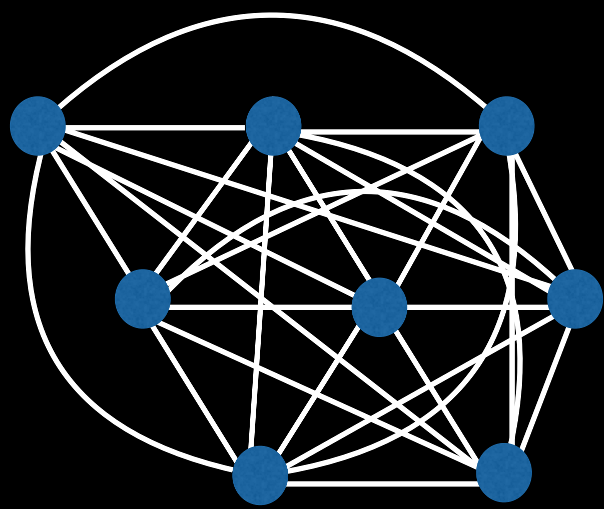
Dynamic Π-Deletion

Π is hereditary (w.r.t induced subgraphs) and includes all cliques

Π-Deletion reduces in poly time to Dynamic Π-Deletion



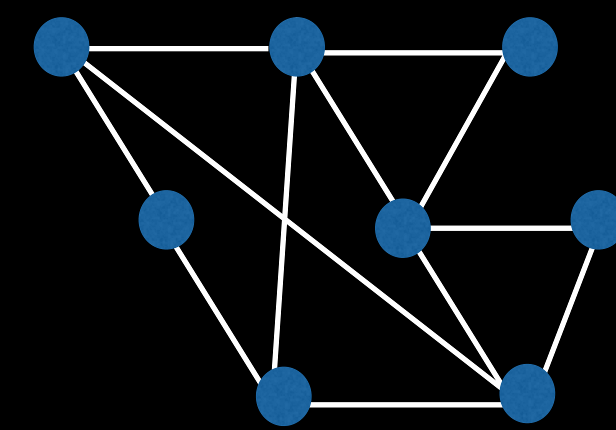
H l



G

$S = \emptyset$

$k = nC_2 - |E(H)|$



H

$r = l$

NP-hard

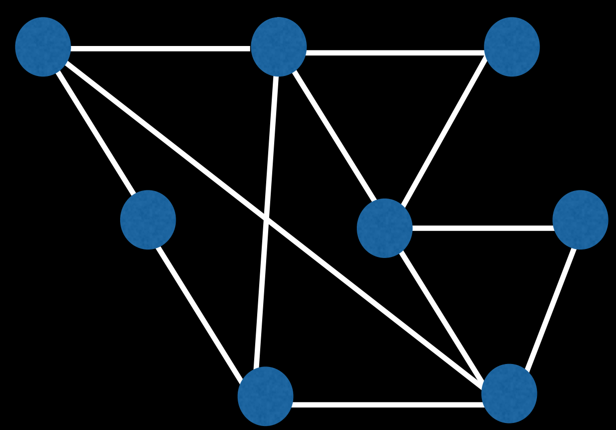
Not FPT w.r.t sol size l

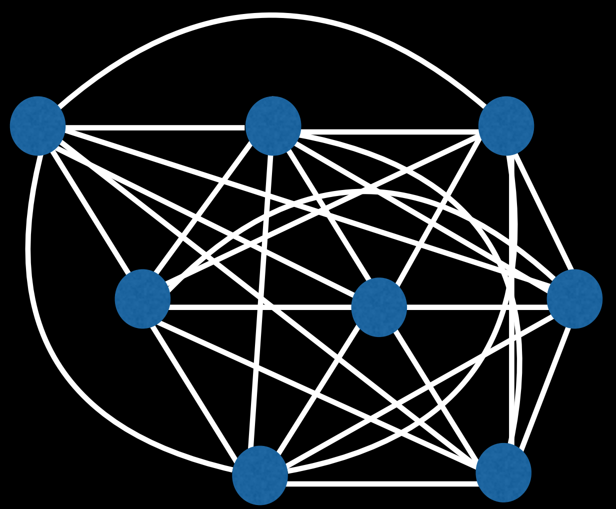
No poly kernel w.r.t sol size l

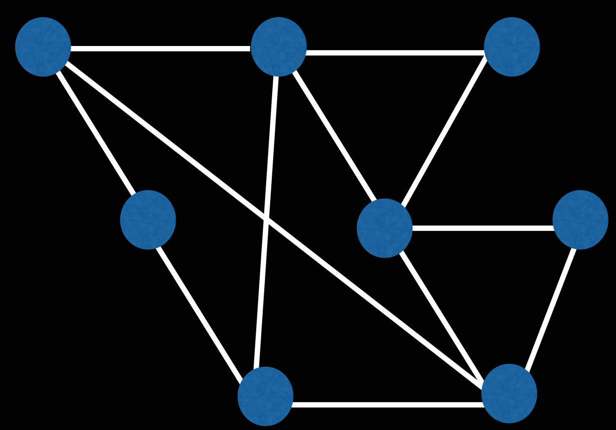
Dynamic Π-Deletion

Π is hereditary (w.r.t induced subgraphs) and includes all cliques

Π-Deletion reduces in poly time to Dynamic Π-Deletion







H l

G

$S = \emptyset$

$k = nC_2 - |E(H)|$

H

$r = l$

NP-hard

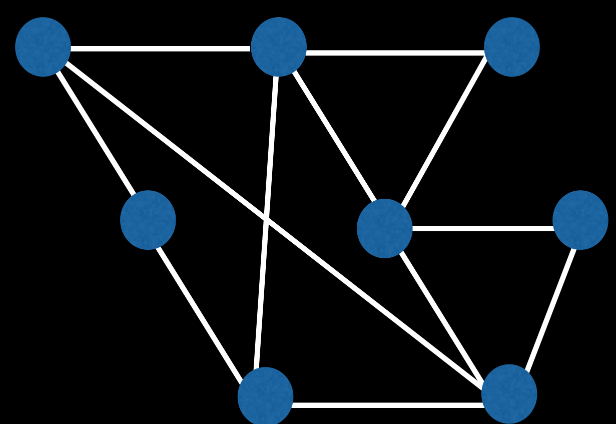
Not FPT w.r.t sol size l $\Rightarrow$

No poly kernel w.r.t sol size l

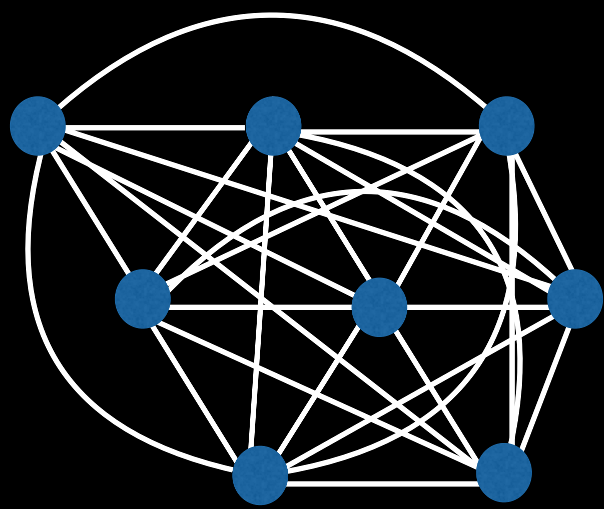
Dynamic Π-Deletion

Π is hereditary (w.r.t induced subgraphs) and includes all cliques

Π-Deletion reduces in poly time to Dynamic Π-Deletion



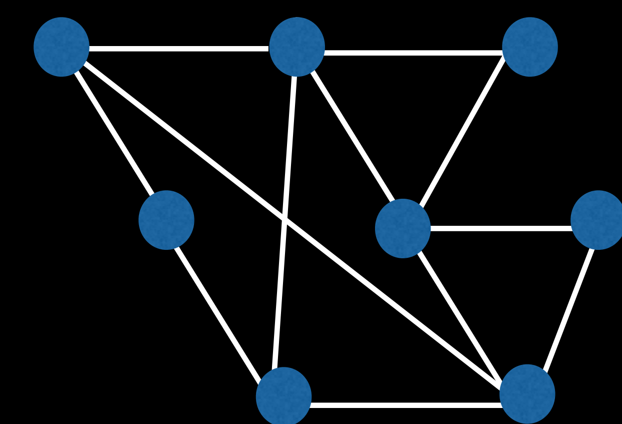
H l



G

$S = \emptyset$

$k = nC_2 - |E(H)|$



H

$r = l$

NP-hard

Not FPT w.r.t sol size l

No poly kernel w.r.t sol size l

⇒

NP-hard

Not FPT w.r.t r

No poly kernel w.r.t r

Dynamic Π-Deletion

Dynamic Π-Deletion

Π is hereditary (w.r.t subgraphs) and membership in Π is poly-time decidable

Dynamic Π-Deletion

Π is hereditary (w.r.t subgraphs) and membership in Π is poly-time decidable

Dynamic Π-Deletion reduces in poly time to Π-Deletion

Dynamic Π-Deletion

Π is hereditary (w.r.t subgraphs) and membership in Π is poly-time decidable

Dynamic Π-Deletion reduces in poly time to Π-Deletion

(G, H, S, k, r)

Dynamic Π-Deletion

Π is hereditary (w.r.t subgraphs) and membership in Π is poly-time decidable

Dynamic Π-Deletion reduces in poly time to Π-Deletion

(G, H, S, k, r)

↙

Dynamic Π-Deletion

Π is hereditary (w.r.t subgraphs) and membership in Π is poly-time decidable

Dynamic Π-Deletion reduces in poly time to Π-Deletion

(G, H, S, k, r)

H-S $\in$ Π

Dynamic Π-Deletion

Π is hereditary (w.r.t subgraphs) and membership in Π is poly-time decidable

Dynamic Π-Deletion reduces in poly time to Π-Deletion

(G, H, S, k, r)

$H - S \in \Pi$ (or)

Dynamic Π-Deletion

Π is hereditary (w.r.t subgraphs) and membership in Π is poly-time decidable

Dynamic Π-Deletion reduces in poly time to Π-Deletion

(G, H, S, k, r)

H-S ∈ Π (or) To find

T ⊇ S s.t H-T ∈ Π

Dynamic Π-Deletion

Π is hereditary (w.r.t subgraphs) and membership in Π is poly-time decidable

Dynamic Π-Deletion reduces in poly time to Π-Deletion

(G, H, S, k, r)

$H - S \in \Pi$ (or) To find

$T \supseteq S$ s.t $H - T \in \Pi$

solve Π-Deletion on (H-S,r)

Dynamic Π-Deletion

Π is hereditary (w.r.t subgraphs) and membership in Π is poly-time decidable

Dynamic Π-Deletion reduces in poly time to Π-Deletion

(G, H, S, k, r)

$H-S \in \Pi$ (or) To find

$T \supseteq S$ s.t $H-T \in \Pi$

solve Π-Deletion on (H-S,r)

FPT w.r.t sol size l

$O^*(f(l))$

Dynamic Π-Deletion

Π is hereditary (w.r.t subgraphs) and membership in Π is poly-time decidable

Dynamic Π-Deletion reduces in poly time to Π-Deletion

(G, H, S, k, r)

$H - S \in \Pi$ (or) To find

$T \supseteq S$ s.t $H - T \in \Pi$

solve Π-Deletion on (H-S,r)

FPT w.r.t sol size l

$O^*(f(l))$

$\Rightarrow$

Dynamic Π-Deletion

Π is hereditary (w.r.t subgraphs) and membership in Π is poly-time decidable

Dynamic Π-Deletion reduces in poly time to Π-Deletion

(G, H, S, k, r)

$H - S \in \Pi$ (or) To find

$T \supseteq S$ s.t $H - T \in \Pi$

solve Π-Deletion on (H-S,r)

FPT w.r.t sol size l

O*(f(l))

$\Rightarrow$

FPT w.r.t r and k

O*(f(r)) algorithm

O*(f(k)) algorithm

Dynamic Π-Deletion

Π is hereditary (w.r.t subgraphs) and membership in Π is poly-time decidable

Dynamic Π-Deletion reduces in poly time to Π-Deletion

(G, H, S, k, r)

$H - S \in \Pi$ (or) To find

$T \supseteq S$ s.t $H - T \in \Pi$

solve Π-Deletion on (H−S,r)

FPT w.r.t sol size l

$O^*(f(l))$

$\Rightarrow$

FPT w.r.t r and k

$O^*(f(r))$ algorithm

$O^*(f(k))$ algorithm

Π includes all independent sets

Dynamic Π-Deletion

Π is hereditary (w.r.t subgraphs) and membership in Π is poly-time decidable

Dynamic Π-Deletion reduces in poly time to Π-Deletion

(G, H, S, k, r)

$H - S \in \Pi$ (or) To find

$T \supseteq S$ s.t $H - T \in \Pi$

solve Π-Deletion on (H-S,r)

FPT w.r.t sol size l

O*(f(l))

$\Rightarrow$

FPT w.r.t r and k

O*(f(r)) algorithm

O*(f(k)) algorithm

Π includes all independent sets

p(l) vertices and q(l) edges kernel

Dynamic Π-Deletion

Π is hereditary (w.r.t subgraphs) and membership in Π is poly-time decidable

Dynamic Π-Deletion reduces in poly time to Π-Deletion

(G, H, S, k, r)

$H-S \in \Pi$ (or) To find

$T \supseteq S$ s.t $H-T \in \Pi$

solve Π-Deletion on (H-S,r)

FPT w.r.t sol size l

O*(f(l)) $\Rightarrow$

FPT w.r.t r and k

O*(f(r)) algorithm

O*(f(k)) algorithm

Π includes all independent sets

p(l) vertices and q(l) edges kernel $\Rightarrow$

Dynamic Π-Deletion

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Dynamic Π-Deletion reduces in poly time to Π-Deletion

(G, H, S, k, r)

$H - S \in \Pi$ (or) To find

$T \supseteq S$ s.t $H - T \in \Pi$

solve Π-Deletion on (H−S,r)

FPT w.r.t sol size l
O*(f(l))

