PATH CONTRACTION FASTER THAN 2^n

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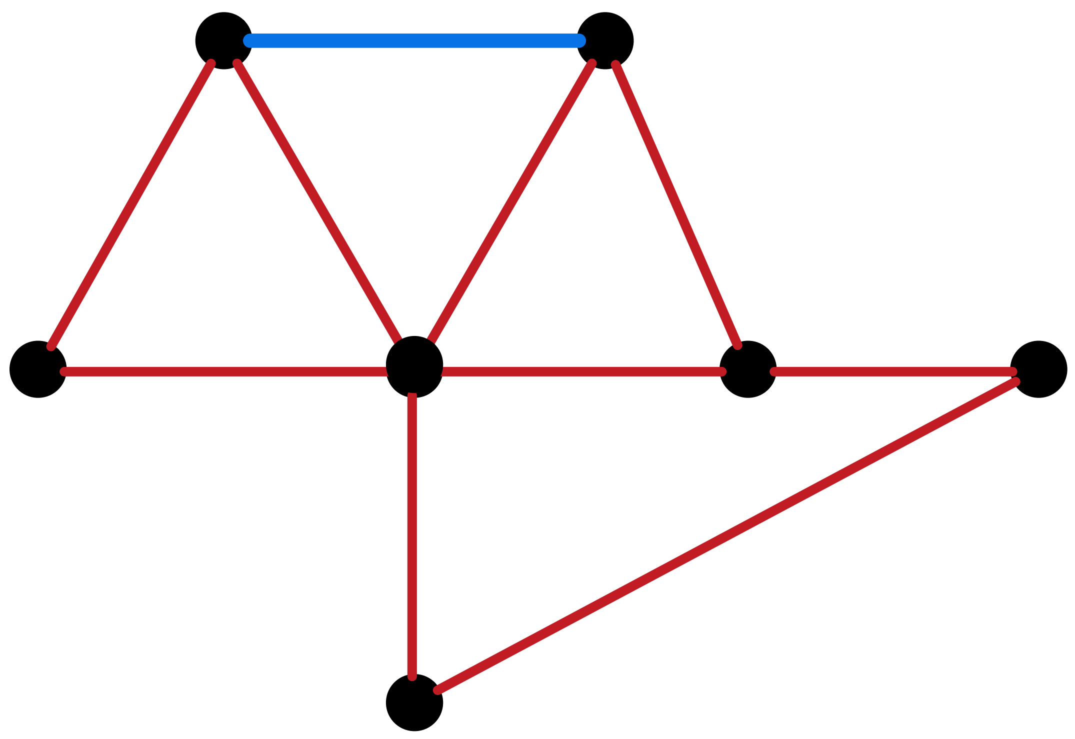
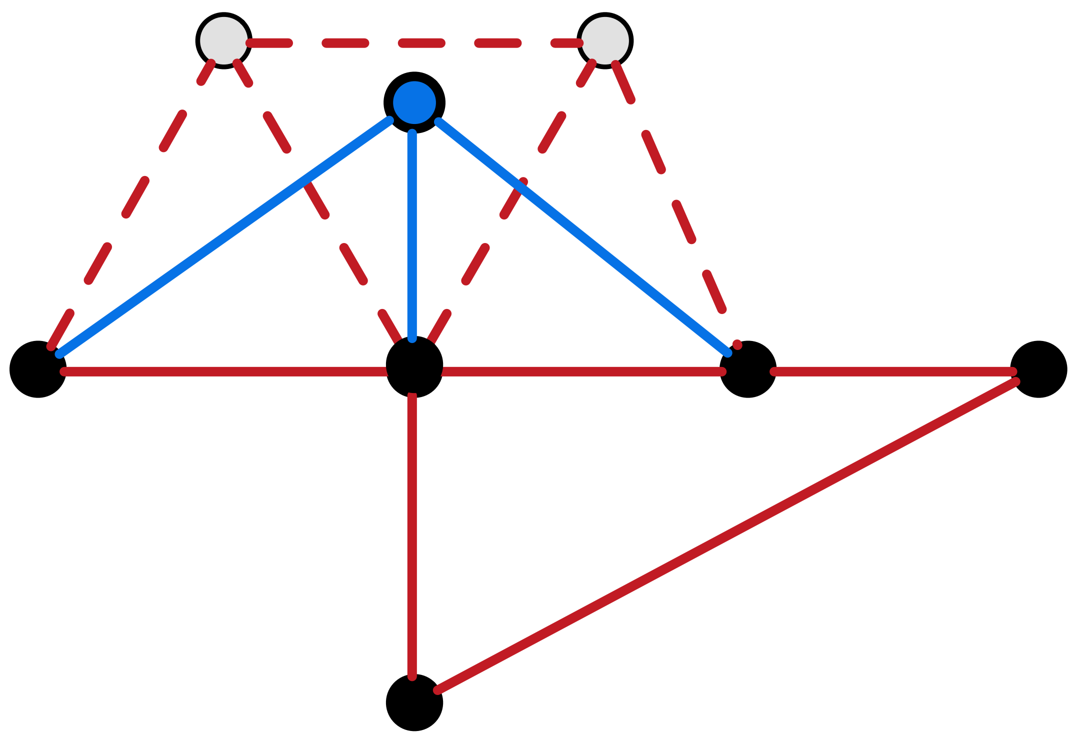
Ben-Gurion University of the Negev

CONTRACTION OF AN EDGE



- Delete the two end-points.

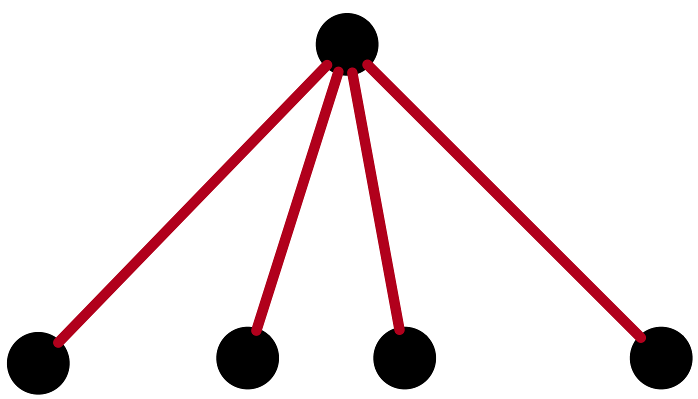
CONTRACTION OF AN EDGE



- Delete the two end-points.
- Add a new vertex adjacent to the neighbors of the deleted vertices.

CONTRACTION TO A PATH

P_t = Path on t vertices



P_3 P_4

Contract a subset E^* of $E(G)$ such that the resulting graph is isomorphic to P_t.

PATH CONTRACTION

Input: Graph G.

Output: Maximum number t, such that G can be contracted to a path on t vertices.

SOME RESULTS ON PATH CONTRACTION

- NP-hard, for every fixed t >= 4.
- Polynomial time solvable for t <= 3.
- The problem has been well studied for restricted input graphs.
- The case when t = 4 is the same as 2-DISJOINT CONNECTED SUBGRAPHS, which admits an algorithm running in time $O^*(1.7804^n)$.

