An FPT Algorithm for Contraction to Cactus

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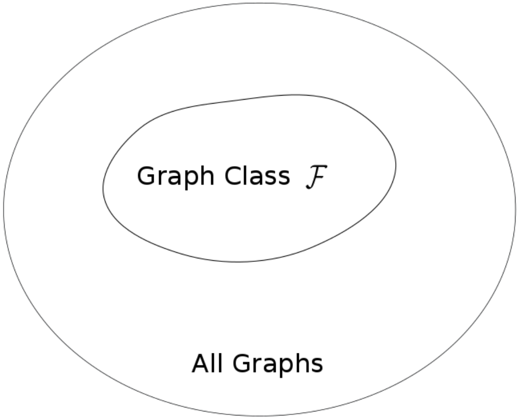
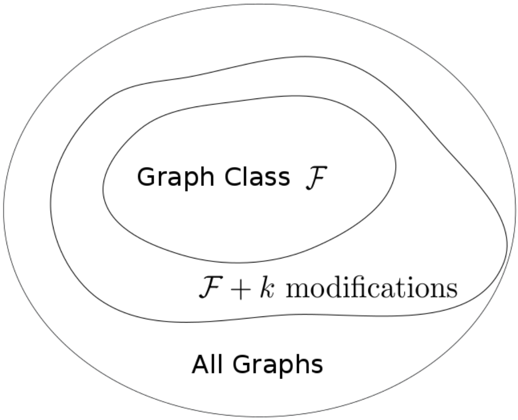

Graph Class $\mathcal{F}$
All Graphs


Graph Class $\mathcal{F}$
$\mathcal{F} + k$ modifications
All Graphs

Graph Modification Problems

$\mathcal{F}$-Modification
Input: A graph G
Question: Can we obtain a graph in $\mathcal{F}$ by *some* modifications in the graph G?

Modification allowed

- Vertex Deletion
- Edge Deletion
- Edge Addition
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Graph Problem	$\mathcal{F}$	Modification
Vertex Cover	Empty graphs	Vertex Deletion
Feedback vertex set	Forests	Vertex Deletion
Odd cycle transversal	Bipartite Graphs	Vertex Deletion
Minimum Fill-In	Chordal Graphs	Edge Addition
Edge Bipartization	Bipartite Graphs	Edge Deletion
Cluster Editing	Cluster Graphs	Edge Addition & Deletion
Tree Contraction	Trees	Edge Contraction

Outline

Parameterized Complexity & Contraction Problems

Graph Contraction Problems

Problem Definition

FPT Algorithm

Parameterized Complexity & Contraction Problems

Parameterized Complexity : Quick Overview

- Goal : Find **better** ways to solve NP-hard problems.
- Associate (*small*) parameter k to each instance I.
- Restrict the combinatorial explosion to the parameter k.
- Parameterized problem (I, k) is *fixed-parameter tractable* (FPT) if there is an algorithm that solves it in time $\mathcal{O}(f(k) \cdot |I|^{\mathcal{O}(1)})$.
- Not all problems (for given parameter) admit such an algorithm Hierarchy of classes : FPT $\subseteq$ W[1] $\subseteq$ W[2] $\ldots$

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Graph Contraction Problems

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$\mathcal{F}$ is a graph class and G/F is graph obtained from G by contracting edges in F

$\mathcal{F}$-CONTRACTION	**Parameter:** k

Input: A graph G and an integer k
Question: Does there exist $F \subseteq E(G)$ of size at most k such that G/F is in $\mathcal{F}$?

$\mathcal{F}$ is polynomial time recongnizable graph class.