⟹

FPT w.r.t r and k
O*(f(r)) algorithm
O*(f(k)) algorithm

Π includes all independent sets

p(l) vertices and q(l) edges kernel

⟹

2p(r) vertices and q(r) edges kernel
2p(k) vertices and q(k) edges kernel

Dynamic Π-Deletion

Π is hereditary (w.r.t subgraphs) and membership in Π is poly-time decidable

Dynamic Π-Deletion reduces in poly time to Π-Deletion

(G, H, S, k, r)

$H - S \in \Pi$ (or) To find

$T \supseteq S$ s.t $H - T \in \Pi$

solve Π-Deletion on (H-S,r)

FPT w.r.t sol size l

O*(f(l))

$\Rightarrow$

FPT w.r.t r and k

O*(f(r)) algorithm

O*(f(k)) algorithm

Π includes all cliques

Dynamic Π-Deletion

Π is hereditary (w.r.t subgraphs) and membership in Π is poly-time decidable

Dynamic Π-Deletion reduces in poly time to Π-Deletion

(G, H, S, k, r)

$H - S \in \Pi$ (or) To find

$T \supseteq S$ s.t $H - T \in \Pi$

solve Π-Deletion on (H-S,r)

FPT w.r.t sol size l

O*(f(l))

⇒

FPT w.r.t r and k

O*(f(r)) algorithm

O*(f(k)) algorithm

Π includes all cliques

p(l) vertices and q(l) edges kernel

Dynamic Π-Deletion

Π is hereditary (w.r.t subgraphs) and membership in Π is poly-time decidable

Dynamic Π-Deletion reduces in poly time to Π-Deletion

(G, H, S, k, r)

$H - S \in \Pi$ (or) To find

$T \supseteq S$ s.t $H - T \in \Pi$

solve Π-Deletion on (H−S,r)

FPT w.r.t sol size l

O*(f(l))

$\Rightarrow$

FPT w.r.t r and k

O*(f(r)) algorithm

O*(f(k)) algorithm

Π includes all cliques

p(l) vertices and q(l) edges kernel $\Rightarrow$

Dynamic Π-Deletion

Π is hereditary (w.r.t subgraphs) and membership in Π is poly-time decidable

Dynamic Π-Deletion reduces in poly time to Π-Deletion

(G, H, S, k, r)

$H - S \in \Pi$ (or) To find

$T \supseteq S$ s.t $H - T \in \Pi$

solve Π-Deletion on $(H-S, r)$

FPT w.r.t sol size l
$O^*(f(l))$

$\Rightarrow$

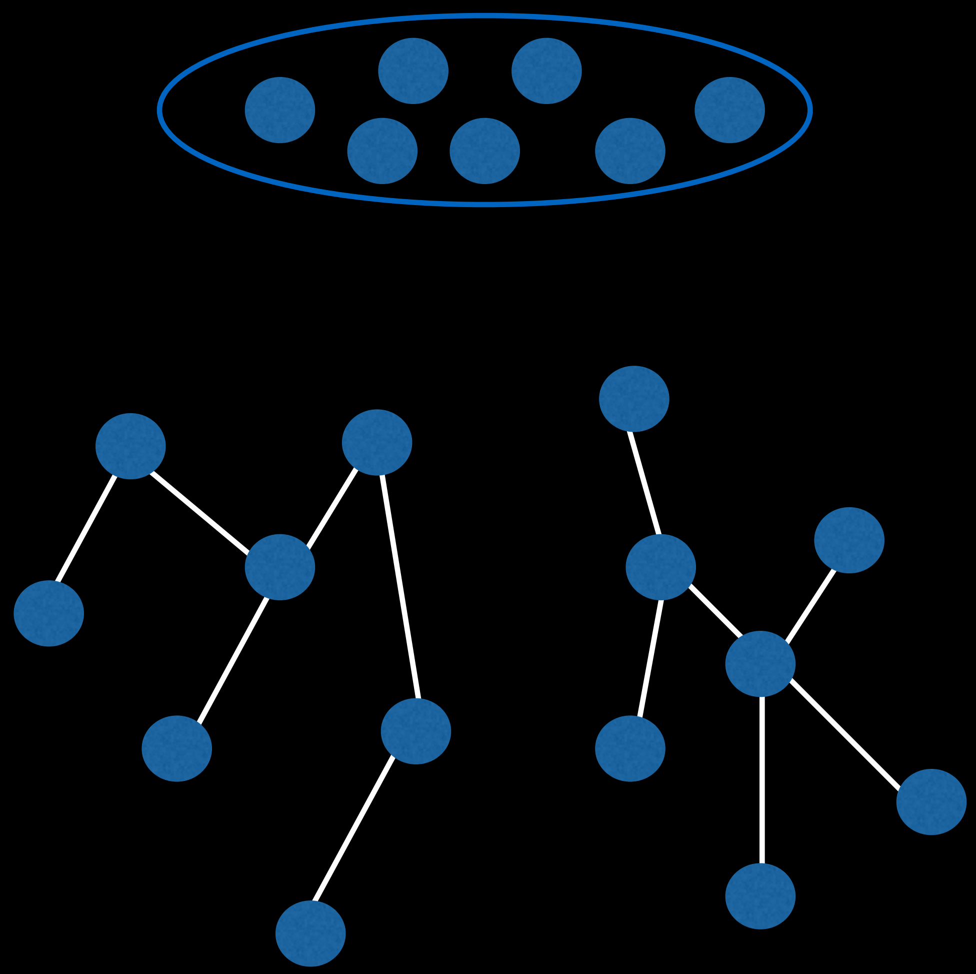
FPT w.r.t r and k
$O^*(f(r))$ algorithm
$O^*(f(k))$ algorithm

Π includes all cliques

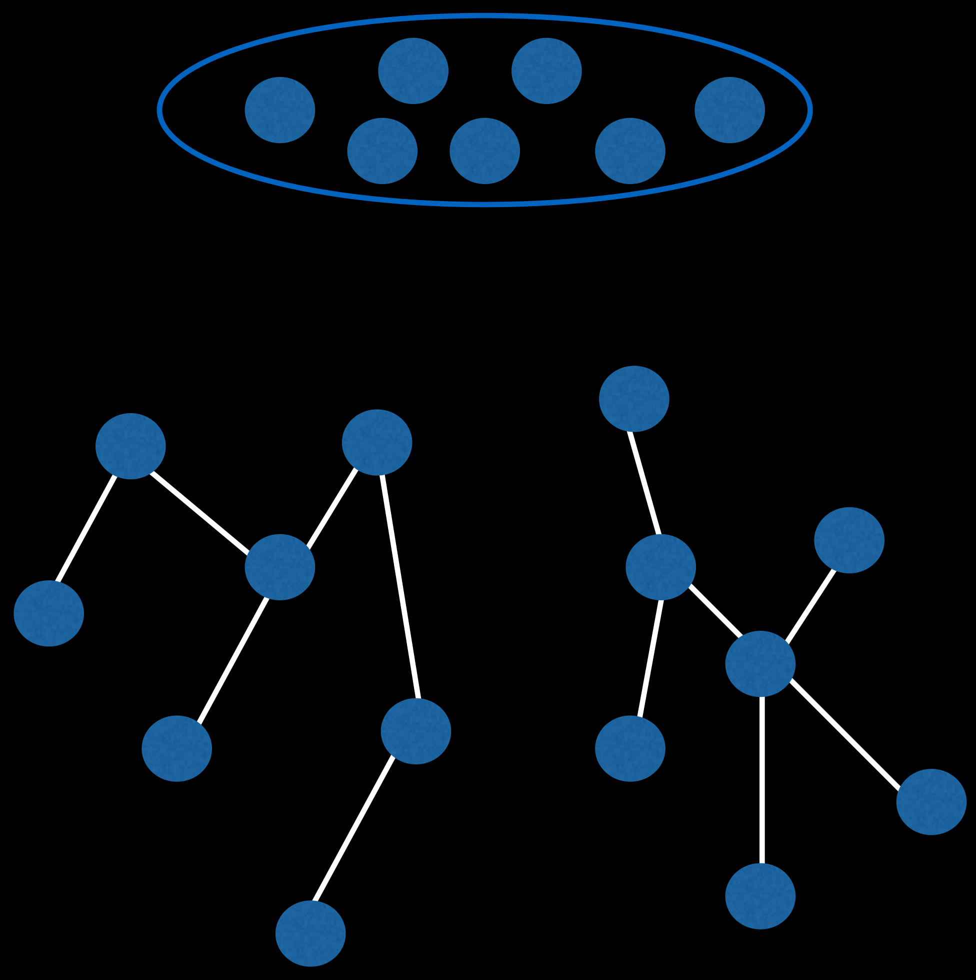
p(l) vertices and q(l) edges kernel $\Rightarrow$ 2p(r) vertices and $q(r)+ p^2(r)$ edges kernel
2p(k) vertices and $q(k)+ p^2(k)$ edges kernel