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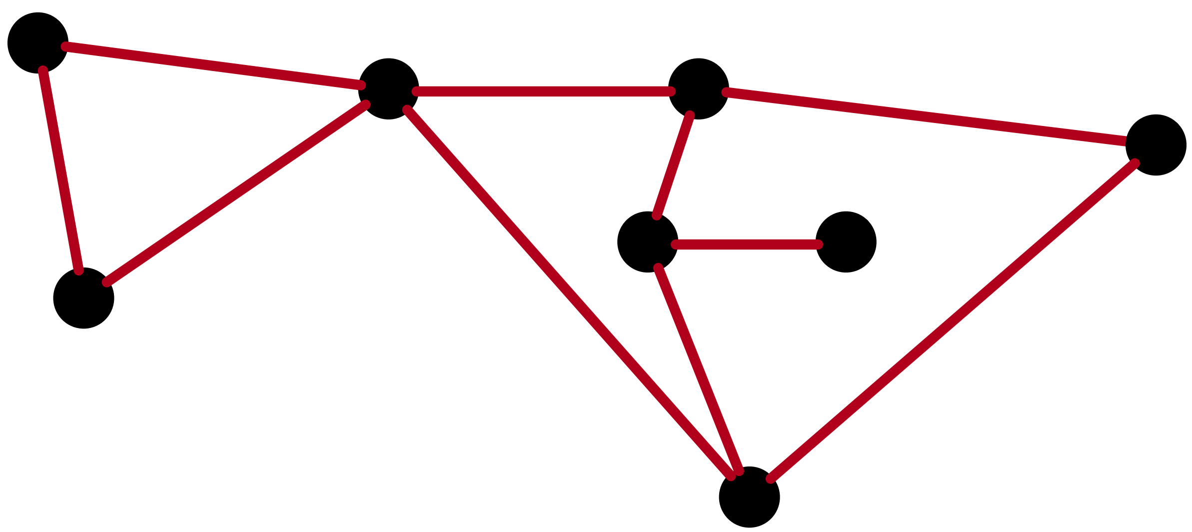
t is the length of the contracted path.

INTERPRETING CONTRACTION AS PARTITIONS

P_t = Path on t vertices



A partition of vertices, where edges exits only between consecutive bags.

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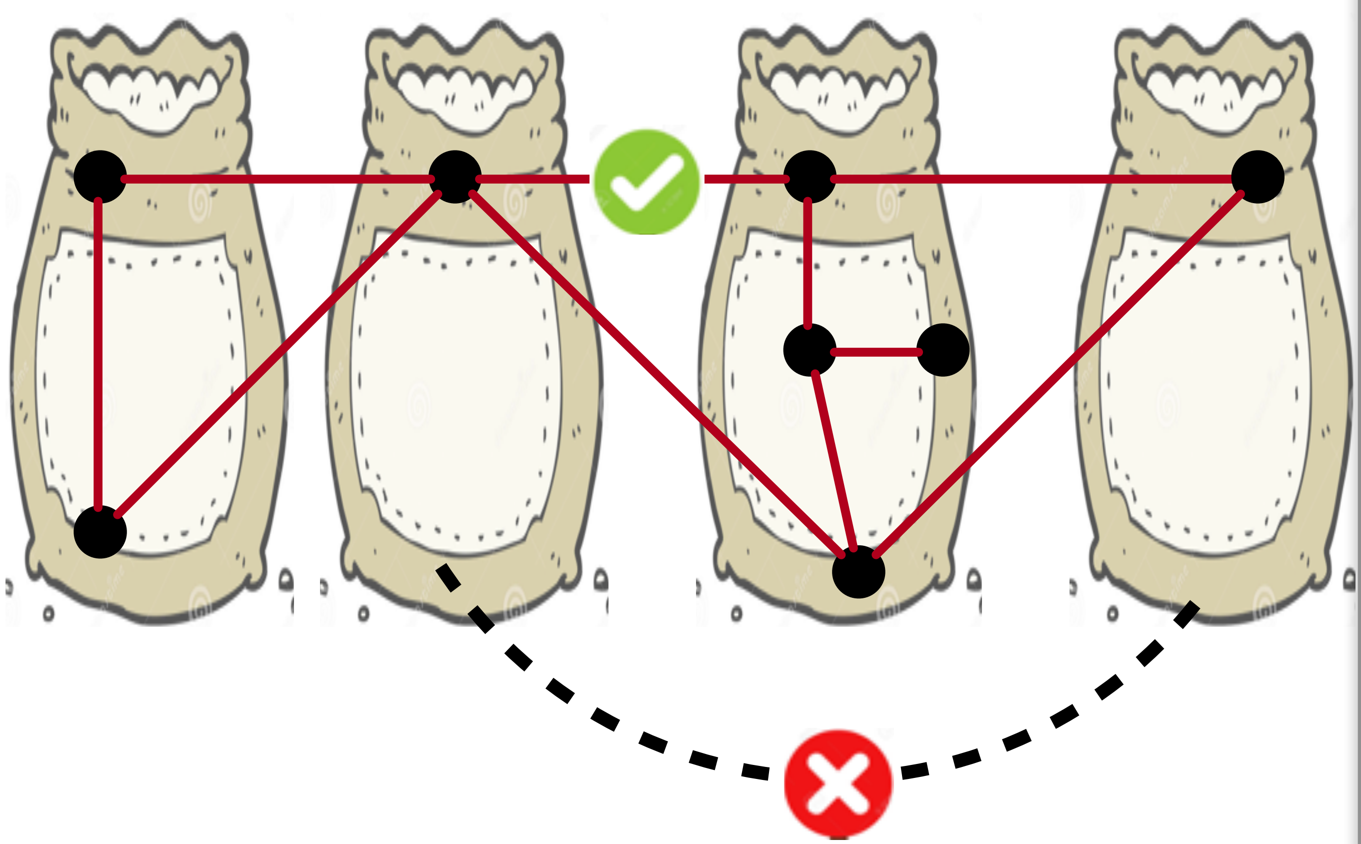

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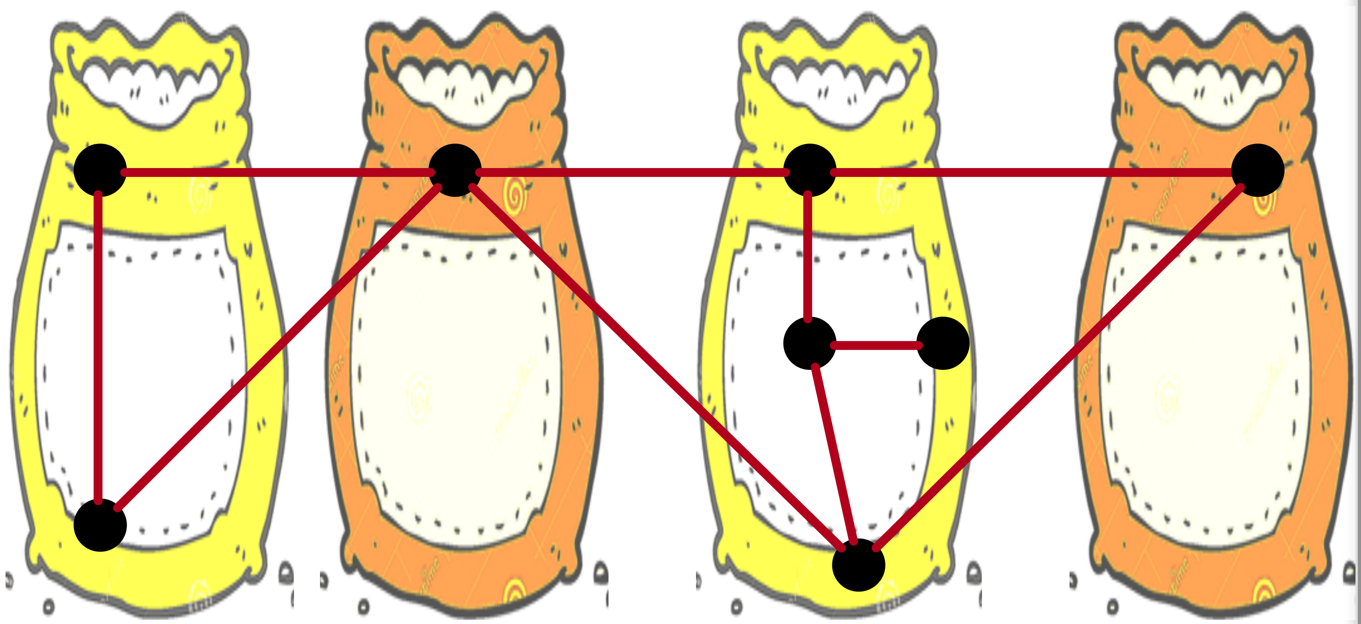
STARTING POINT OF THE RESULT

Best known algorithm prior to our result was a very simple algorithm, for a deceptively simple problem.



- For every coloring of V(G) with two colors:
  - Contract each connected component.
  - If the above is a path, store the number of vertices in it.
- Return the **maximum** over the stored numbers.