$\mathcal{F}$-Contraction: Parameterized Complexity

[HvtHL+12]	Tree Contraction	4^k
	Path Contraction	$2^{k+o(k)}$
[GvtHP13]	Planar Contraction	FPT
[CG13]	Clique Contraction	$2^{\mathcal{O}(k \log k)}$
[HvtHLP13]	Bipartite Contraction	FPT
[GM13]		$2^{\mathcal{O}(k^2)}$
[GKPT13]	$\mathcal{F}_{mindeg \geq d}$Contraction	FPT (k, d)

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- *[ALSZ17]* $\mathcal{F}$ *is Split Graphs*

Starting Point

$\mathcal{F}$-Contraction is FPT when $\mathcal{F}$ is Trees (which are C_3-free) but $W[2]$-hard when $\mathcal{F}$ is family of C_ℓ-free graphs ($\ell \geq 4$).

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What are the superclasses of Tree which admits an FPT algorithm?

Problem Definition

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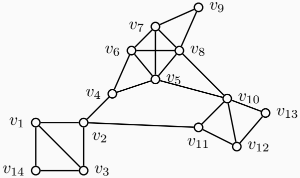
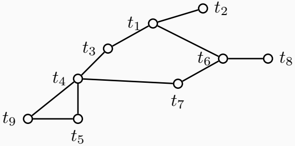
  Every graph mentioned here is 2-connected.

Contraction as a Partition Problem

$\mathcal{F}$-Contraction as a Partition Problem





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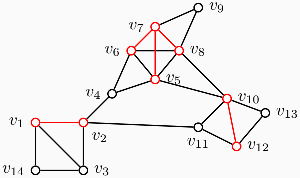

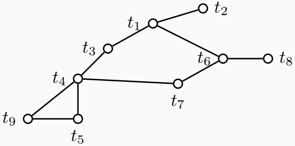

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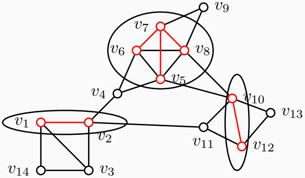
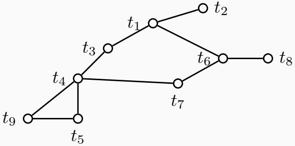
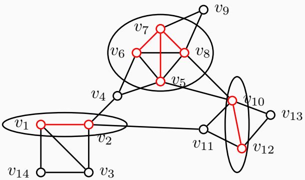
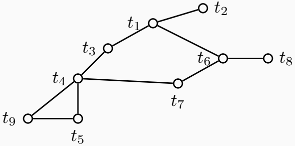




G is contractible to T if there exists a partition of $V(G)$ into

Witness Structure : Definition





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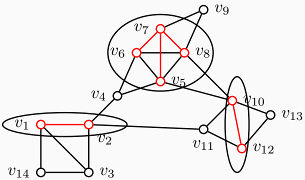

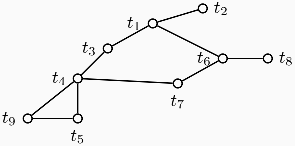

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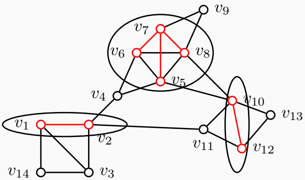

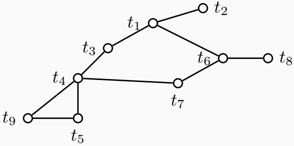

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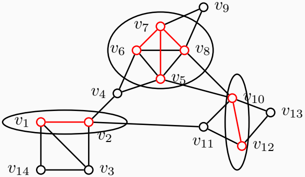

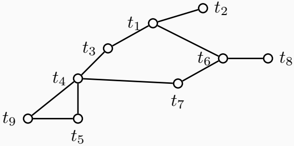

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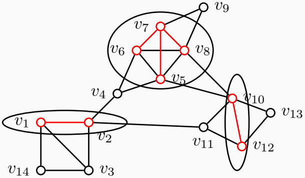

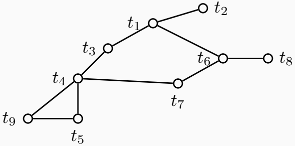

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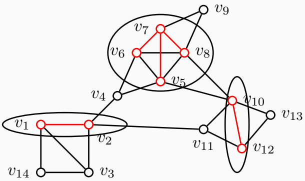