Dynamic Feedback Vertex Set

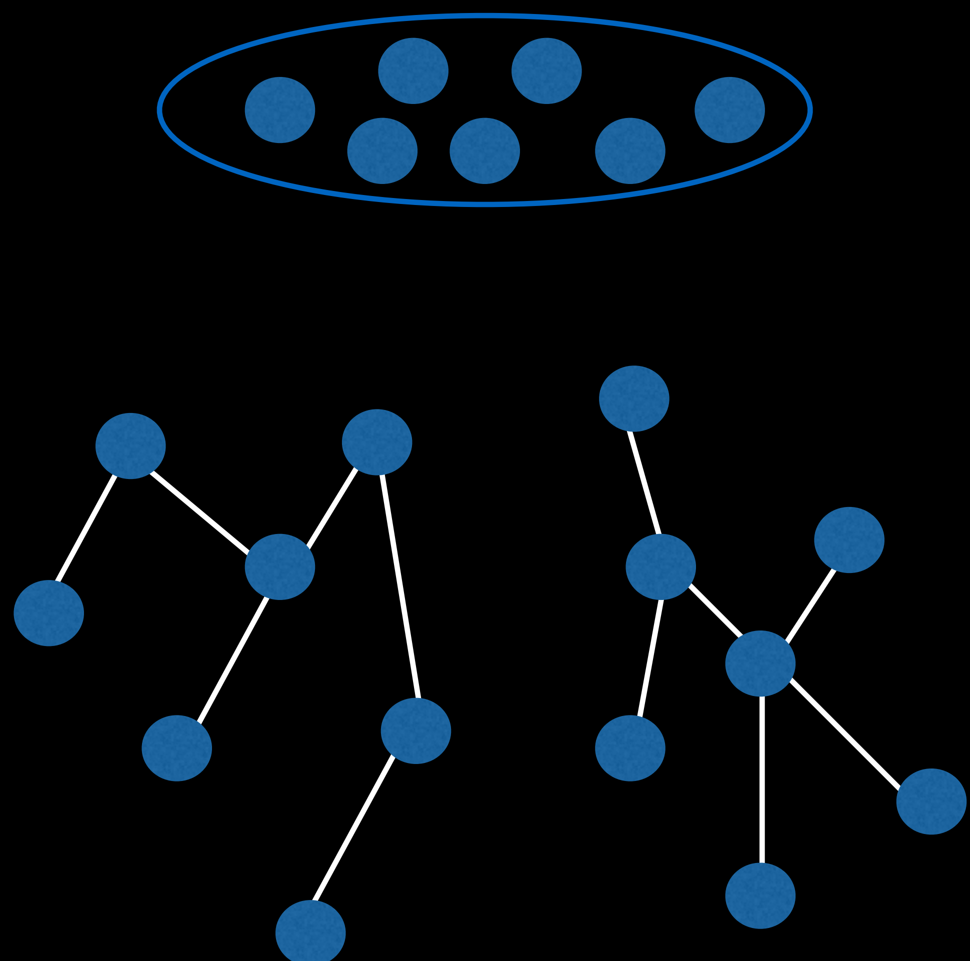
Dynamic Feedback Vertex Set

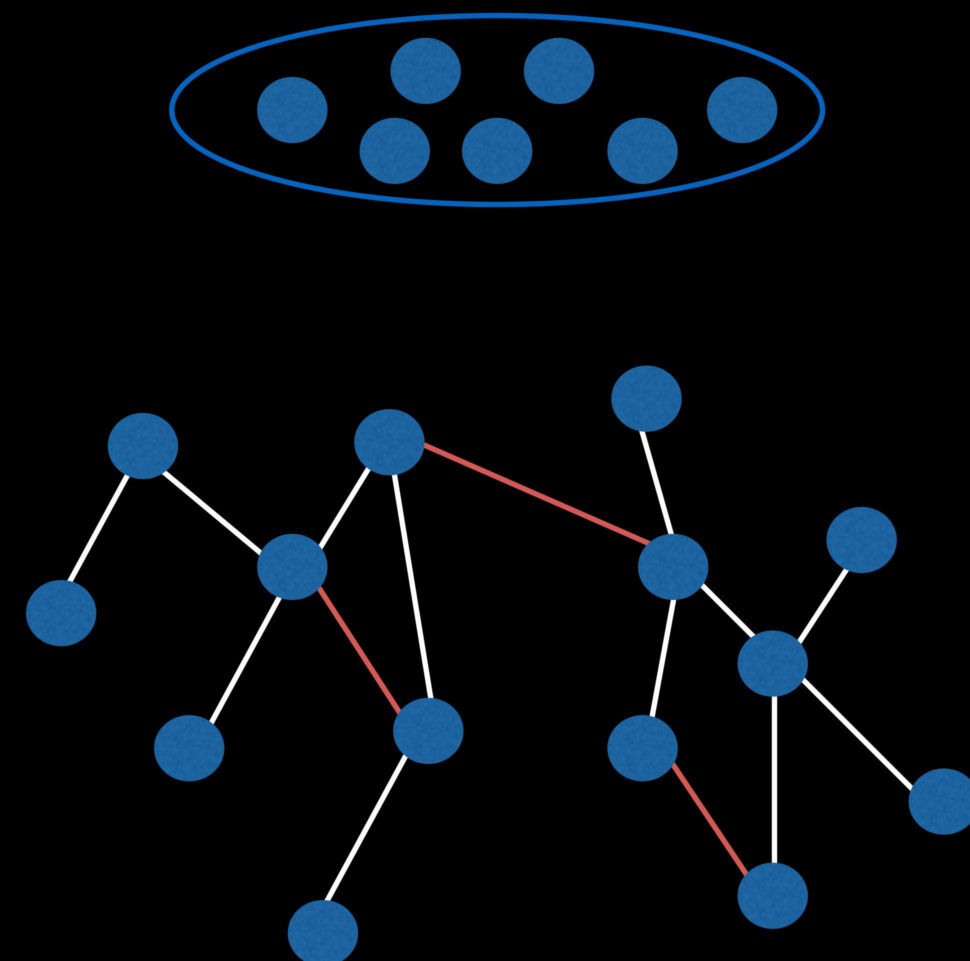


Dynamic Feedback Vertex Set



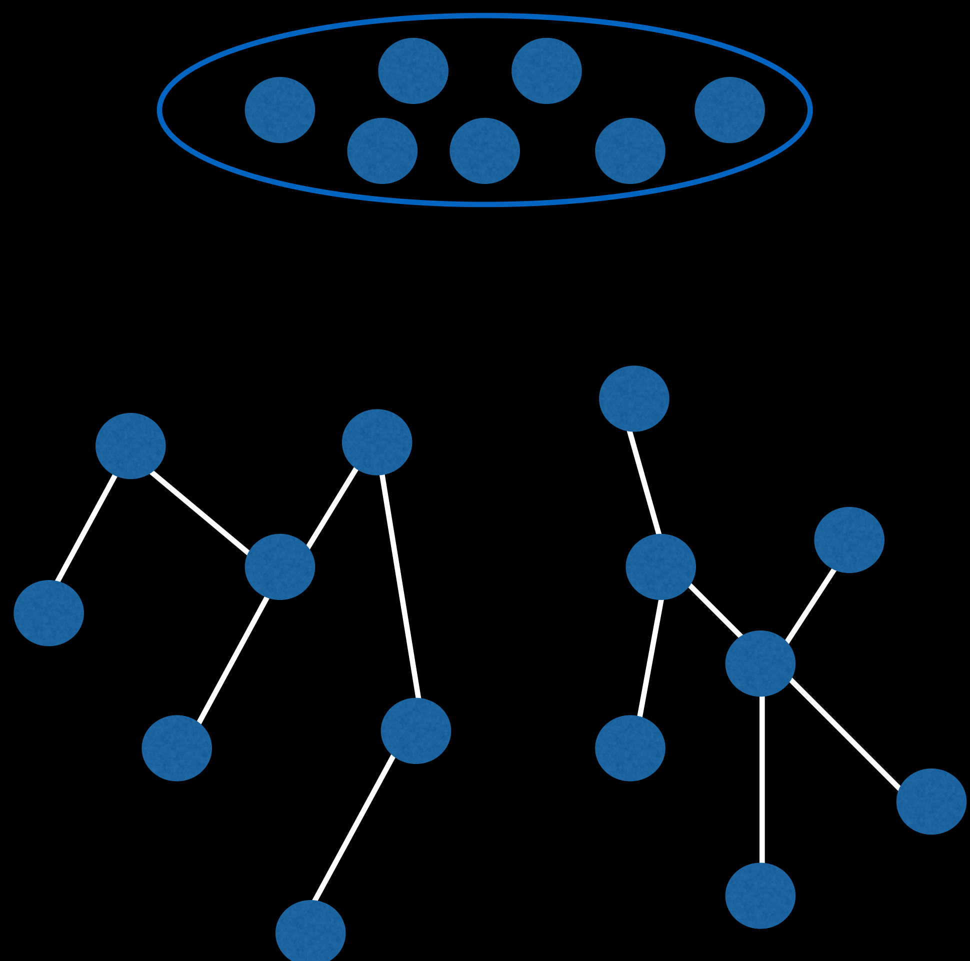
Dynamic Feedback Vertex Set

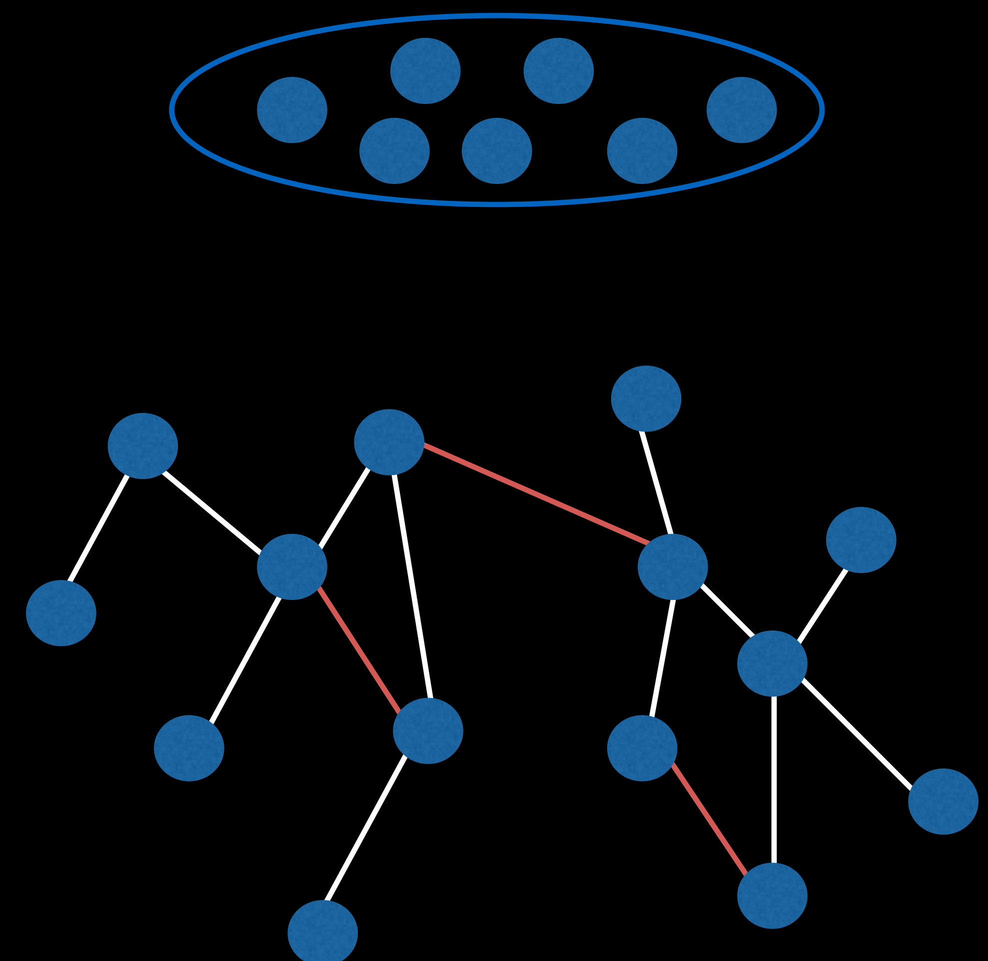




Forest + ≤ k edges

Dynamic Feedback Vertex Set

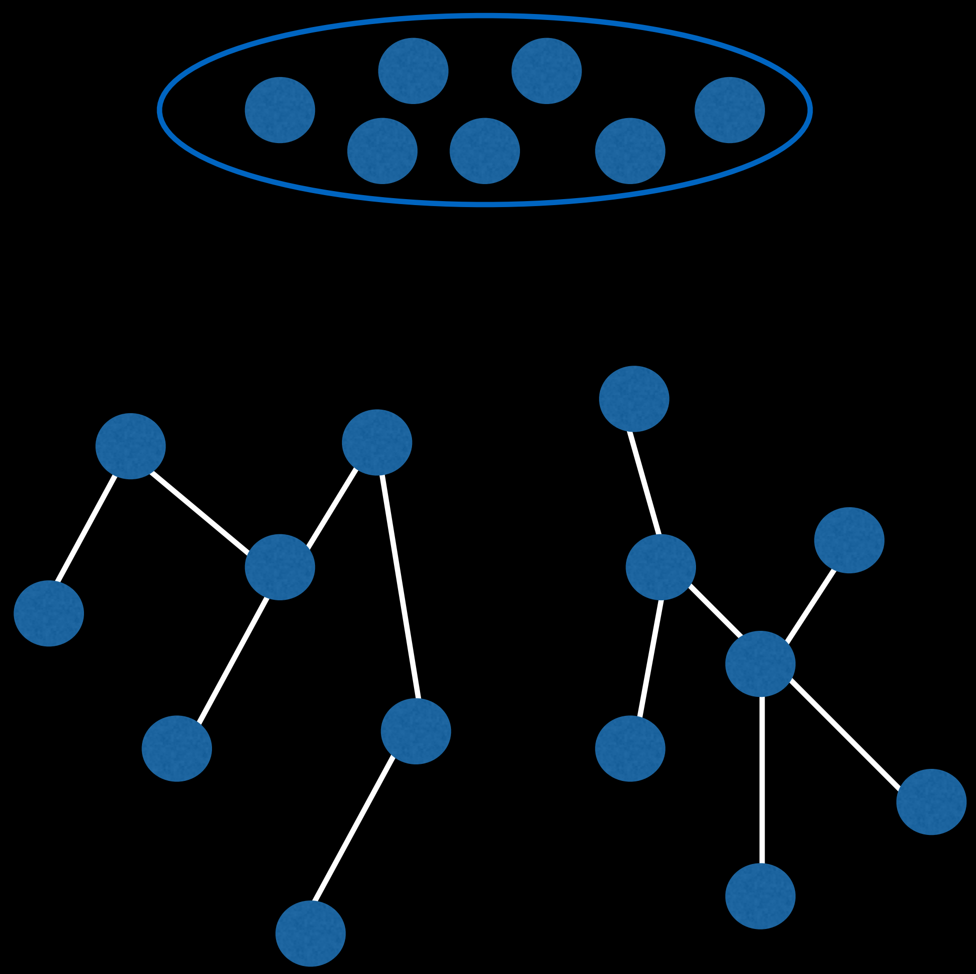


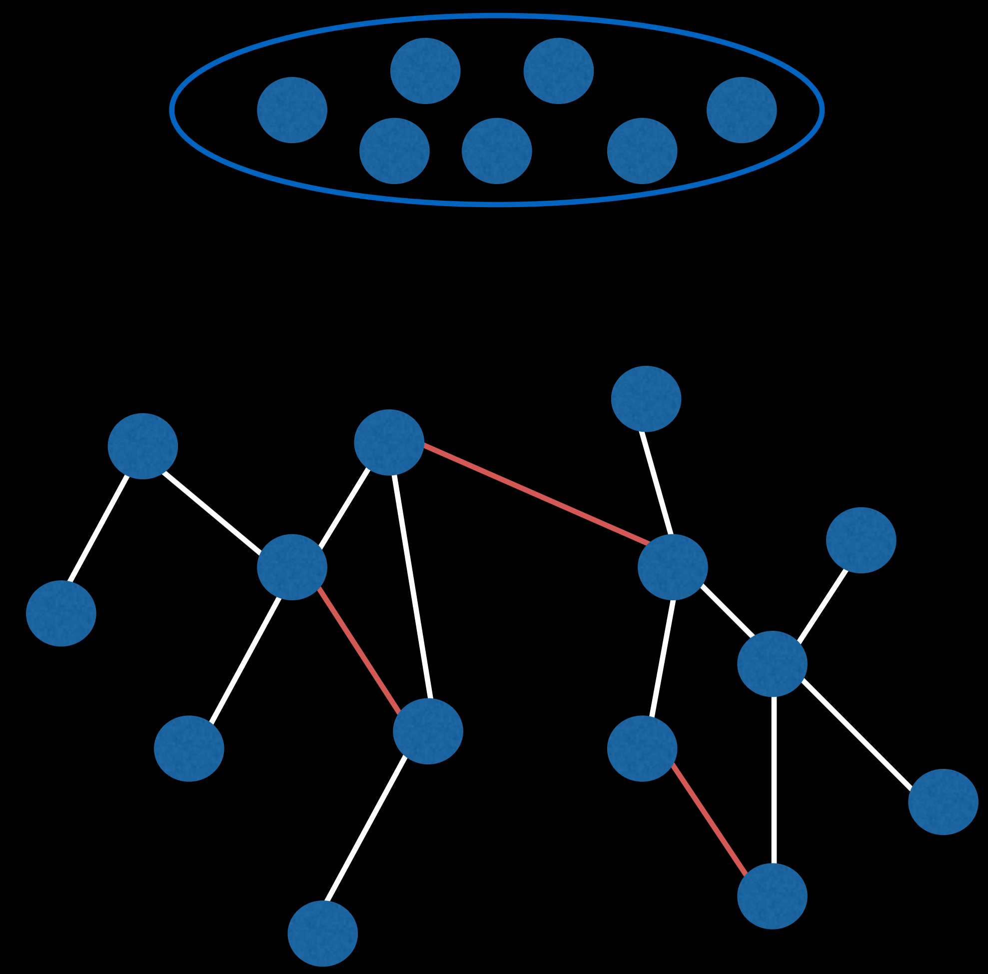


Forest + ≤ k edges