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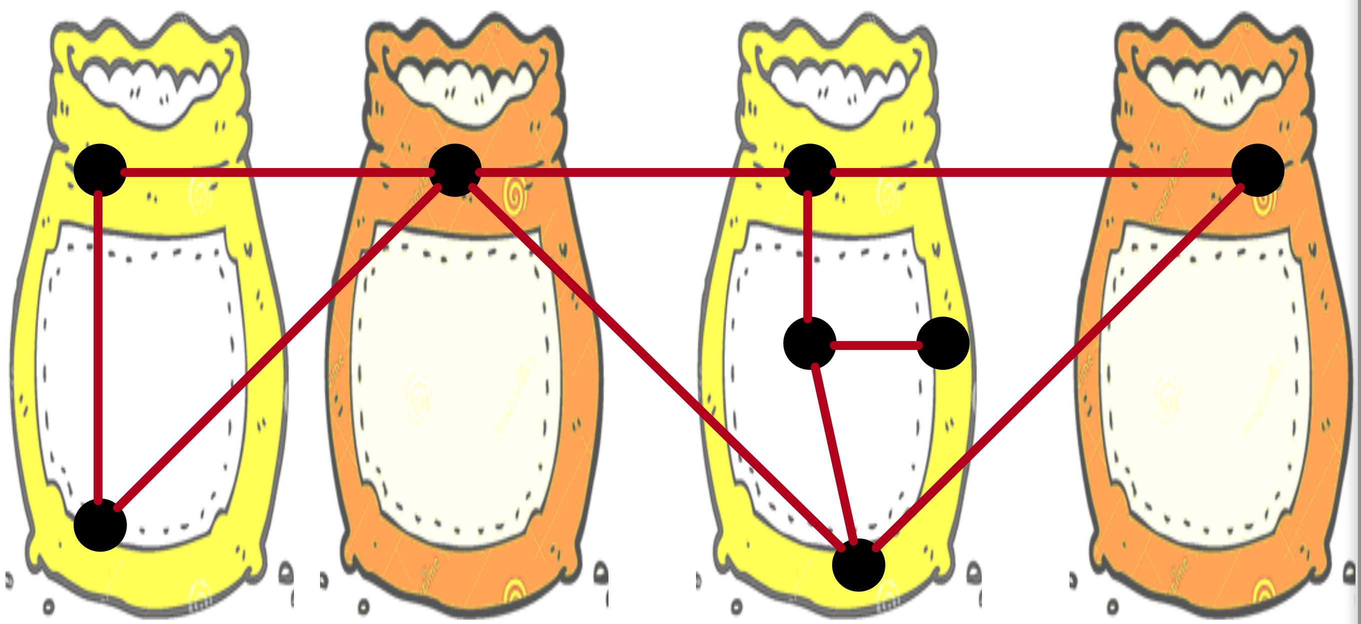

$O^*(2^n)$ algorithm!

- For every coloring of V(G) with two colors:
  - Contract each connected component.
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OUR RESULT

Can we break the 2^n barrier? YES!

PATH CONTRACTION admits an algorithm running in time c^n, where $c<2$.

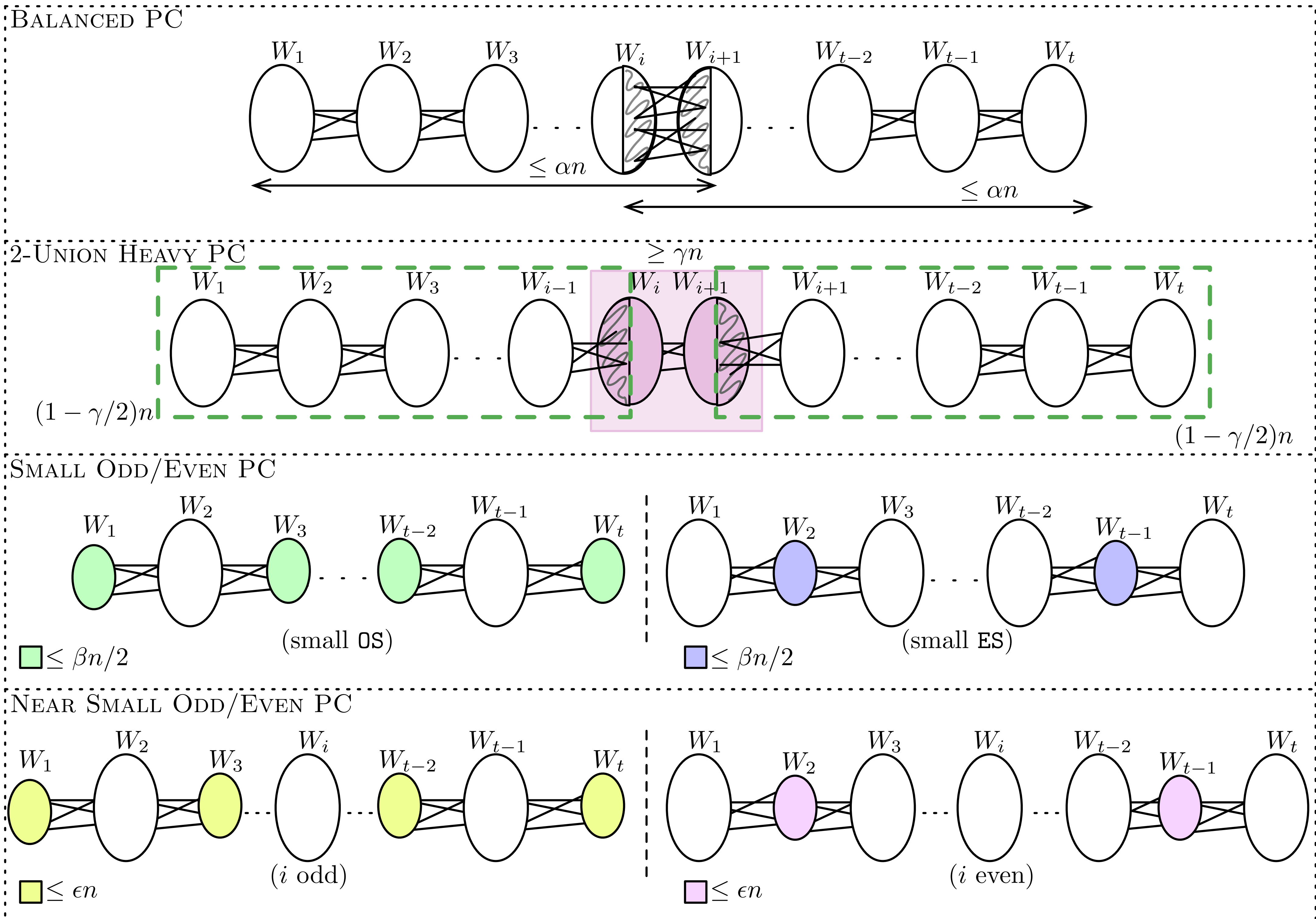
Answers an open question of van't Hof et. al [TCS'09]

OUR METHODS

For a deceptively simple problem, breaking the brute force barrier is quite non-trivial.

- To obtain our result, we design **four** different algorithms for PATH CONTRACTION, and apply the **most relevant** one for the input.

DIFFRENT ALGORITHMS FOR DIFFERENT CASES



By setting numbers appropriately, one of the algorithms is always relevant for the given input.