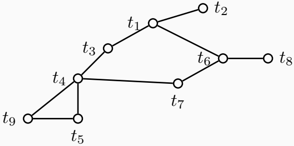

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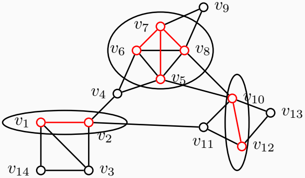

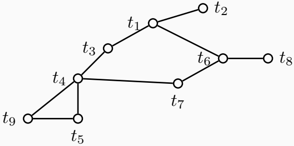

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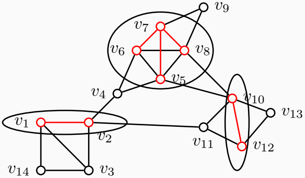
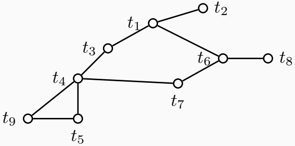
FPT Algorithm

Properties of a cactus T

1. The vertices of T can be properly colored using 3 colors.
2. Every vertex of degree at least 3 is a cut-vertex.
3. The graph obtained from T by subdividing any edge is a cactus.
4. The graph obtained from T by short-circuiting any degree 2 vertex is a cactus.

Few Definitions





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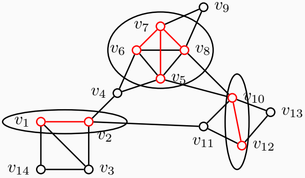

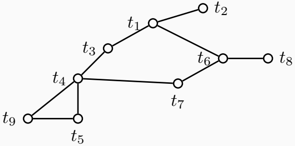

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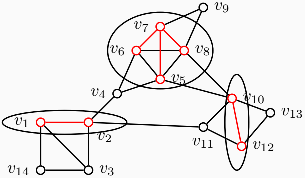

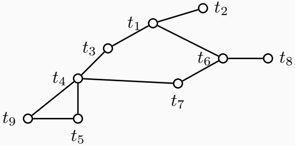

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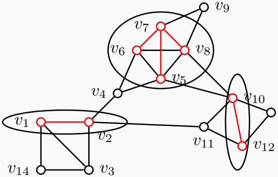

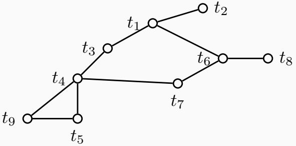

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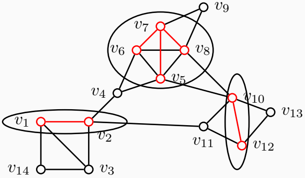

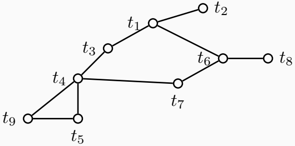

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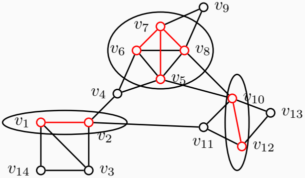

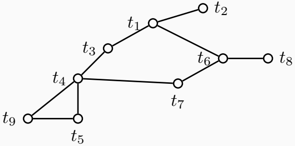

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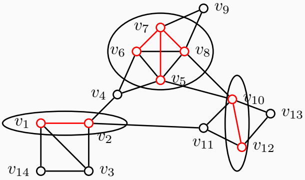
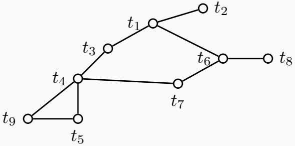
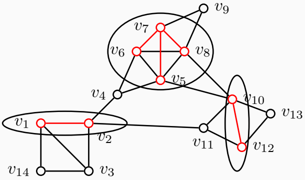


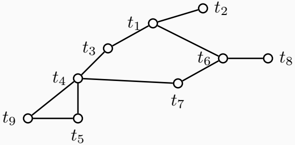


Lemma

There are at most 6k important vertices.