O(k) edges kernel

Dynamic Feedback Vertex Set





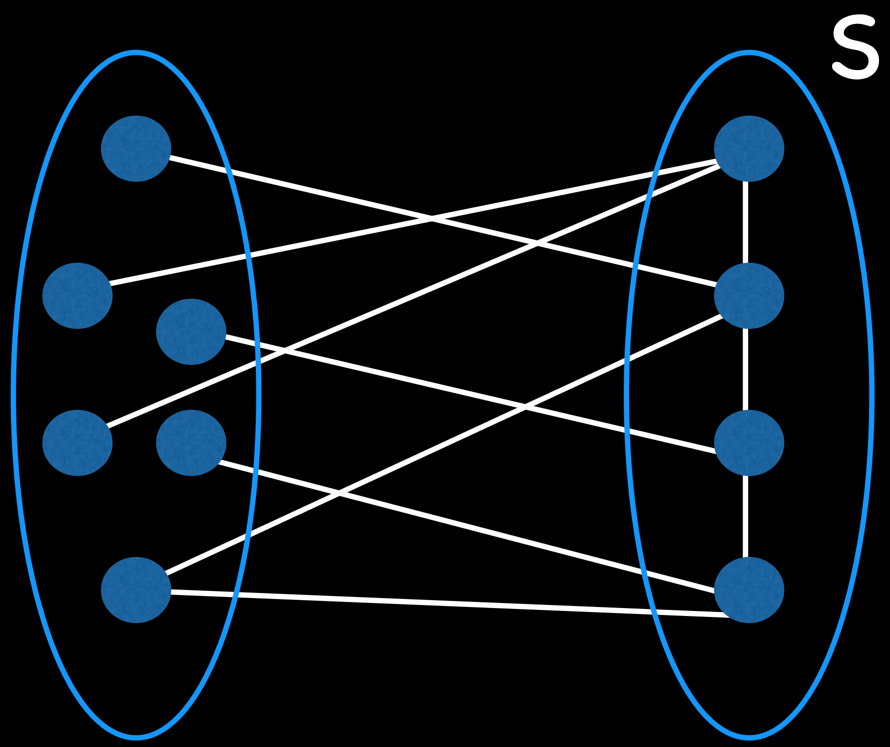
Forest + ≤ k edges

O(k) edges kernel

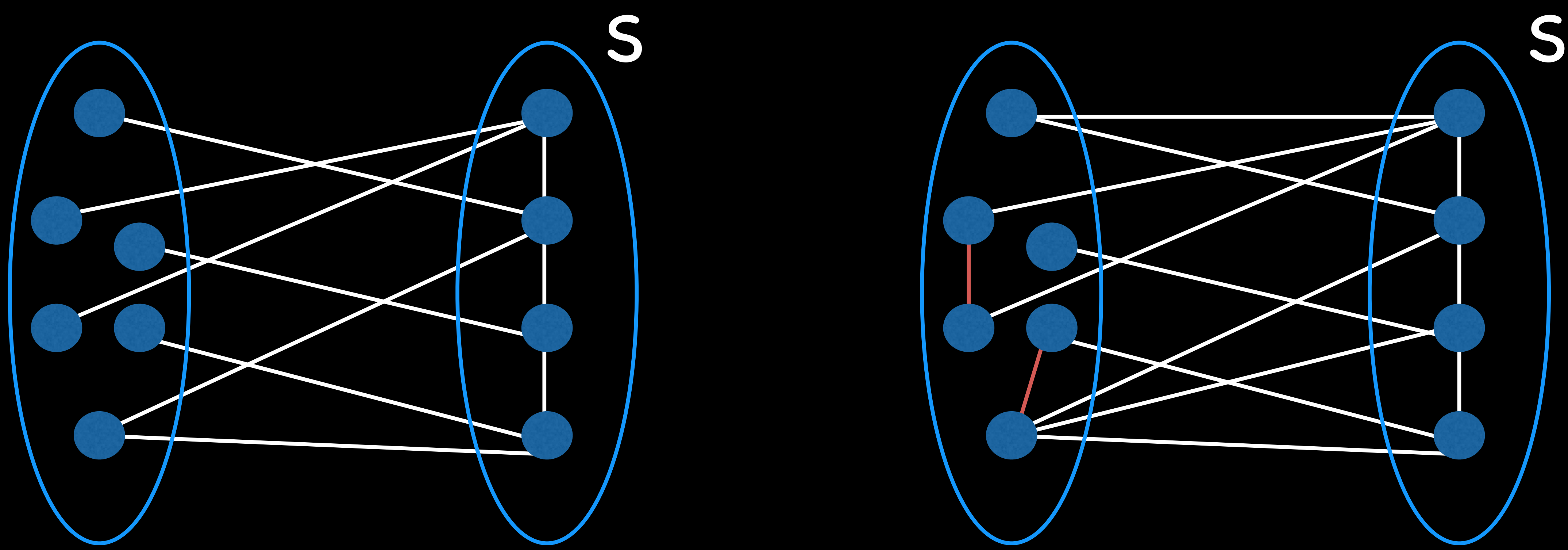
1.6667^k randomized algorithm

Dynamic Vertex Cover

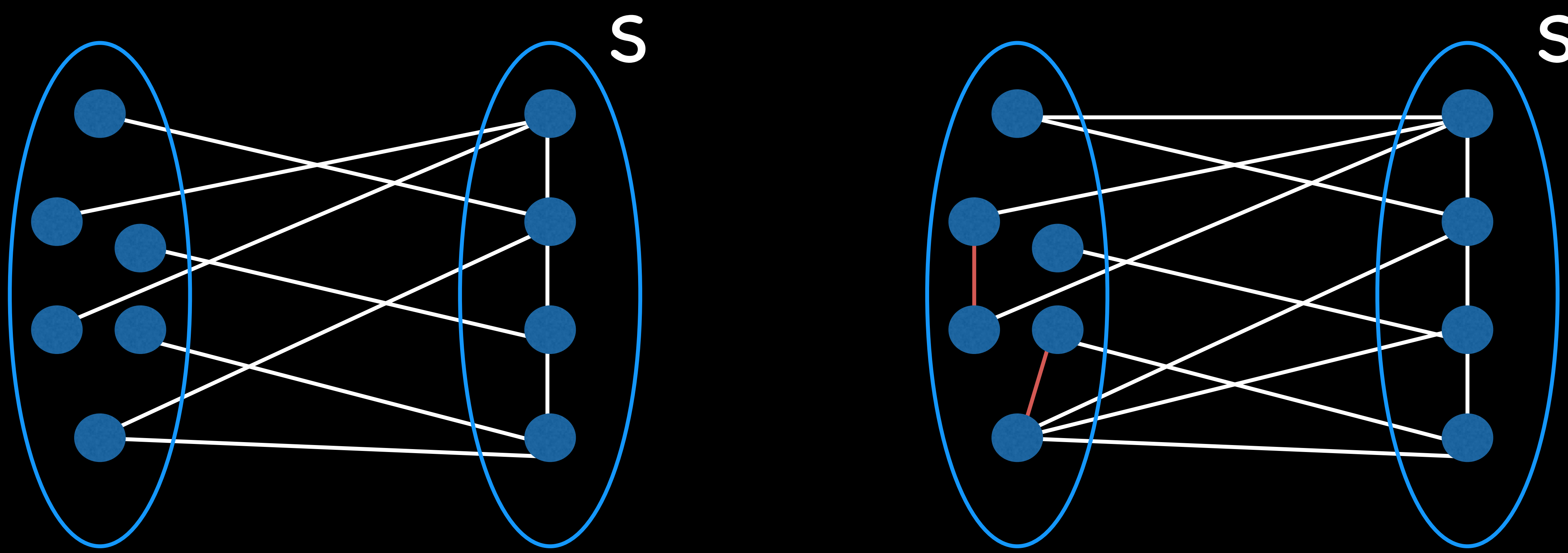
Dynamic Vertex Cover



Dynamic Vertex Cover

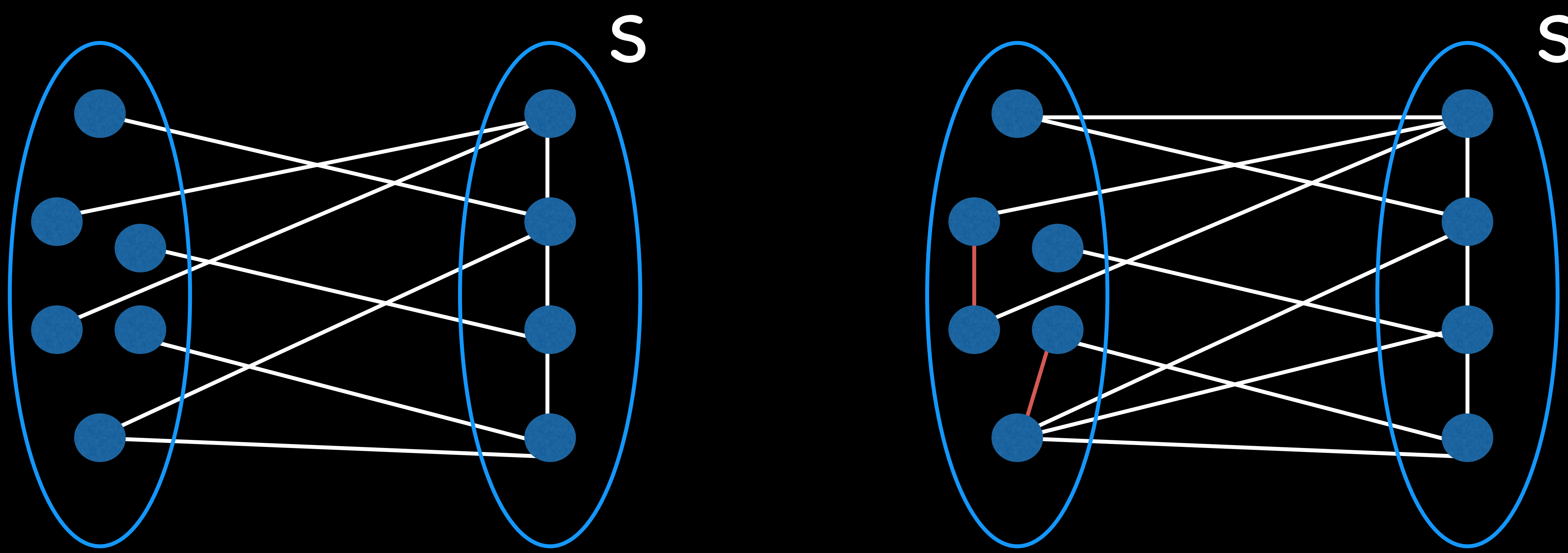


Dynamic Vertex Cover



At most k edges are not covered by S

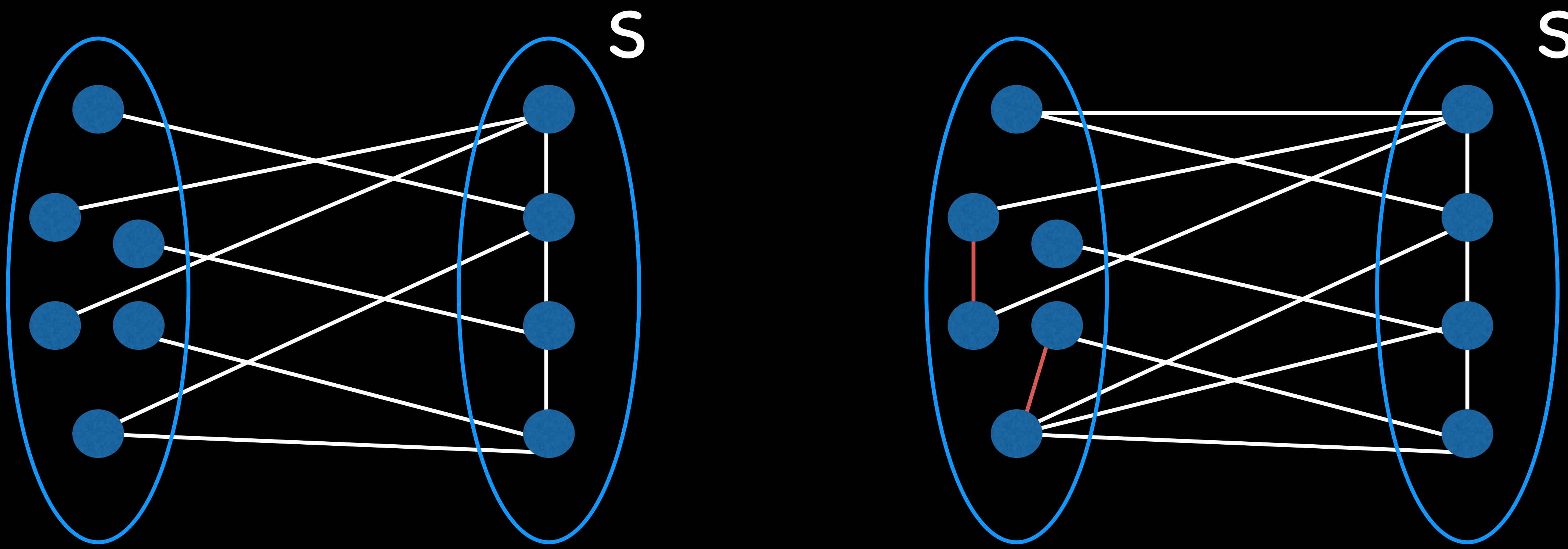