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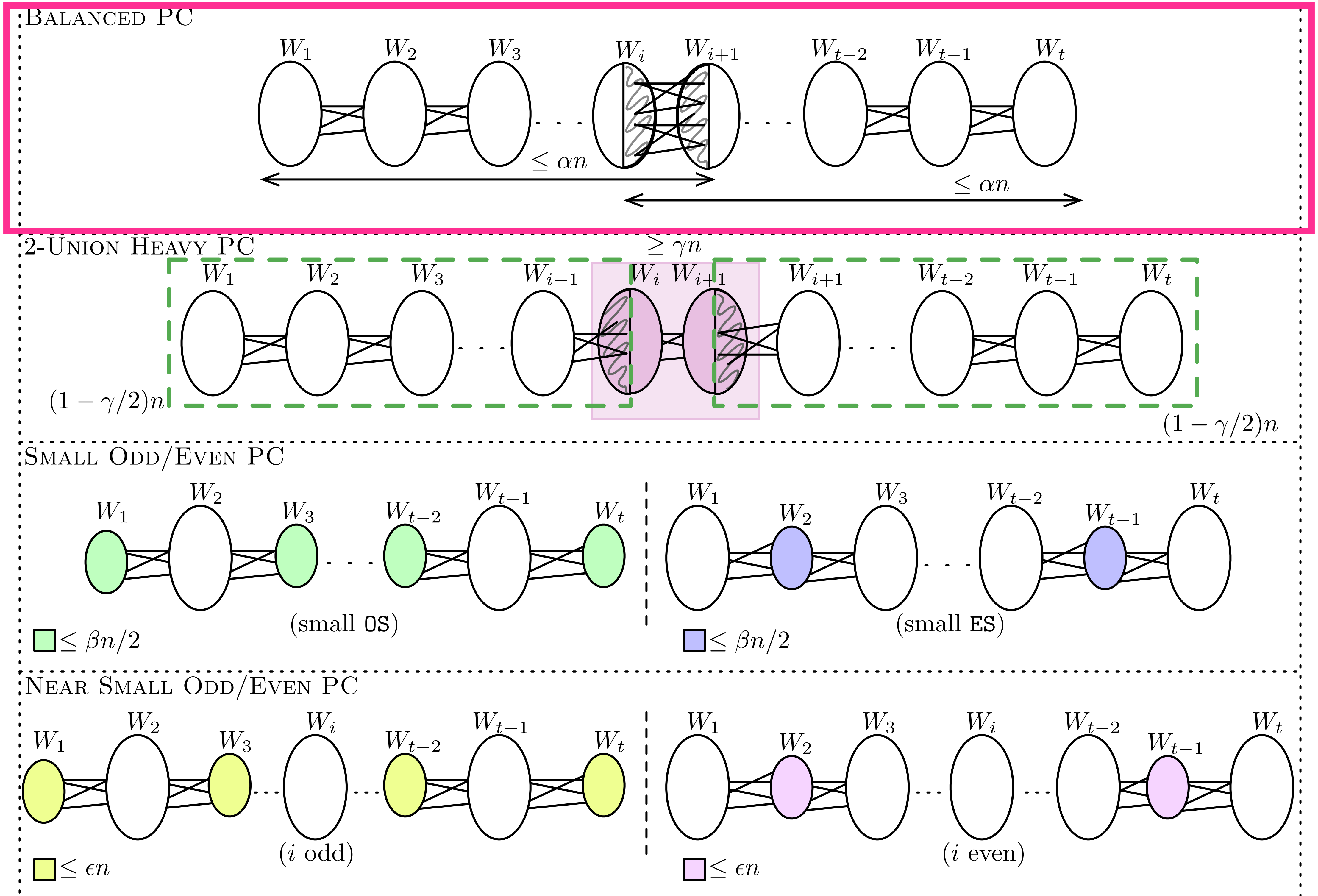

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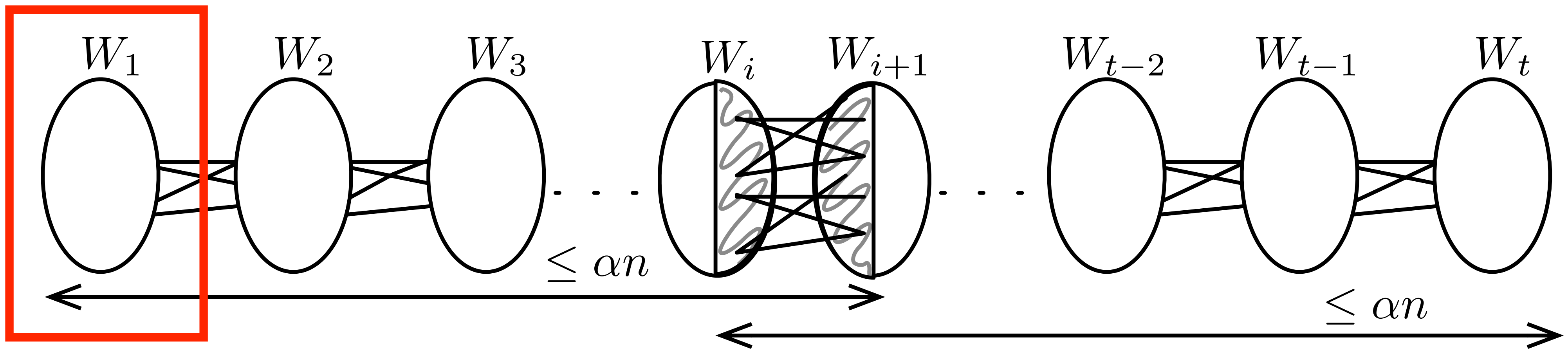
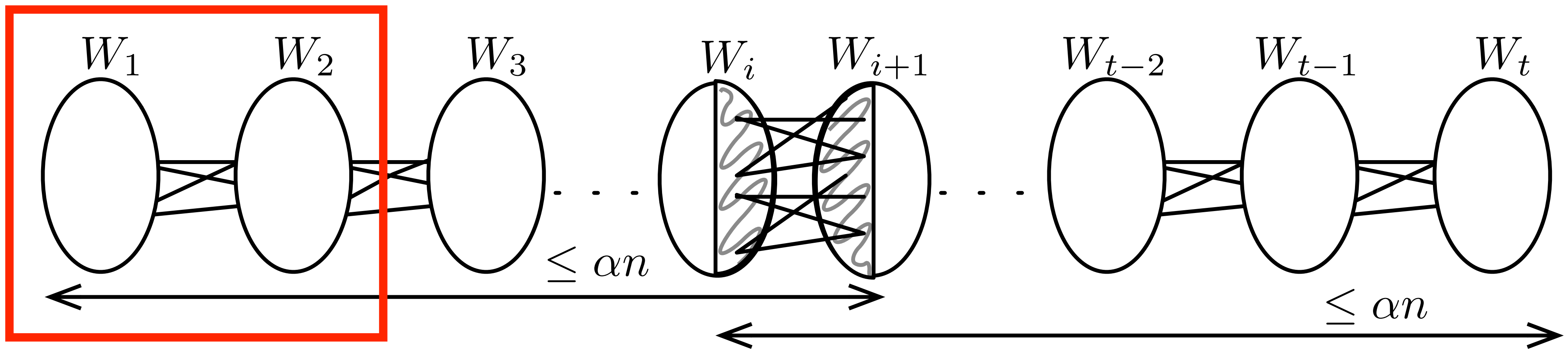
BALANCED PC



Connected, with small closed neighborhood.

BALANCED PC



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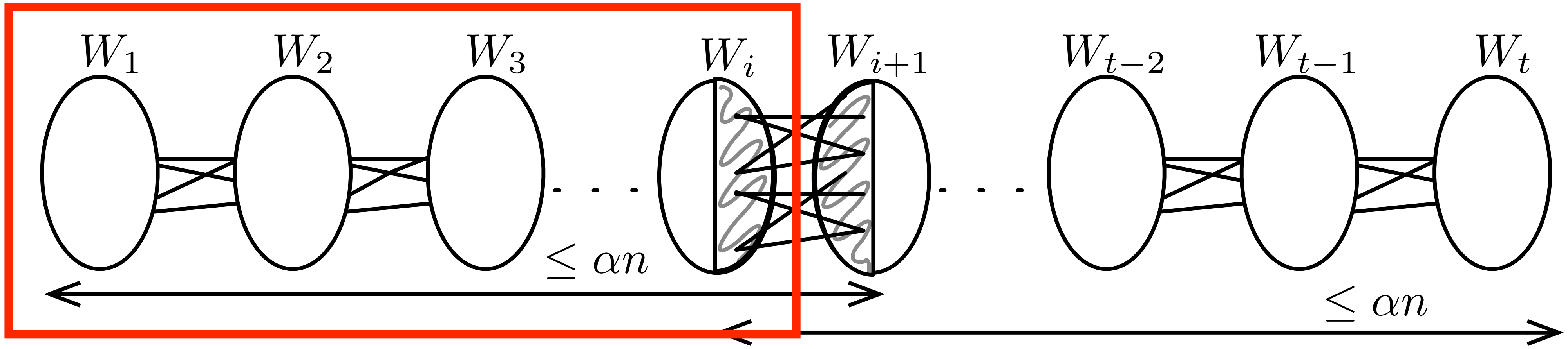