At most $2k$ important vertices contained in big bags (Ex $\{v_1, v_2\}$)





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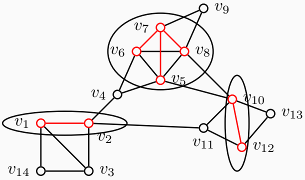

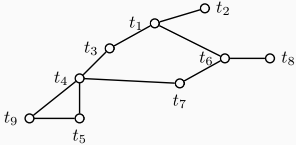

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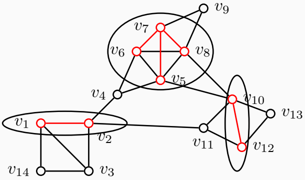

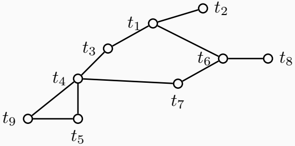

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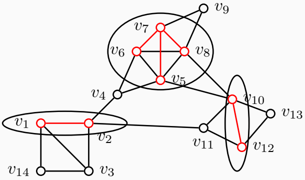

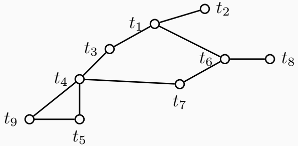

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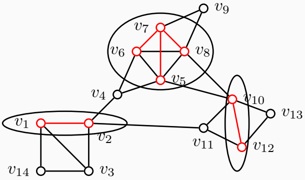

v_9
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v_6
v_8
v_5
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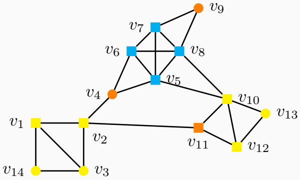
v_2
v_{11}
v_{12}
v_{14}
v_3



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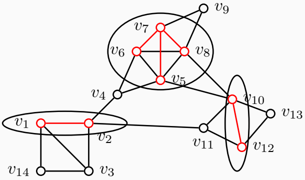
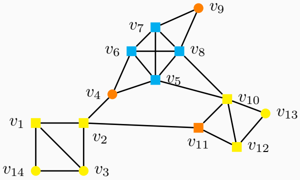
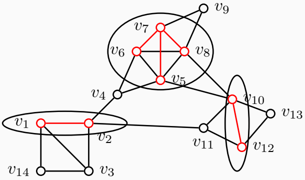
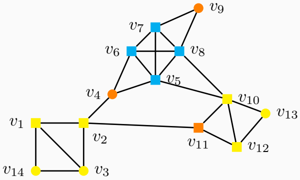




Compatible Coloring $\phi : V(G) \to [3]$ is compatible (wrt $\mathcal{W}$) if





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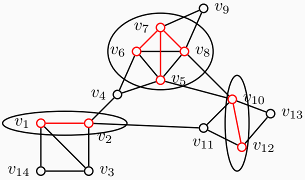

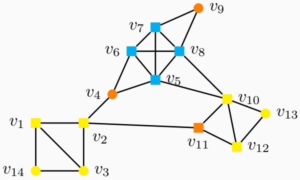

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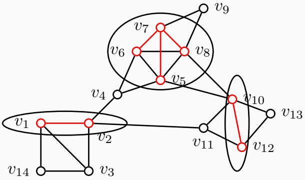

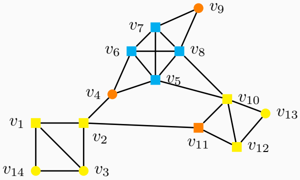

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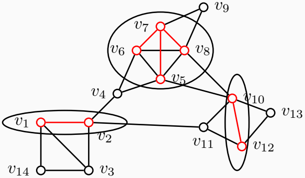

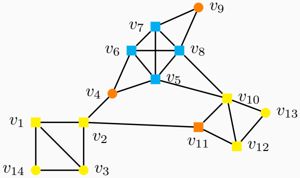

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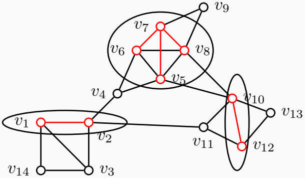