Dynamic Vertex Cover



At most k edges are not covered by S

Graph has at most 2k vertices and k edges

Dynamic Vertex Cover

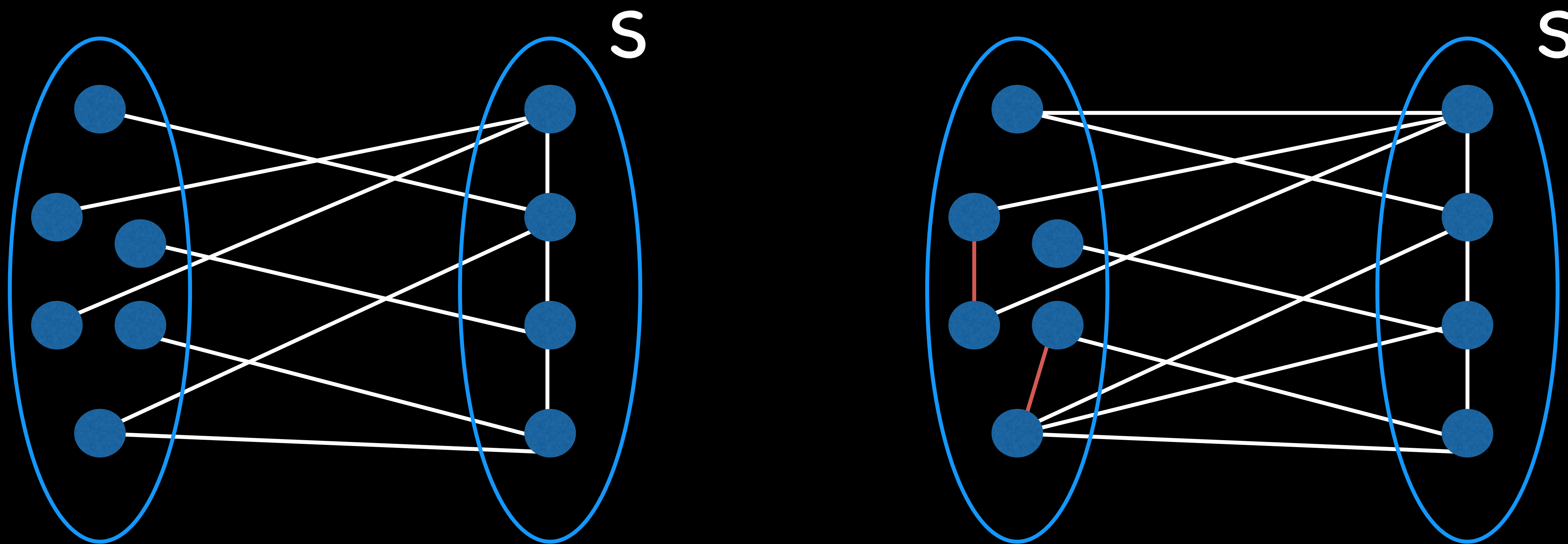


At most k edges are not covered by S

Graph has at most 2k vertices and k edges

$O^*(1.174^k)$ poly space algorithm

Dynamic Vertex Cover



At most k edges are not covered by S

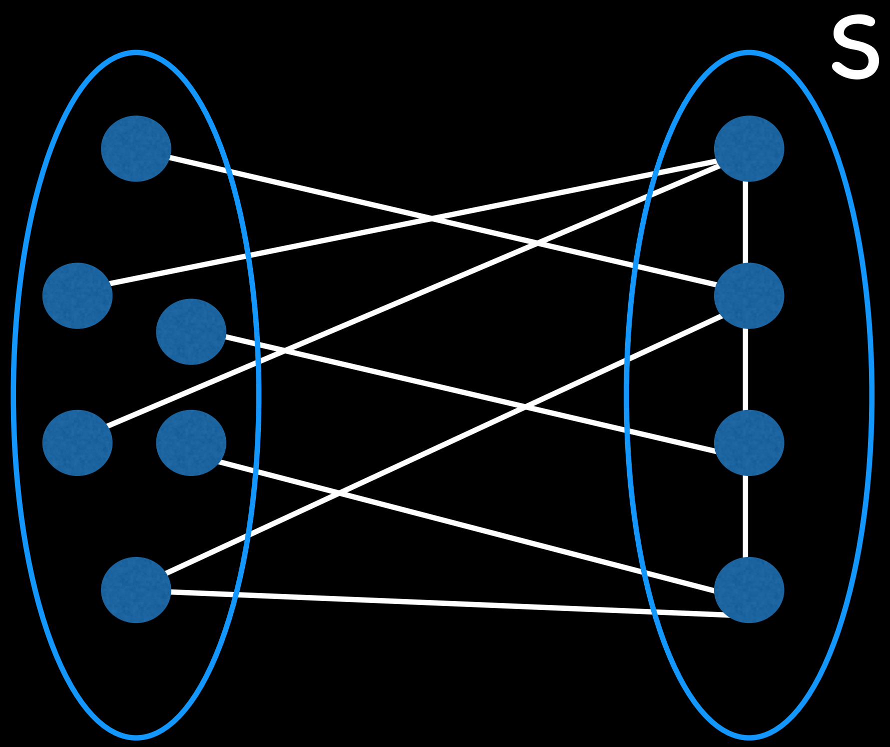
Graph has at most 2k vertices and k edges