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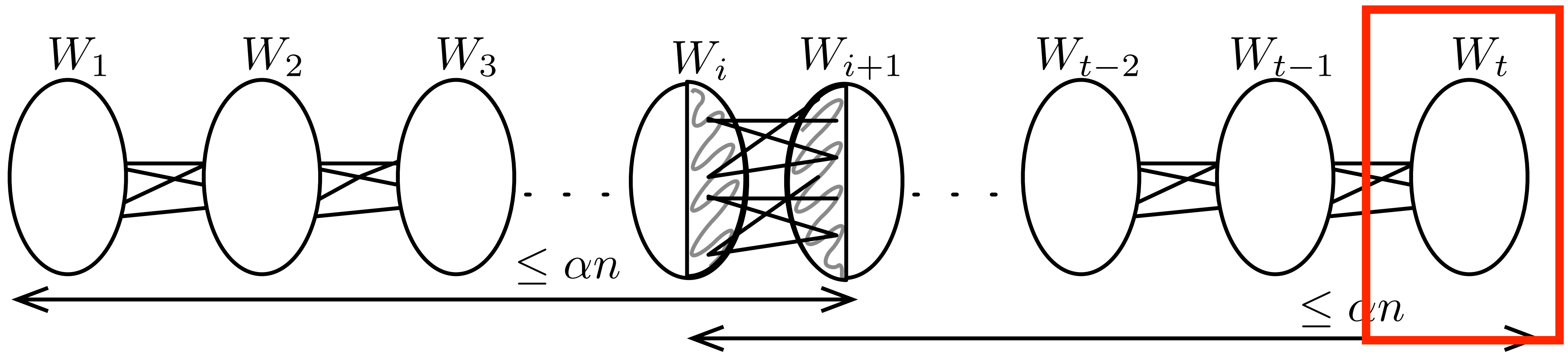

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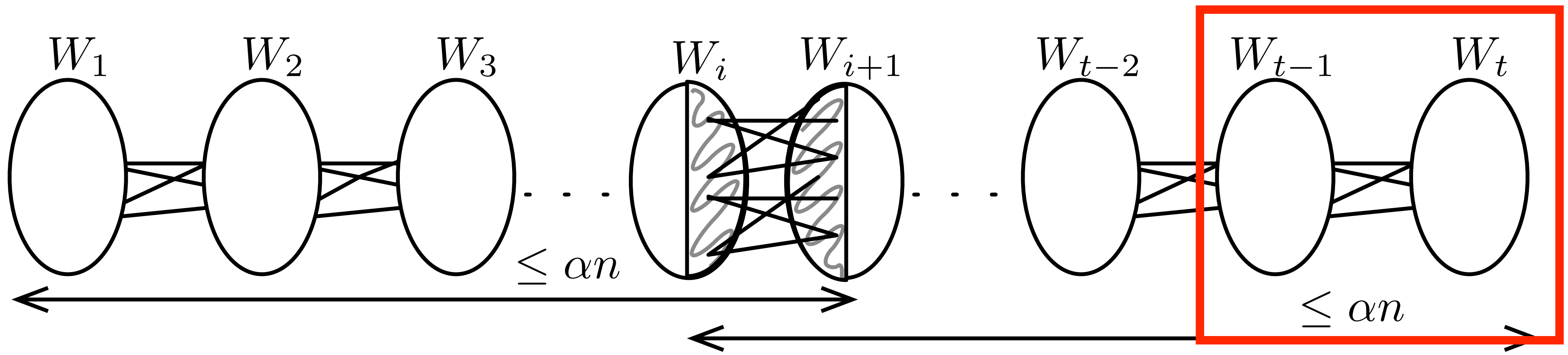

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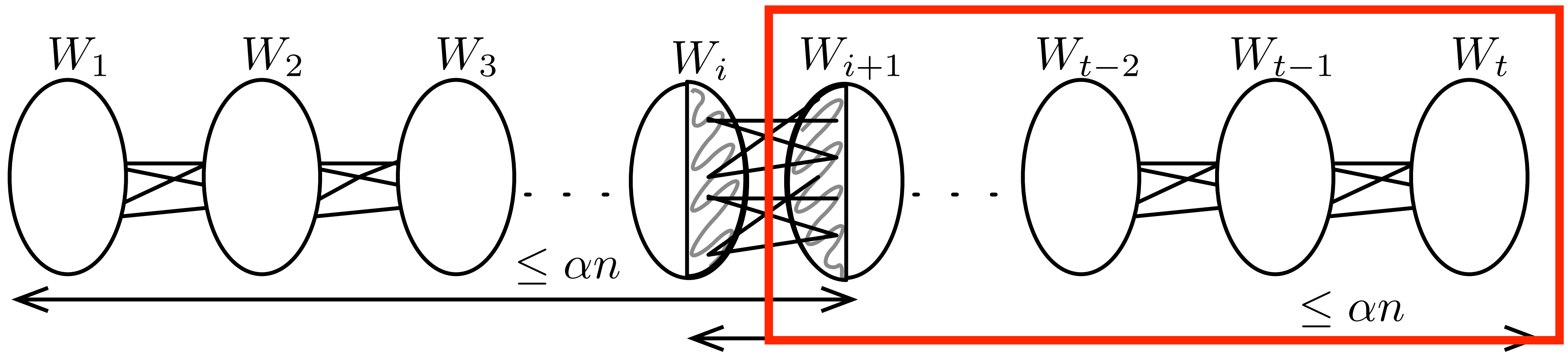

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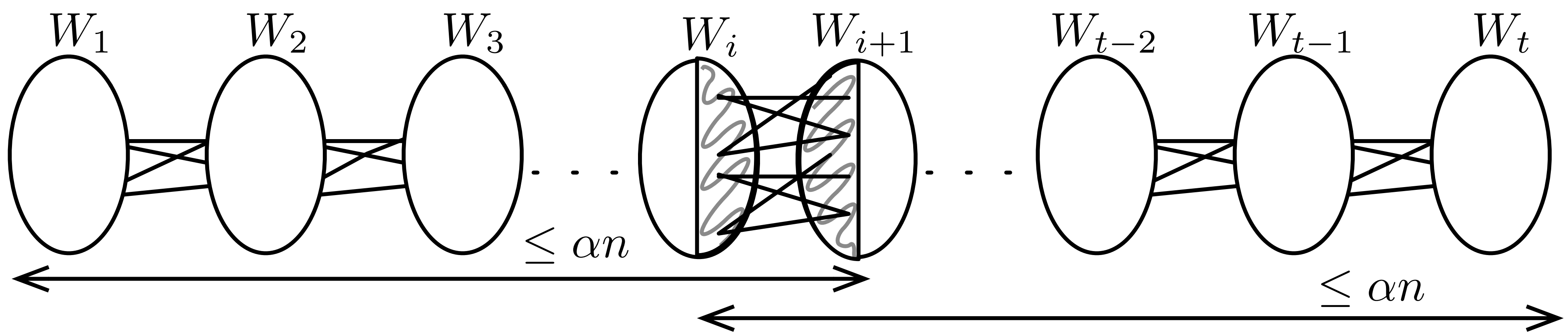
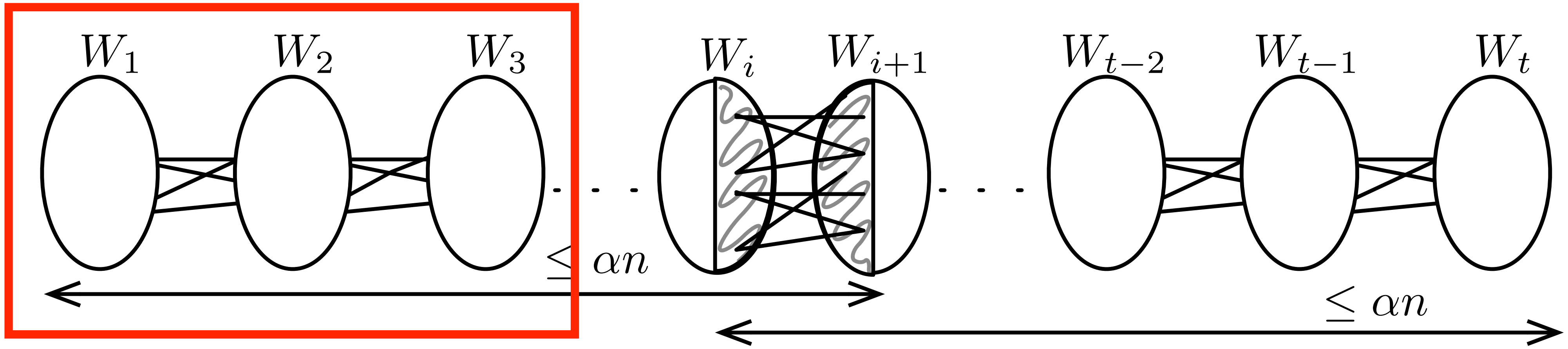