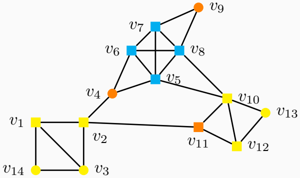

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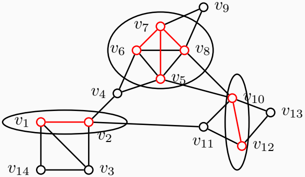

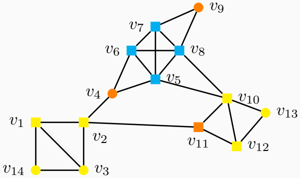

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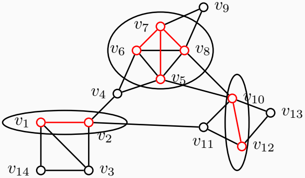

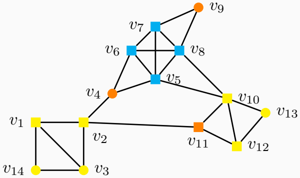

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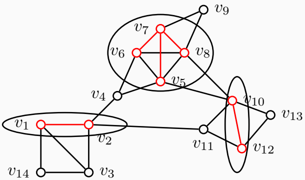
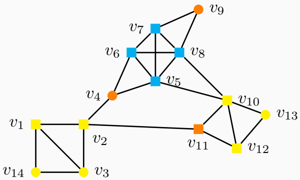
Properties of Compatible Coloring





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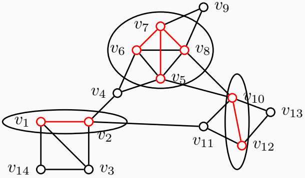

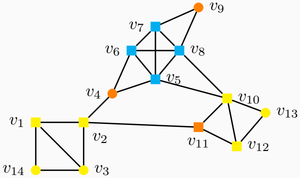

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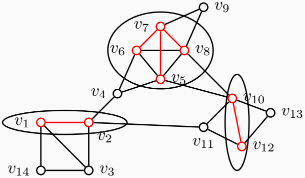

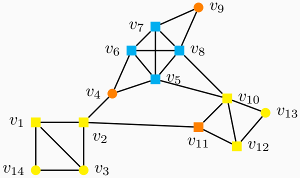

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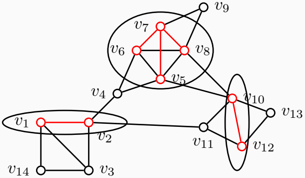

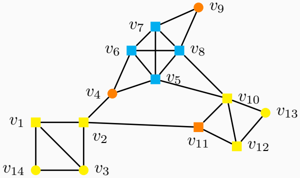

$X \subseteq V(G)$ a colored component of

Properties of Compatible Coloring

For every color component X in $\mathcal{X}$:

- all vertices of X are in small bags in $\mathcal{W}$, OR
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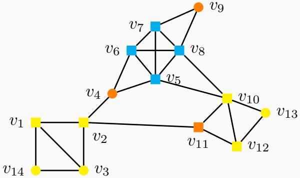
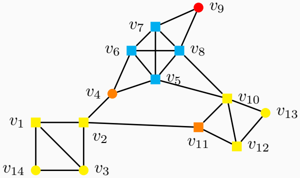
Hence, for any two components X, Y of ϕ, no edge uv in $E(X, Y)$ is in optimum solution.