$O^*(1.174^k)$ poly space algorithm

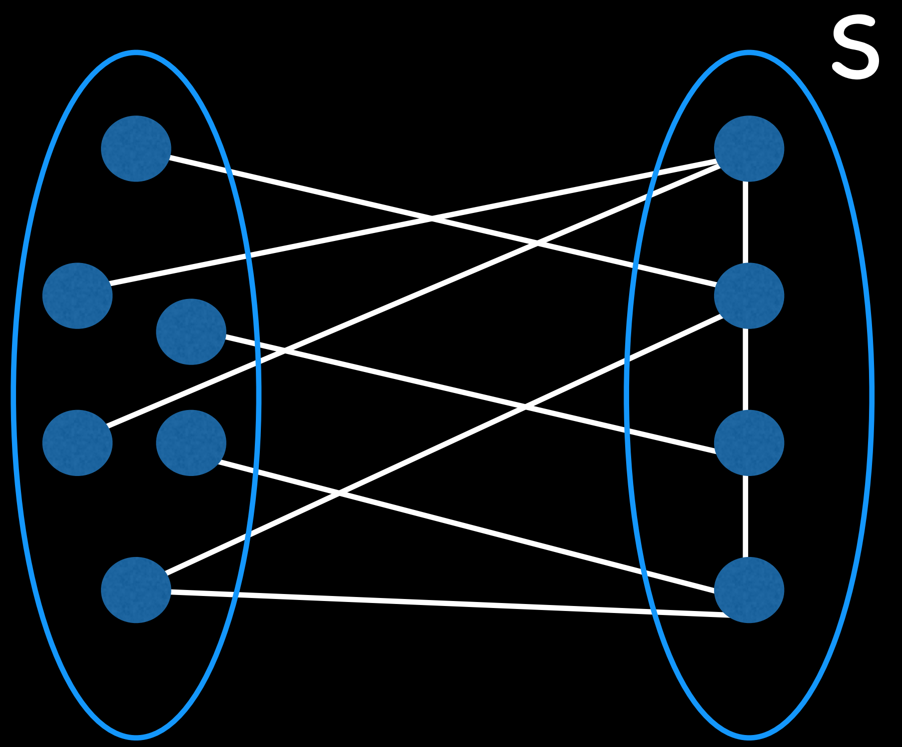
$O^*(1.1277^k)$ expo space algorithm

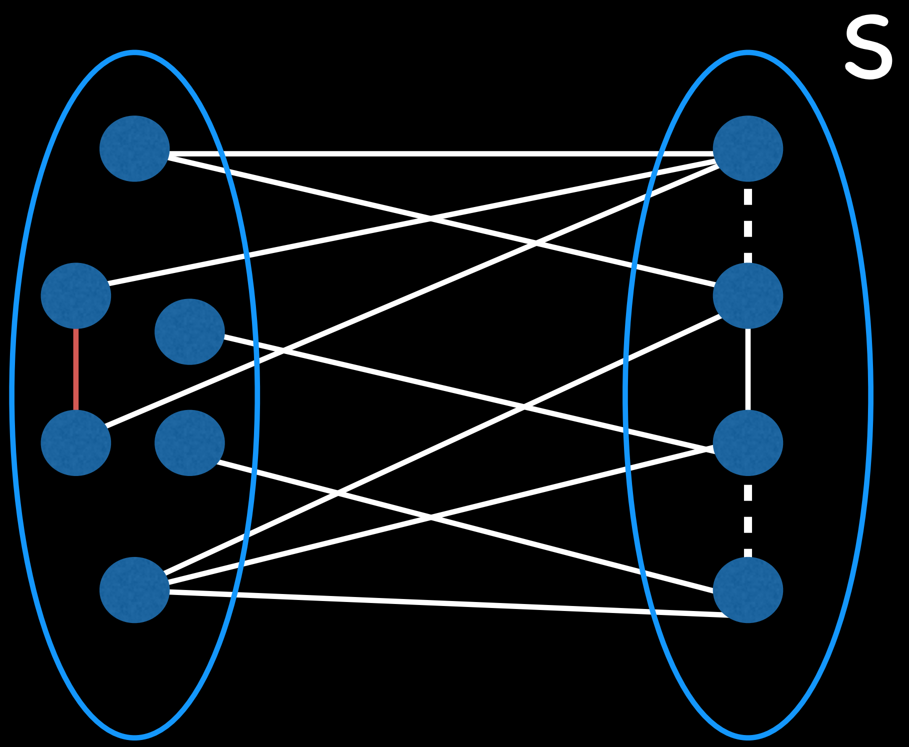
Dynamic Connected Vertex Cover

Dynamic Connected Vertex Cover

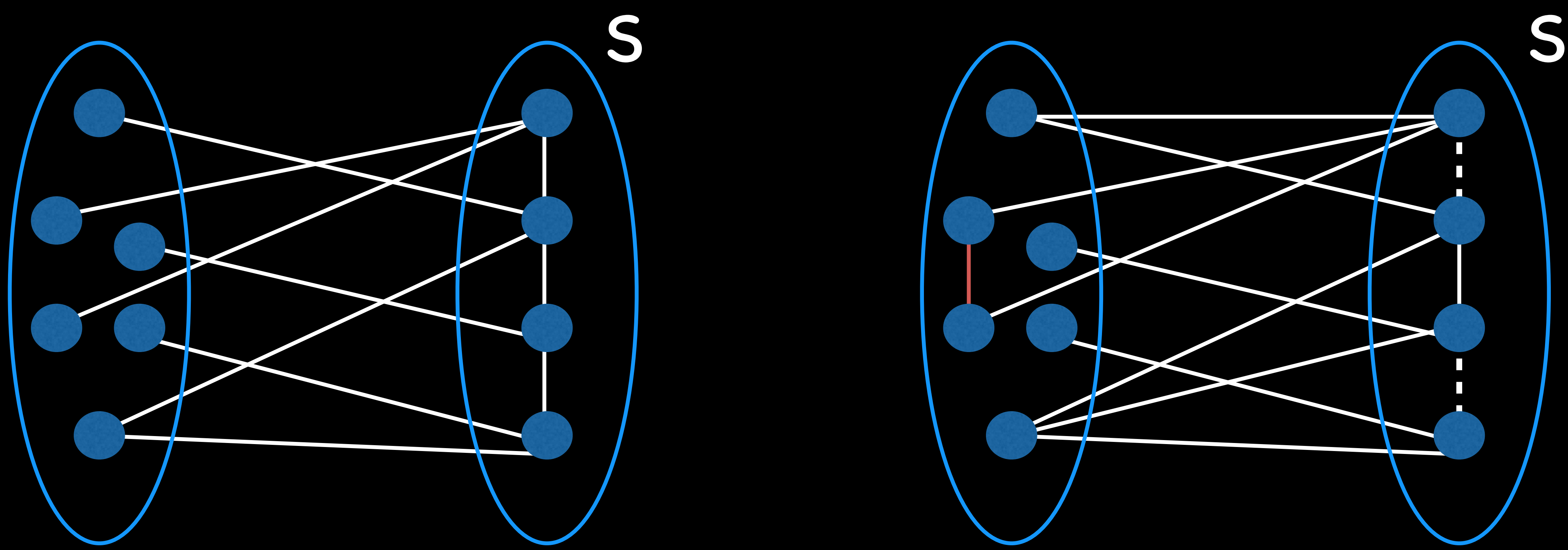


Dynamic Connected Vertex Cover



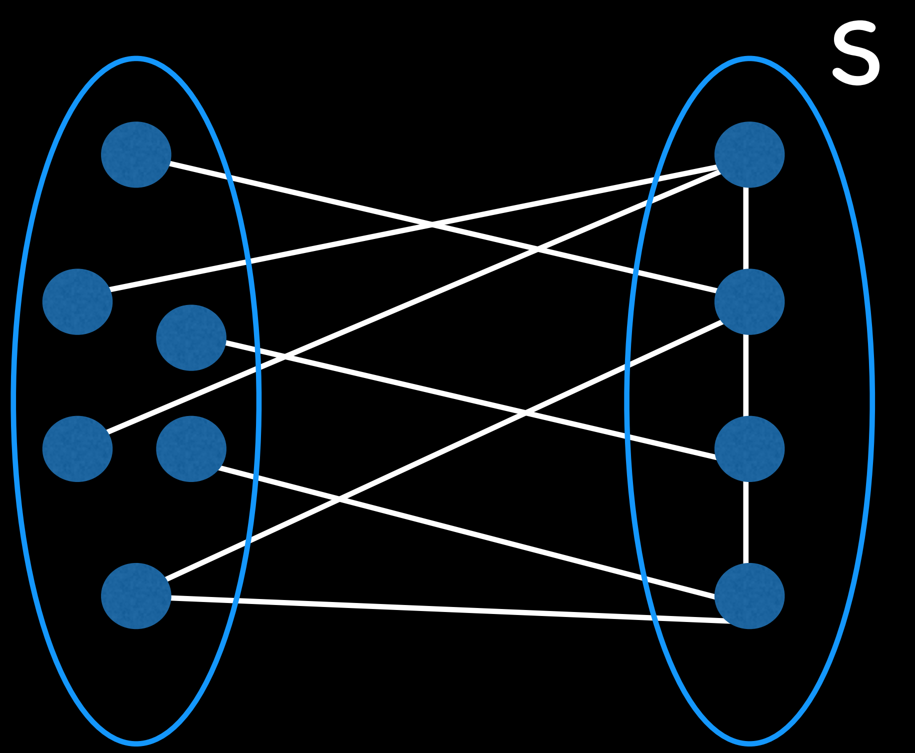


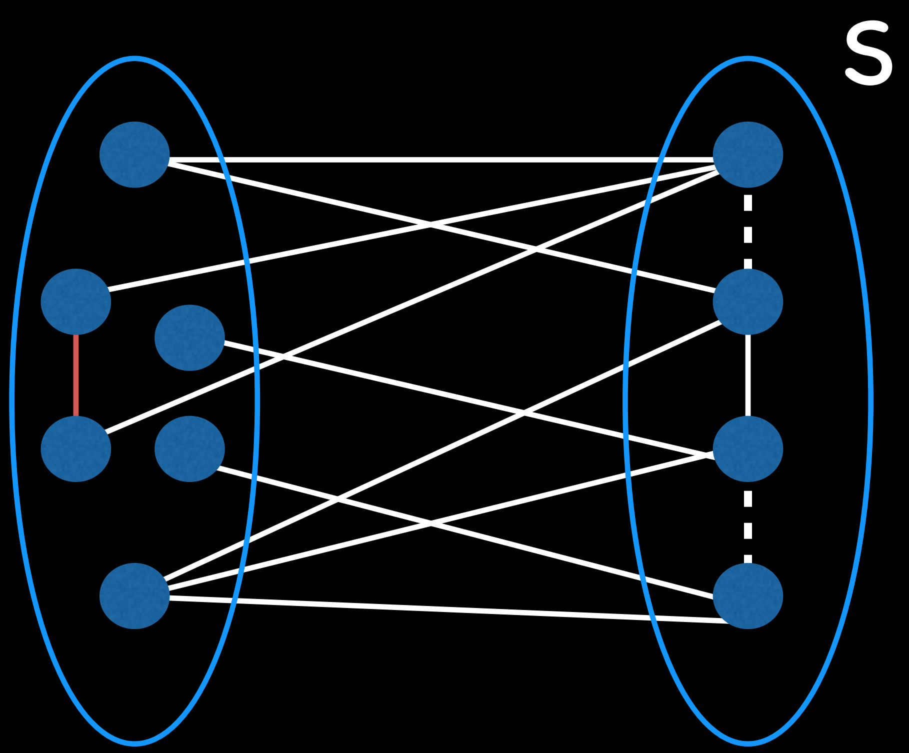
Dynamic Connected Vertex Cover



At most k_1 edges are not covered by S that has at most k_2 components

Dynamic Connected Vertex Cover

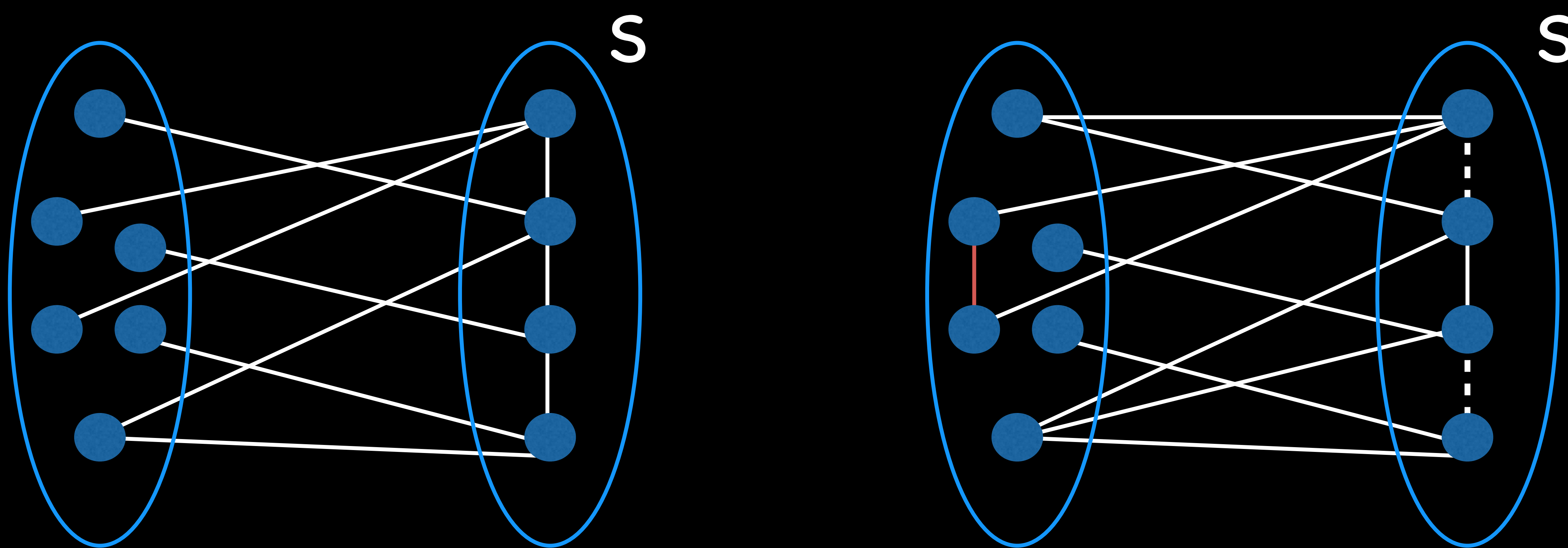




At most k_1 edges are not covered by S that has at most k_2 components

A group Steiner tree problem with parameter $k_1+k_2 \leq k$

Dynamic Connected Vertex Cover



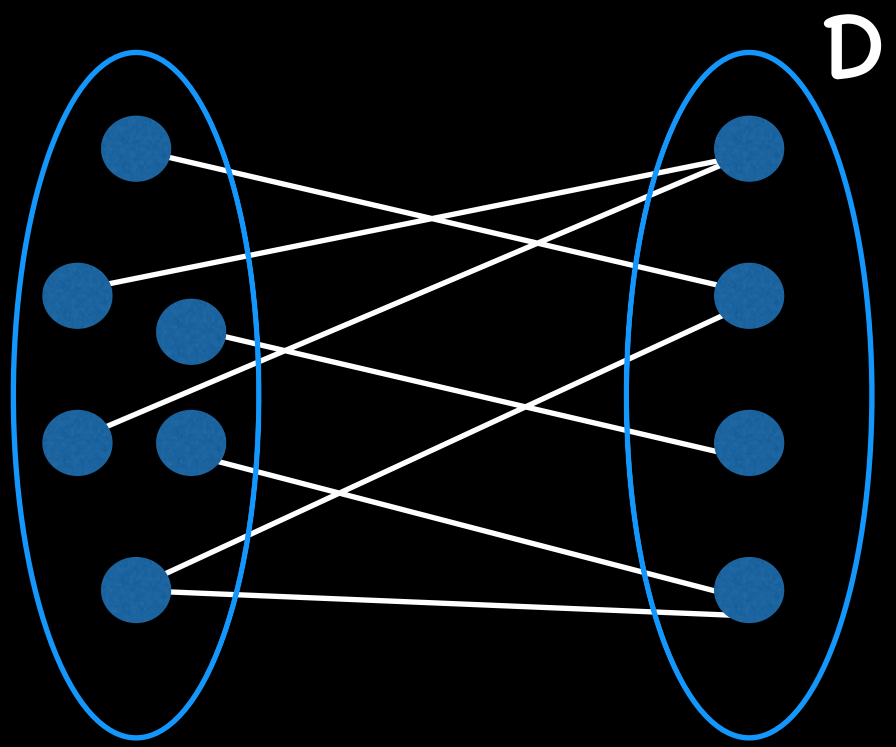
At most k_1 edges are not covered by S that has at most k_2 components