- Enumerate all connected sets with small closed neighborhood.

[Better than 2^n]

BALANCED PC



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- Compute optimal solution for each such set.

[Using Dynamic Programming]

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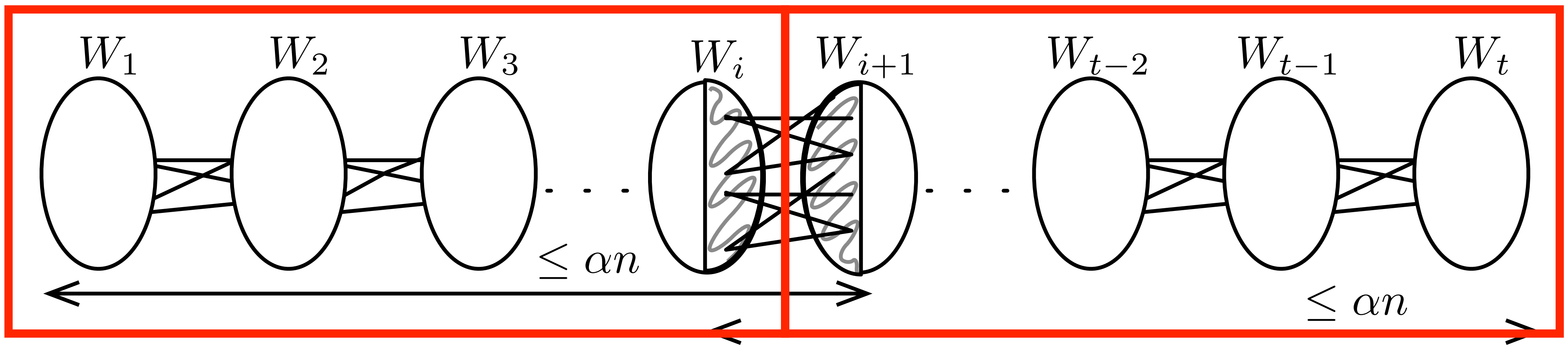

- Enumerate all connected sets with small closed neighborhood.

  [Better than 2^n]

- Compute optimal solution for each such set.

  [Using Dynamic Programming]

- Combine the results.

  [For the correct combination, this gives the solution]

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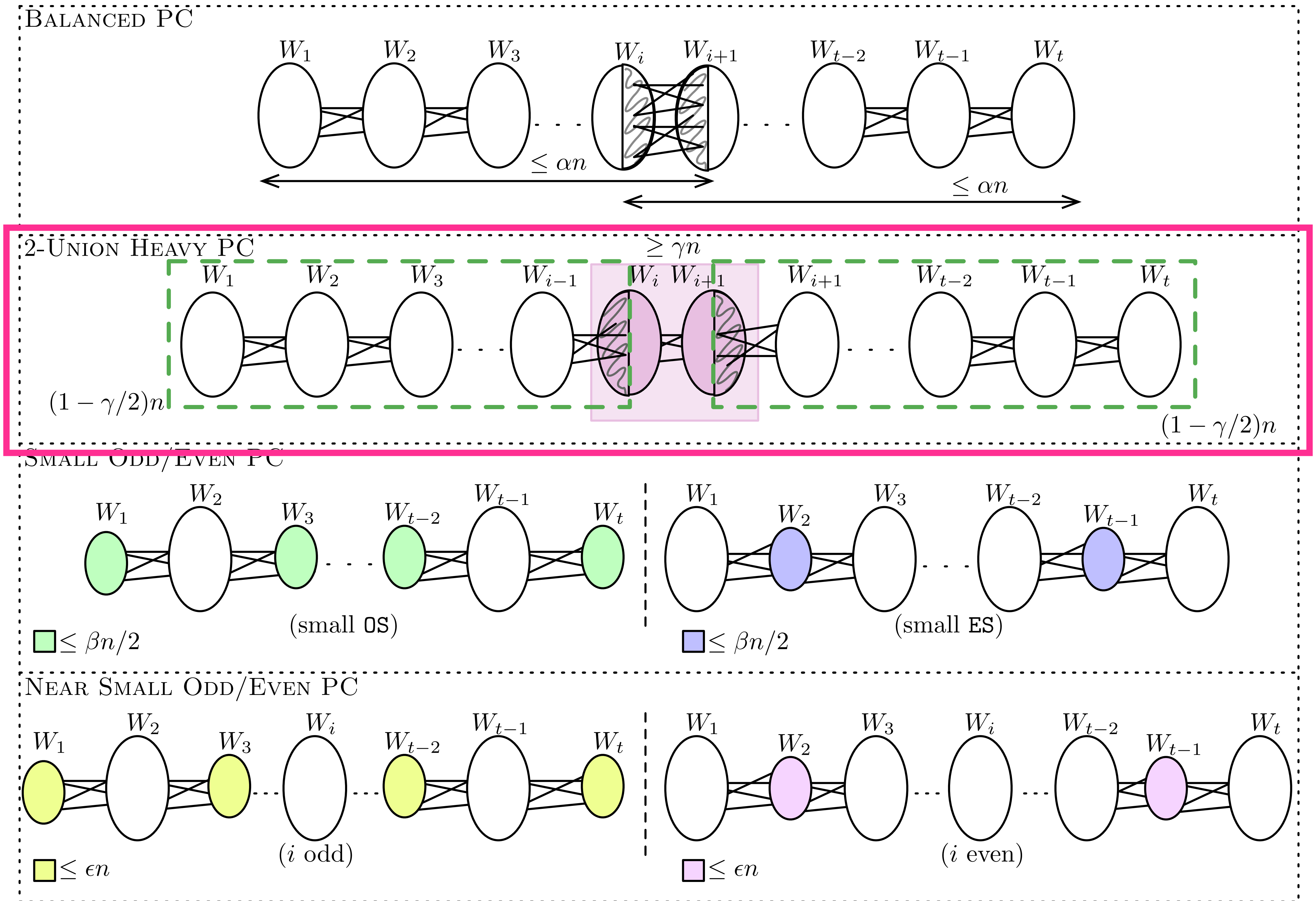
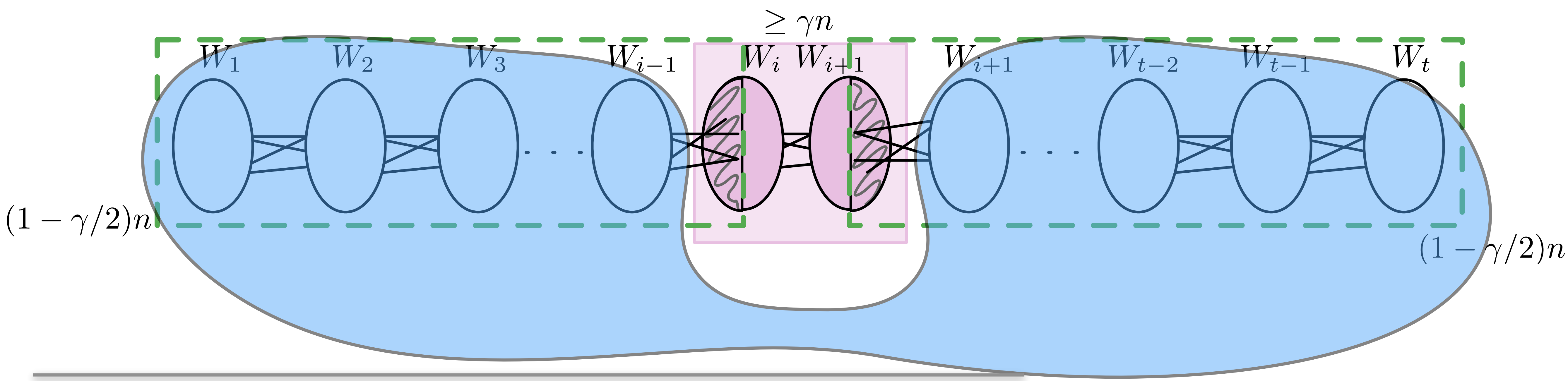