Identifying Vertices in Pendant Cycles and Leaves





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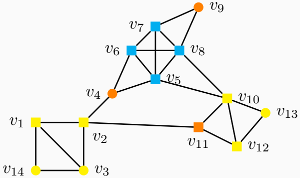

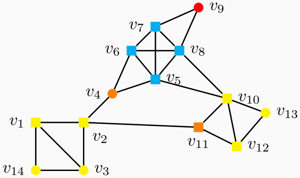

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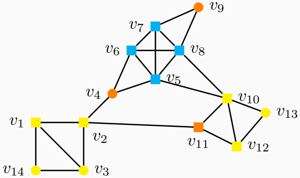
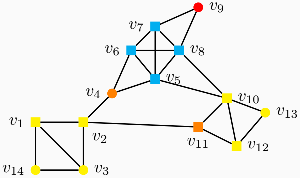
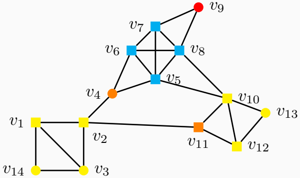
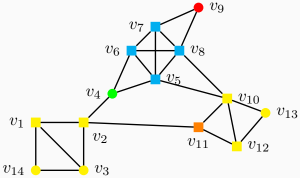




Lemma

Identifying Vertices in Between Two Big bags





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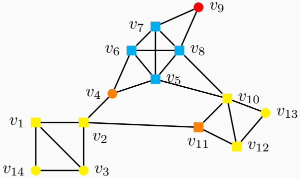

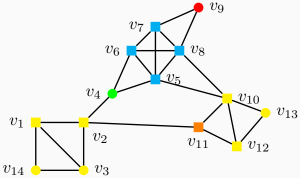

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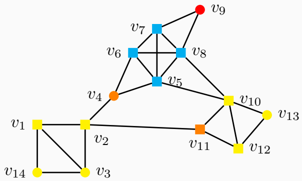

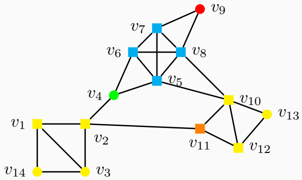

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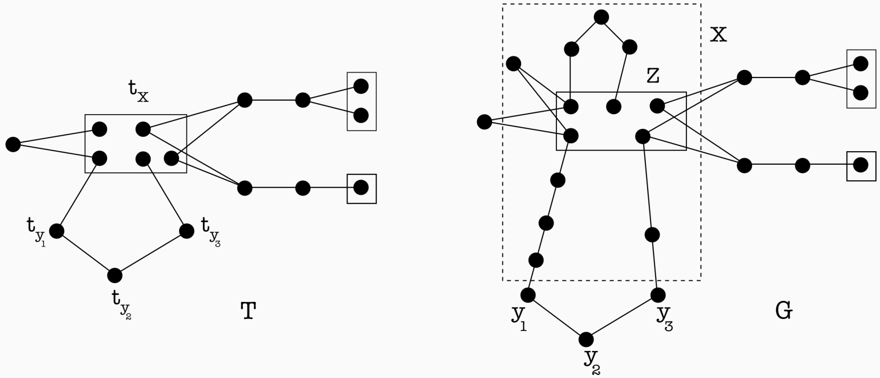
At this stage, every colored component X contains exactly one big witness set $W(t)$ but we don't know it explicitly.

Extract witness sets



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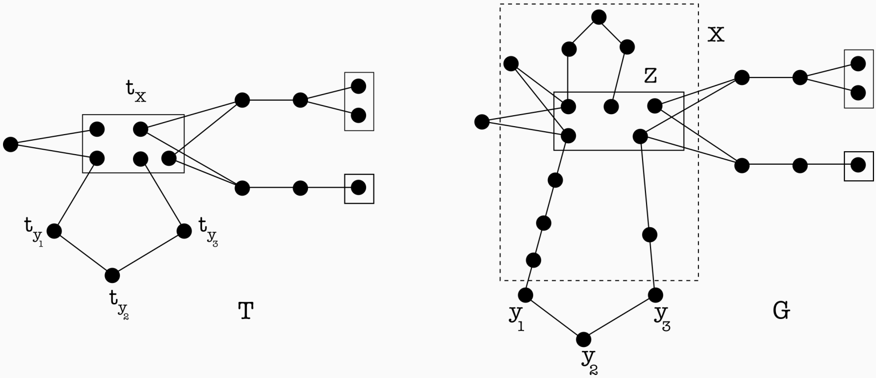