A group Steiner tree problem with parameter $k_1+k_2 \leq k$

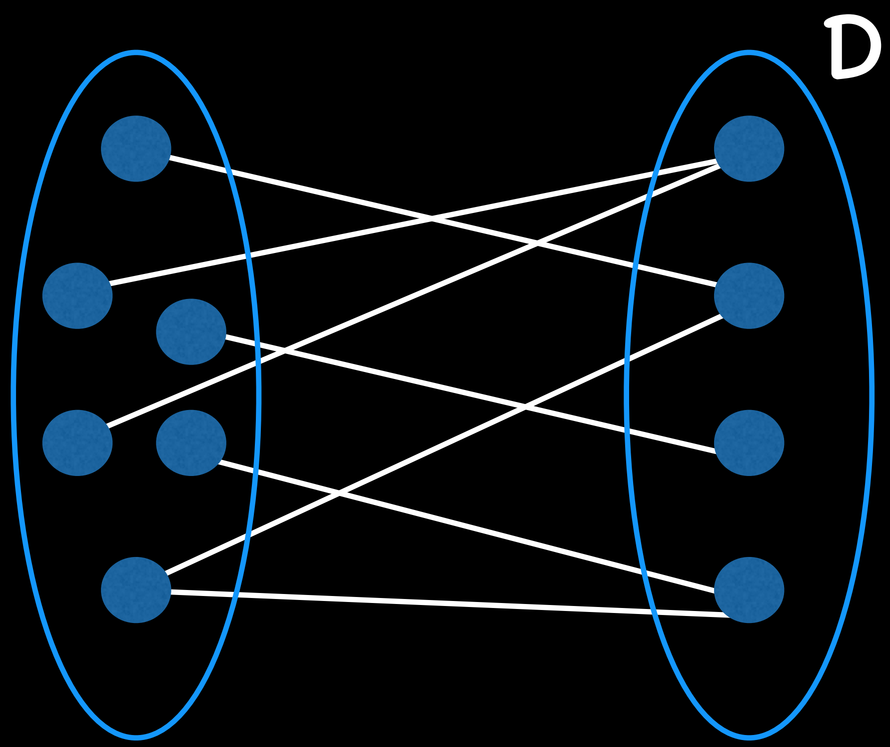
$O^*(2^k)$ algorithm

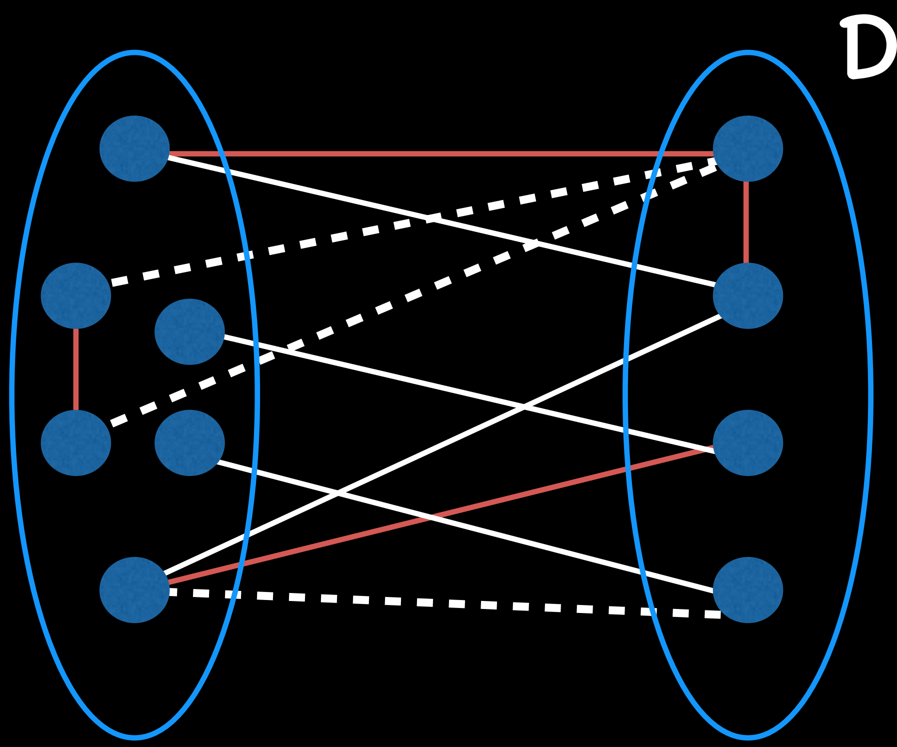
Dynamic Dominating Set

Dynamic Dominating Set

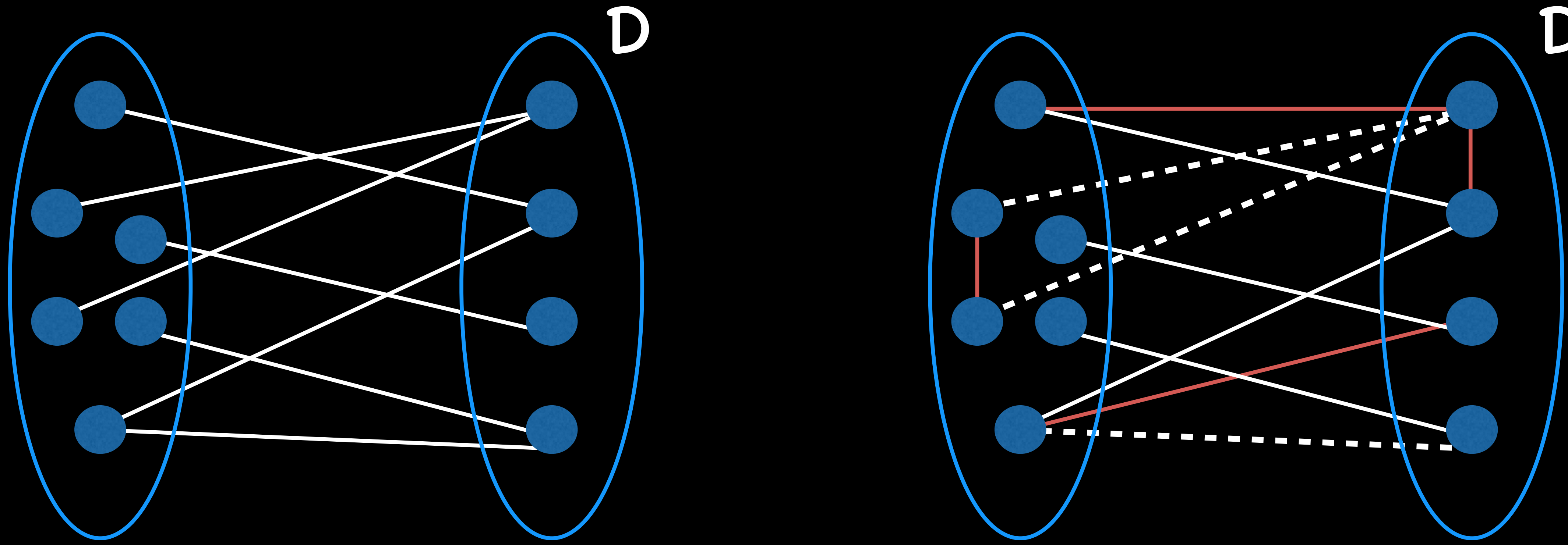


Dynamic Dominating Set



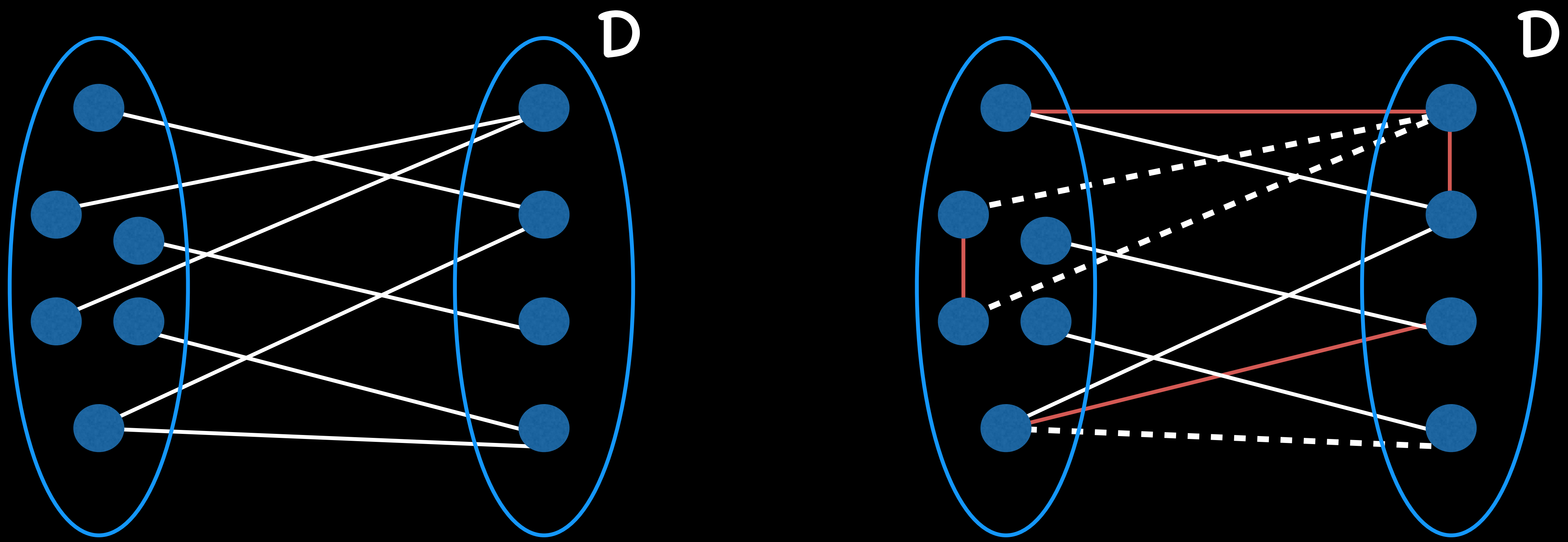


Dynamic Dominating Set



At most k vertices are not dominated by D

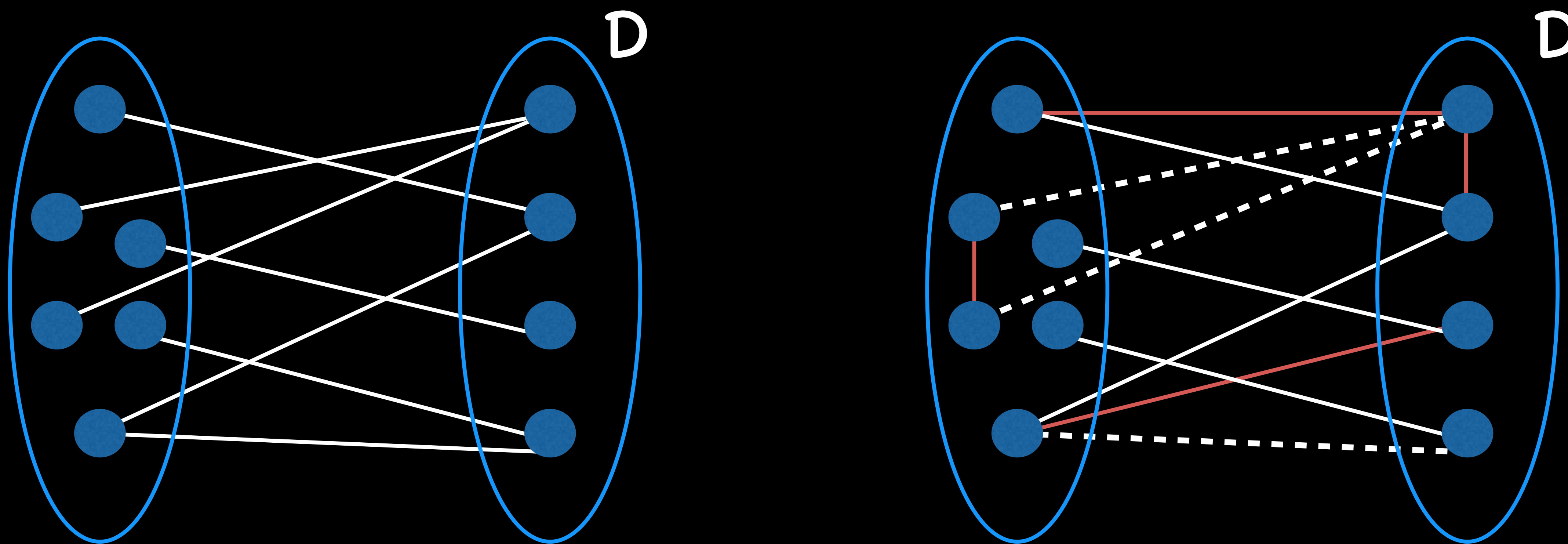
Dynamic Dominating Set



At most k vertices are not dominated by D

A set cover instance on k-element universe

Dynamic Dominating Set

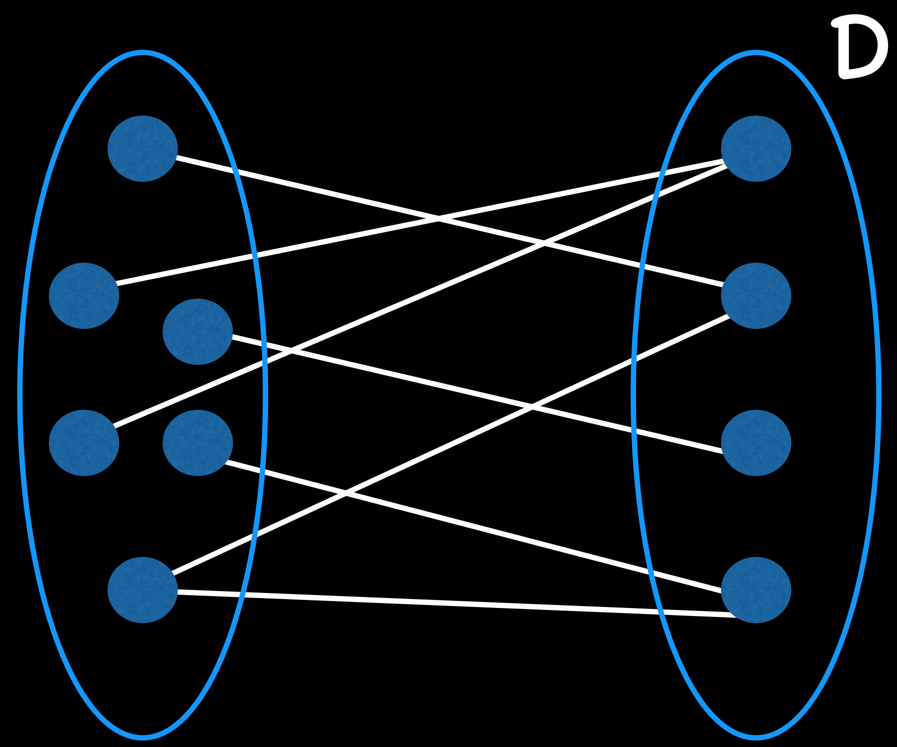


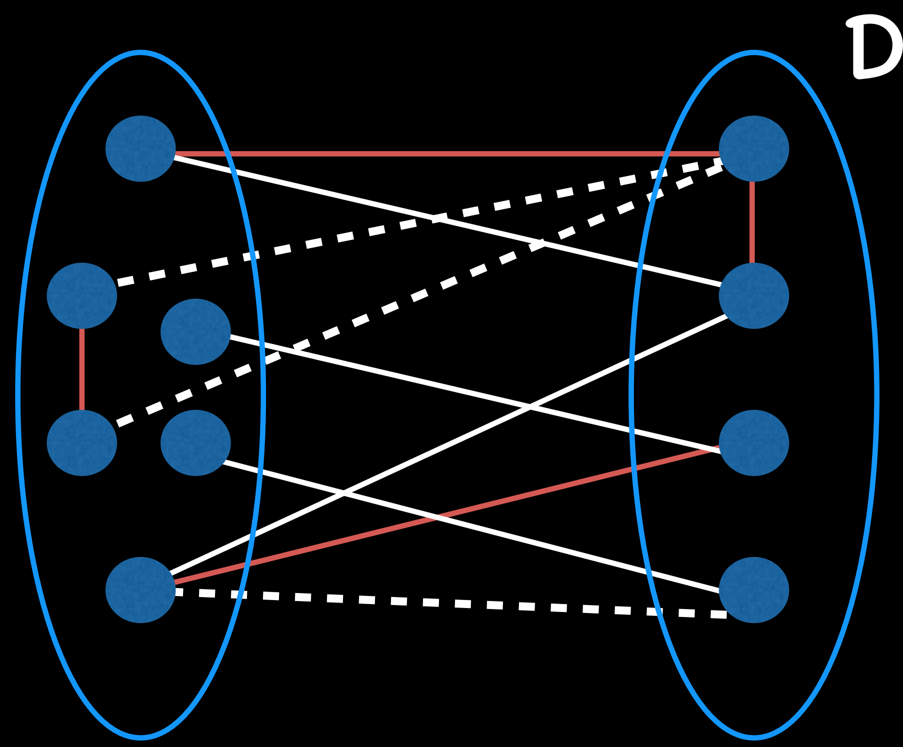
At most k vertices are not dominated by D

A set cover instance on k-element universe

$O^*(2^k)$ algorithm

Dynamic Dominating Set





At most k vertices are not dominated by D

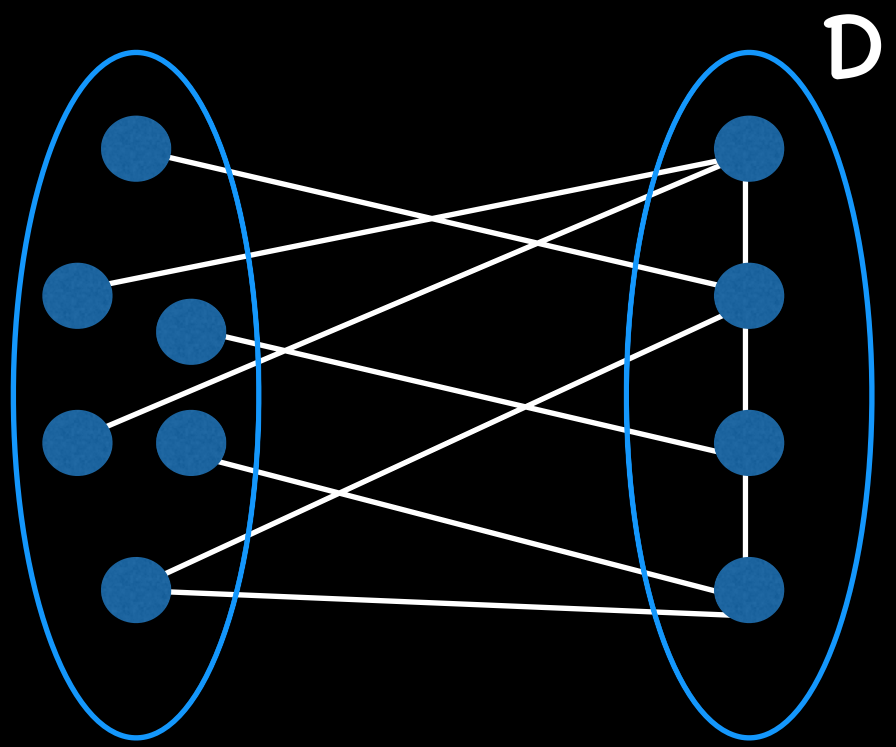
A set cover instance on k-element universe