By setting numbers appropriately, one of the algorithms is always relevant for the given input.

2-UNION HEAVY PC



- Find correct blue set, by trying **all possibilities**. Now solve the following partitioning problem.

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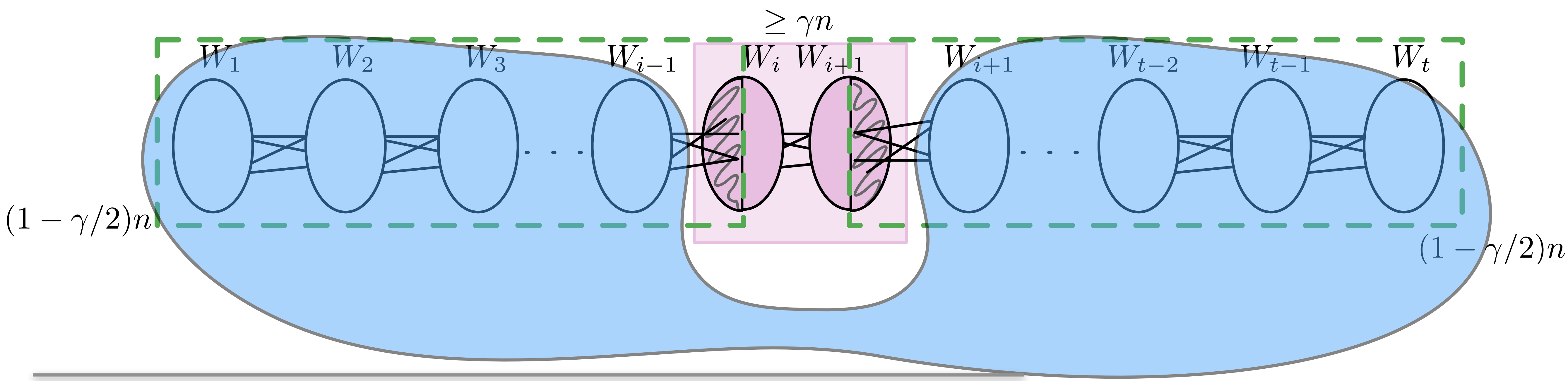

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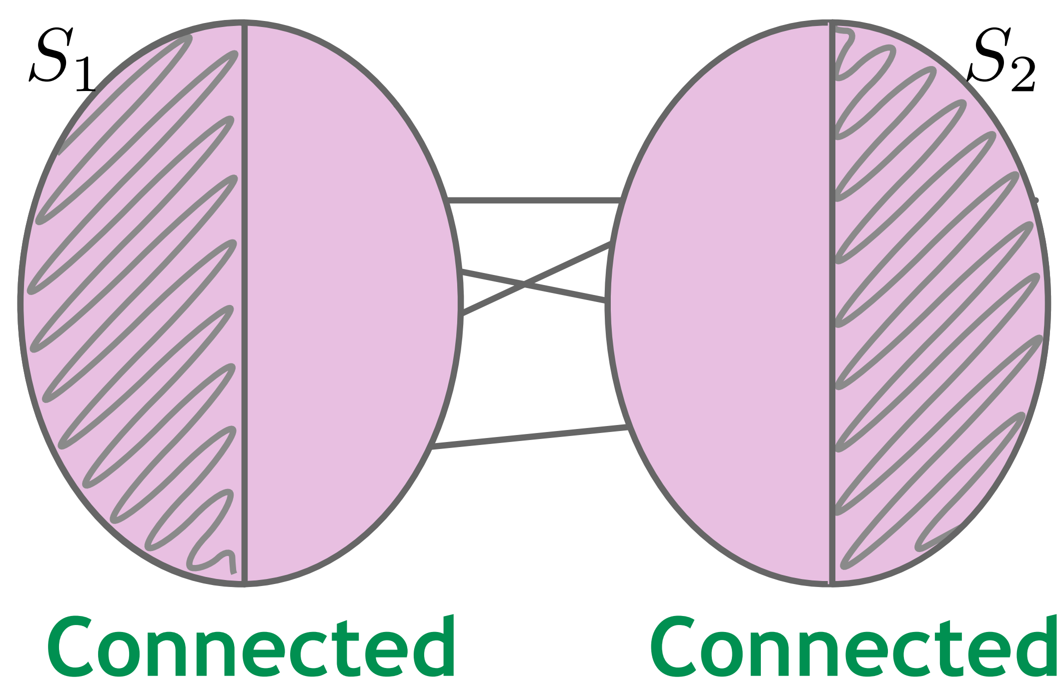
2-DISJOINT CONNECTED SUBGRAPHS

$O^*(1.7804^n)$

[Known]



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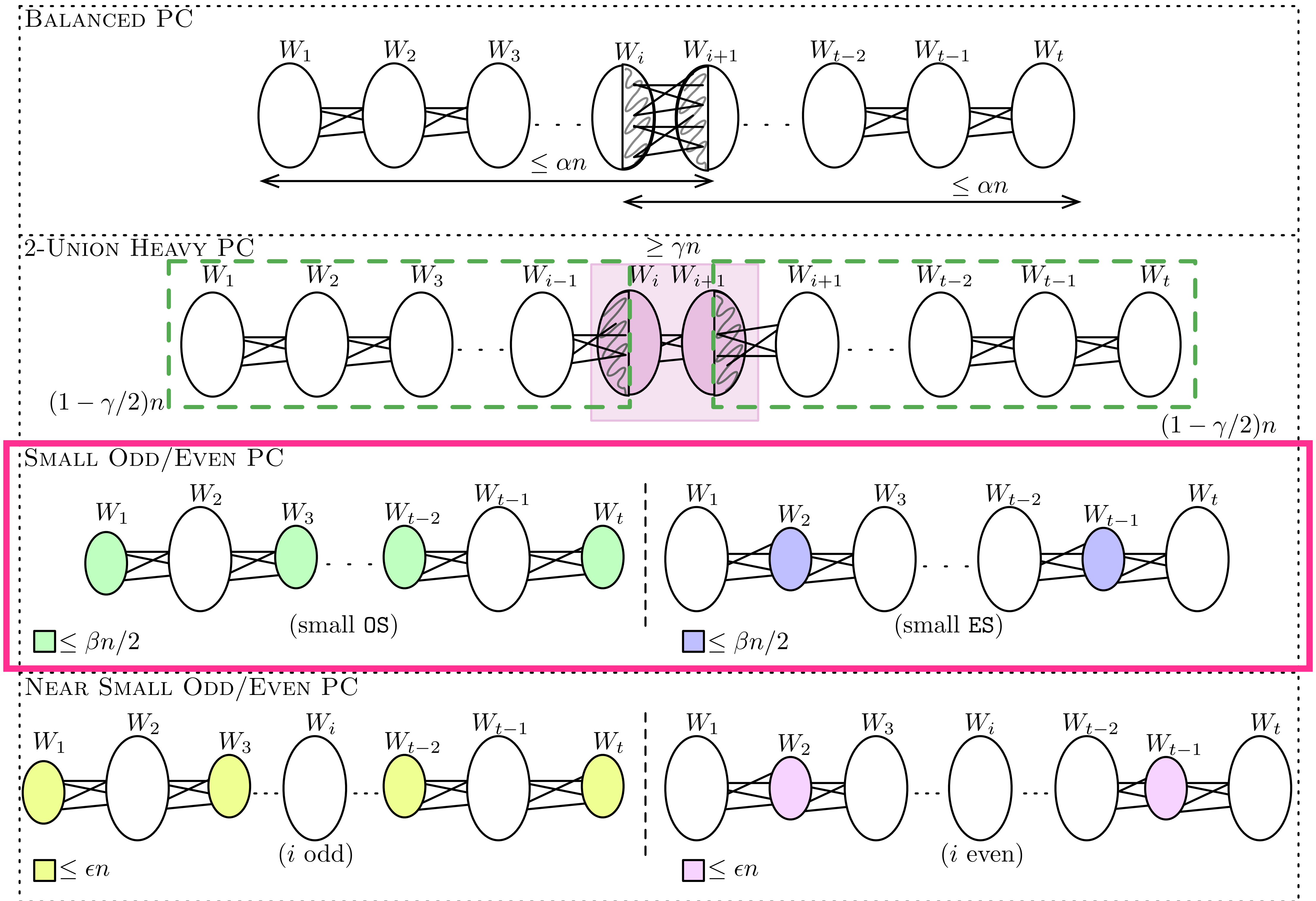