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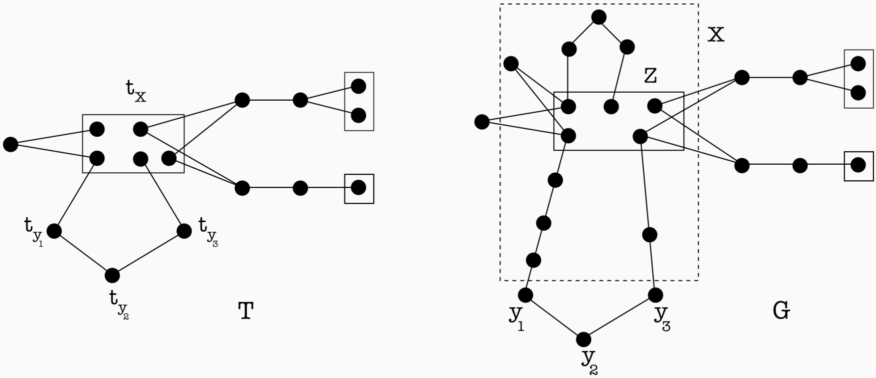

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We argue that contracting Z is as good as contracting $W(t)$.

Extract witness sets

To argue that contracting Z is as good as contracting $W(t)$, we need to prove:

1. $|Z| \leq |W(t)|$
2. Characterize $W(t)$ in graph $G[X]$ and ensure that set Z has that property.

Once we identify such Z in X, we replace X by Z and singleton set for every vertex in $X \setminus Z$ in $\mathcal{X}$.

Characterize $W(t)$ in graph $G[X]$

A ***core*** of a graph G is a set $Z \subseteq V(G)$ such that every connected component of $G - Z$ is either an isolated vertex or a simple path whose neighborhood is contained in Z.

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If a core Z is a connected set in G, then we call it a *connected core* of G.

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Lemma

For a colored component X in $\mathcal{X}$, if $W(t)$ is the big witness set contained in X then $W(t)$ is a connected core of $G[\hat{X}]$.

$\hat{X}$ is superset of X which contains vertices in the connected components of $G - X$ that are either isolated vertices or a simple path in G.

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Every vertex in $W(t)$ has at least one of the two responsibility:

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We introduce **Marking Scheme** to mark vertices which are in $W(t)$ because of external constraints.

Characterize $W(t)$ in graph $G[X]$

Marking-Scheme For a colored component X in $\mathcal{X}$,

1. If there exists y in $N(X)$ such that $\phi(y) = 5$ then mark all the vertices in $N(y) \cap X$.
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Lemma

Every marked vertex in X is in big witness set $W(t)$.

Finding Connected Core

Lemma

Given a connected graph G and a subset Q of its vertices, one can compute a minimum connected core of G which has at most k vertices and contains Q in $\mathcal{O}^*(6^k)$ time if it exists.

FPT Algorithm

Input: A *2-connected graph* G and an integer k

Output: A set F of k edges in G such that G/F is a cactus

Generate random coloring $\phi : V(G) \rightarrow \{1, 2, 3\}$ and construct $\mathcal{X}$.

Recoloring I, II.

for *each* $X \in \mathcal{X}$ **do**

  Apply Marking Scheme to obtain marked vertices $Y_X \subseteq X$

  $Z_X \leftarrow$ minimum connected core of $(G[\hat{X}], Y_X)$

  In $\mathcal{X}$, replace X by Z_X and small witness set for every vertex in $X \setminus Z_X$.

if *a spanning forest* F *of* $\mathcal{X}$ *has* $\leq k$ *edges* **then**

  return F

Step 1: Color vertices of input graph uniformly at random with 3 colors.

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Theorem

CACTUS CONTRACTION *is* FPT *with running time* $\mathcal{O}^*(c^k)$.

Thank you!

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