$O^*(2^k)$ algorithm

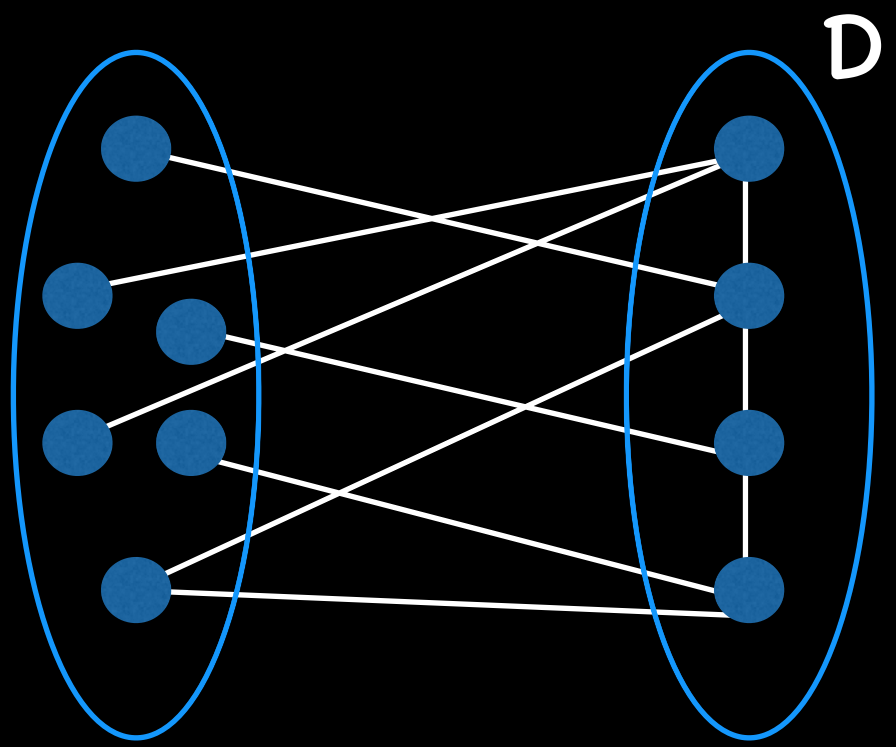
Tight under the Set Cover Conjecture

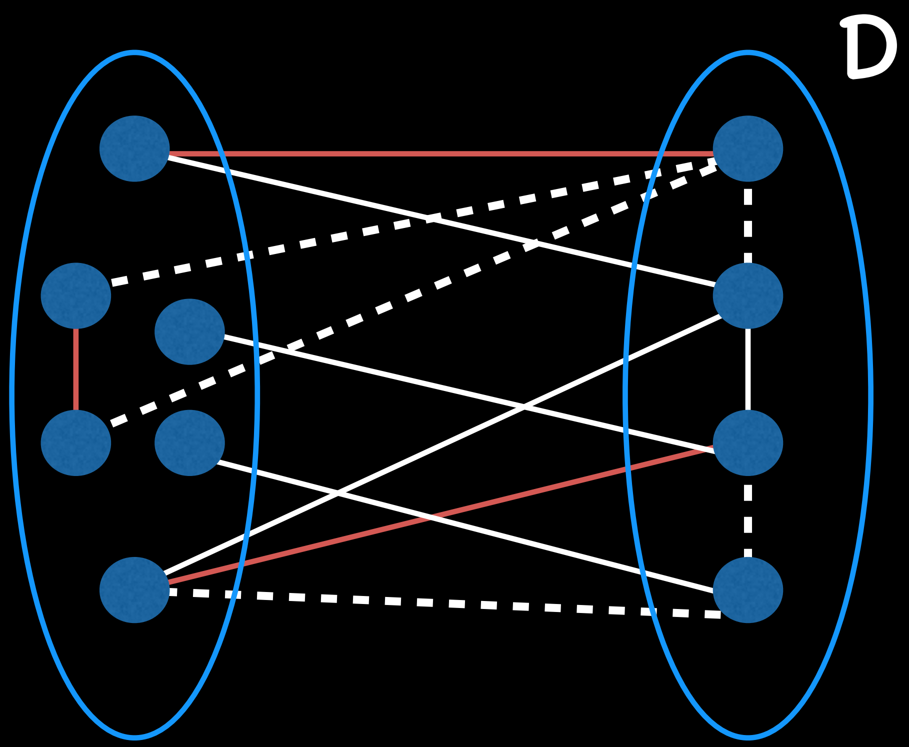
Dynamic Connected Dominating Set

Dynamic Connected Dominating Set

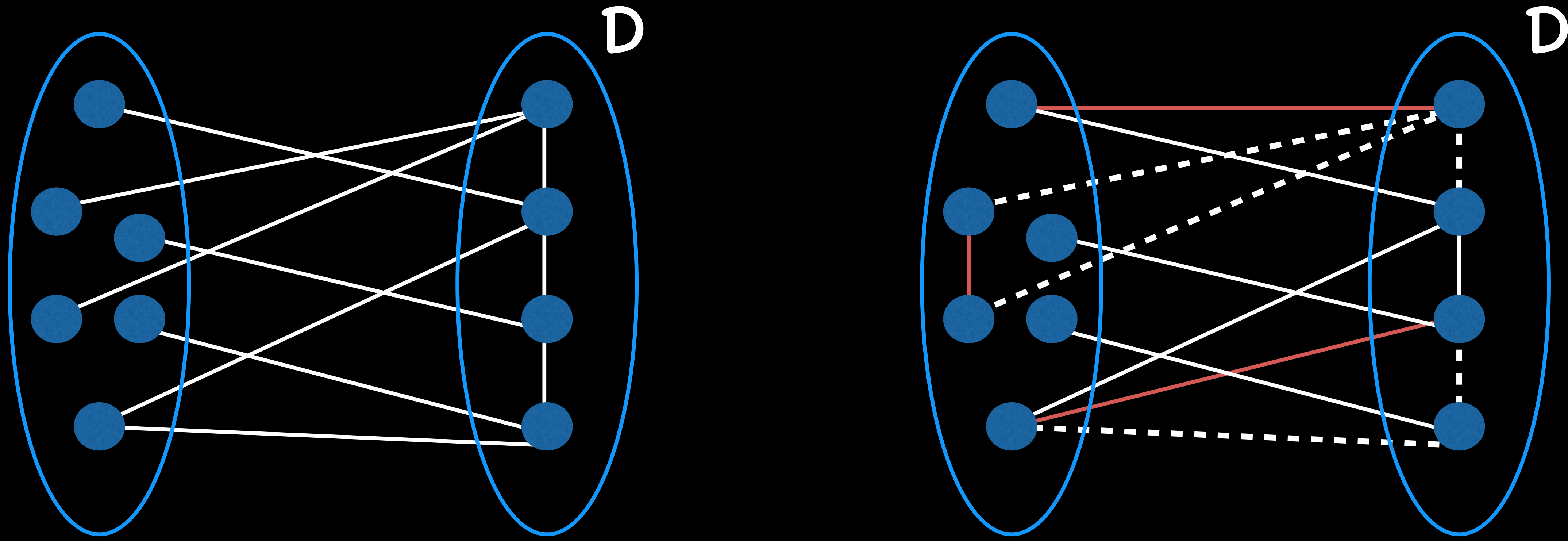


Dynamic Connected Dominating Set



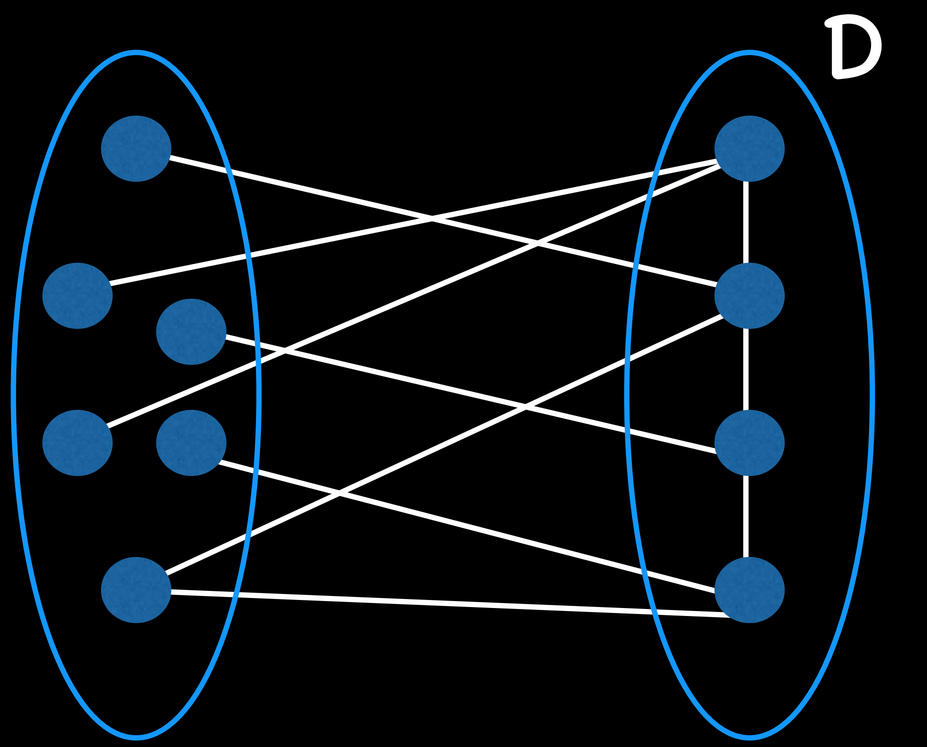


Dynamic Connected Dominating Set



At most k_1 vertices are not dominated by D that has at most k_2 components

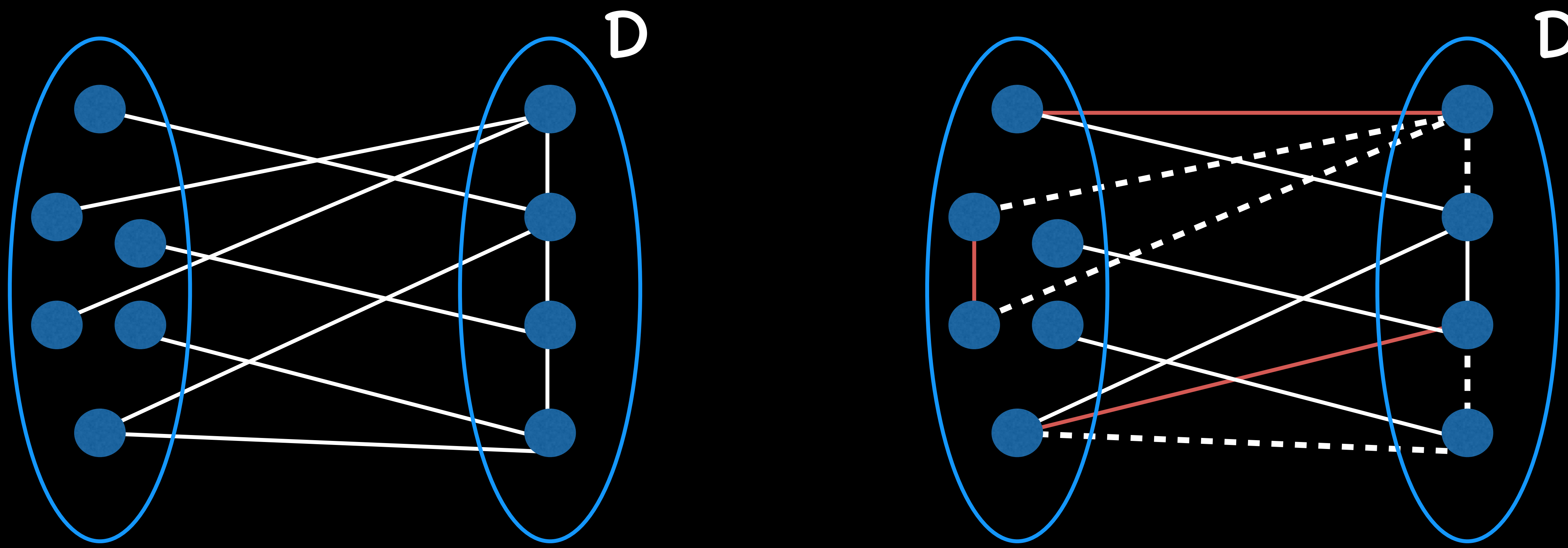
Dynamic Connected Dominating Set



At most k_1 vertices are not dominated by D that has at most k_2 components

A group Steiner tree problem with parameter $k_1+k_2 \leq k$

Dynamic Connected Dominating Set



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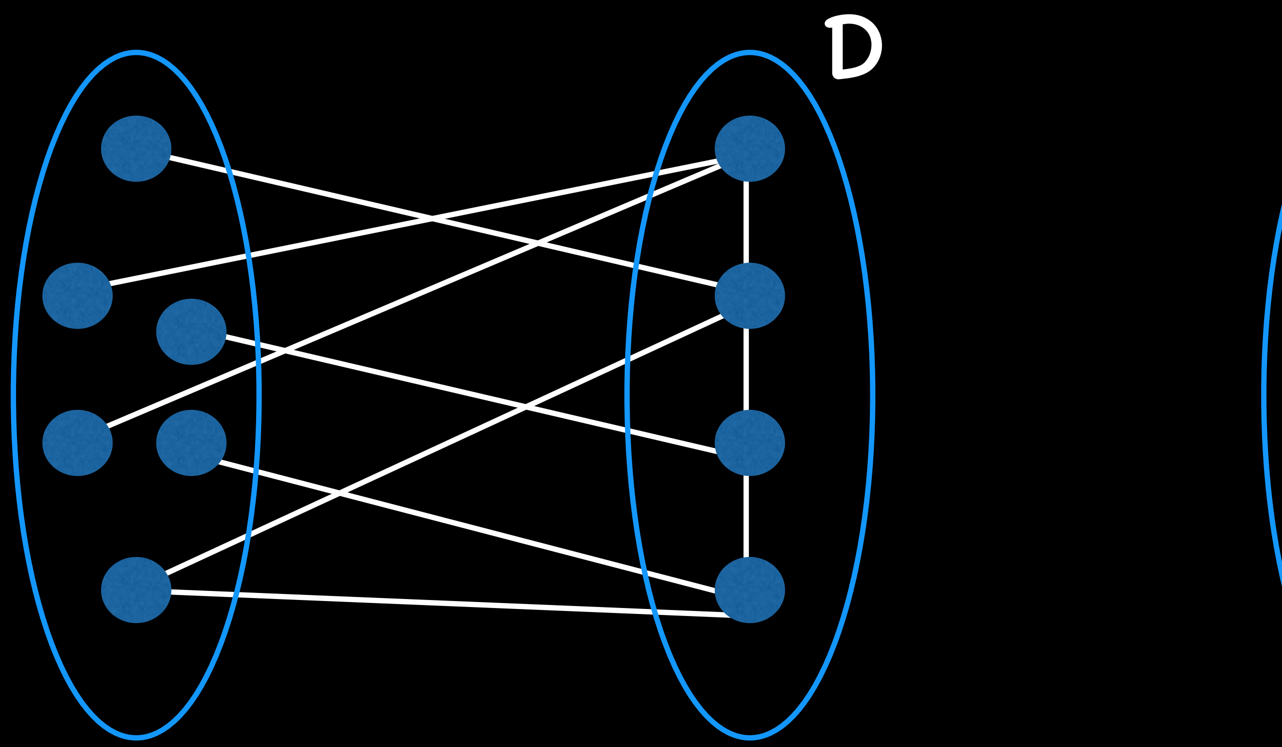
$O^*(2^k)$ algorithm

Dynamic Connected Dominating Set



At most k_1 vertices are not dominated by D that has at most k_2 components

A group Steiner tree problem with parameter $k_1+k_2 \leq k$

$O^*(2^k)$ algorithm

Tight under the Set Cover Conjecture

Concluding Remarks

- Viewed as extending partial solutions
- Other interesting parameters
  - $=k_1$ edge additions and $=k_2$ edge deletions
  - treewidth, vertex cover
- Relation to reconfiguration problems and online problems
- Interesting data structures

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Thank you :)

Questions?