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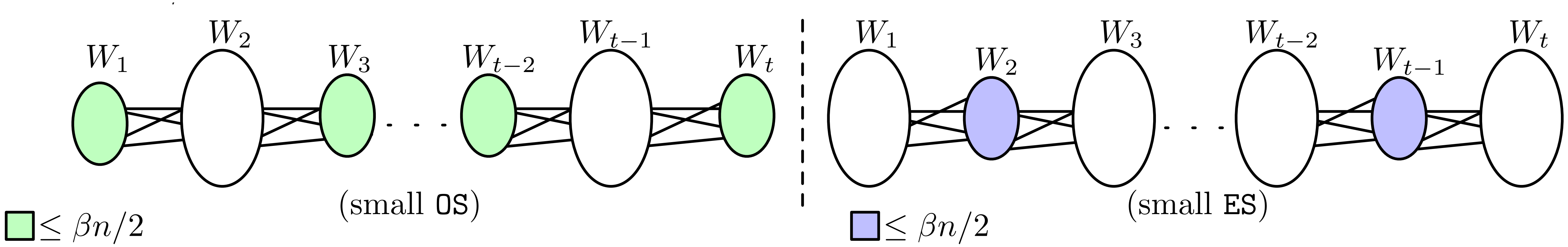
SMALL ODD/EVEN PC



- Find the odd/even set, whichever is smaller, by trying all possibilities.
  - Contract each connected component.
  - If the above is a path, store the number of vertices in it.
- Return the **maximum** over the stored numbers.

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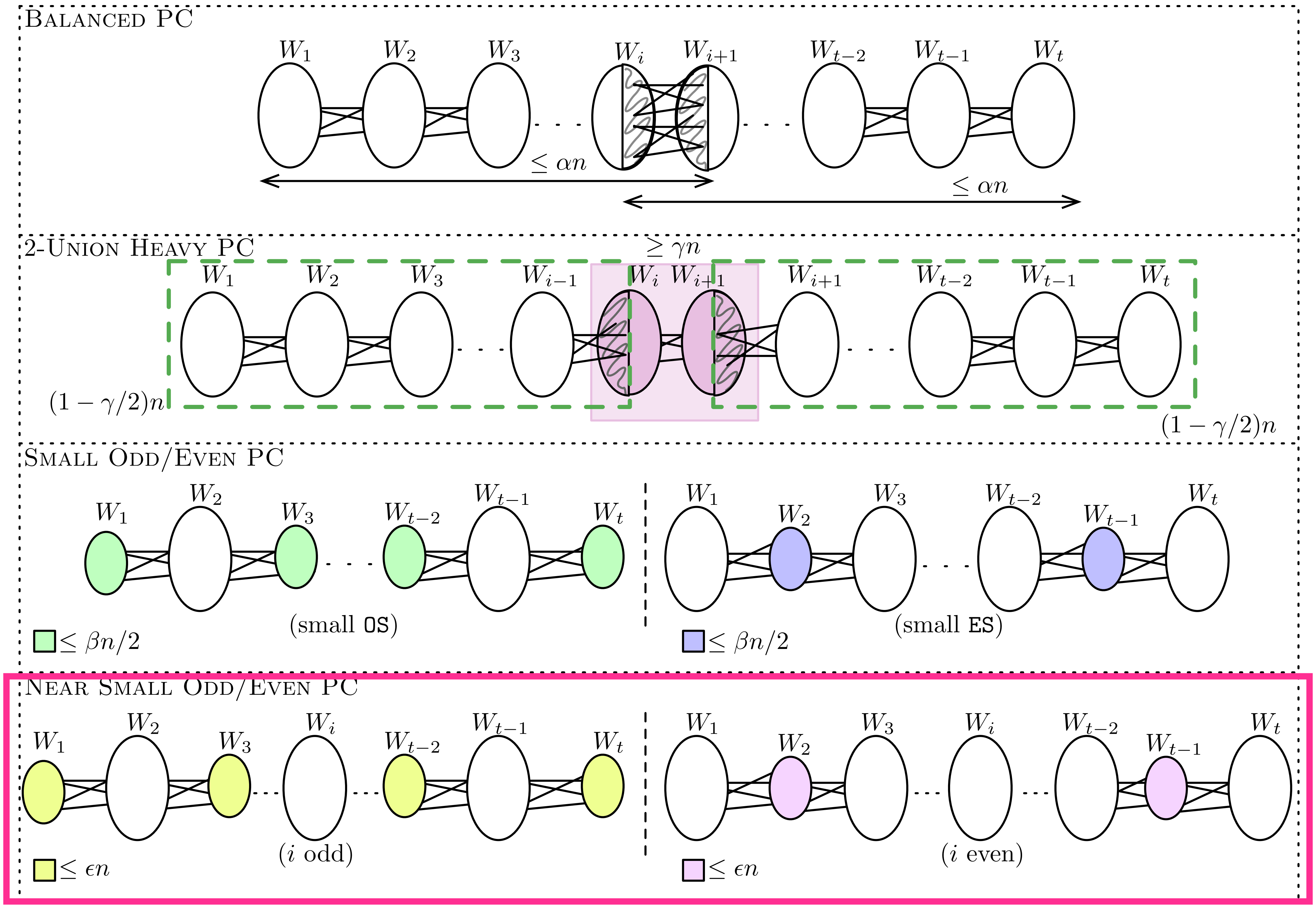

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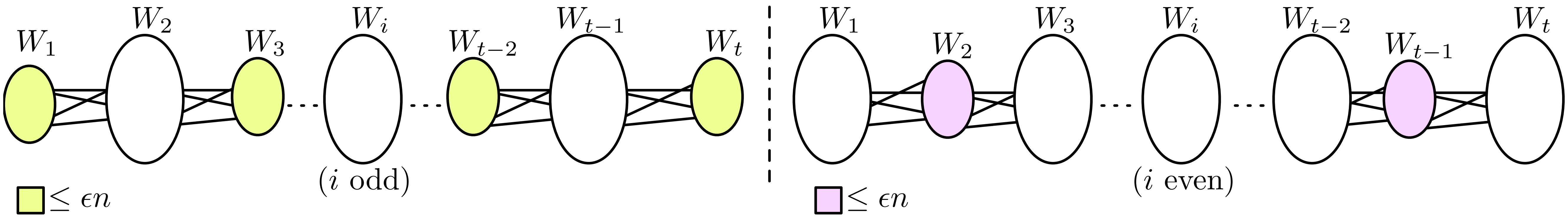
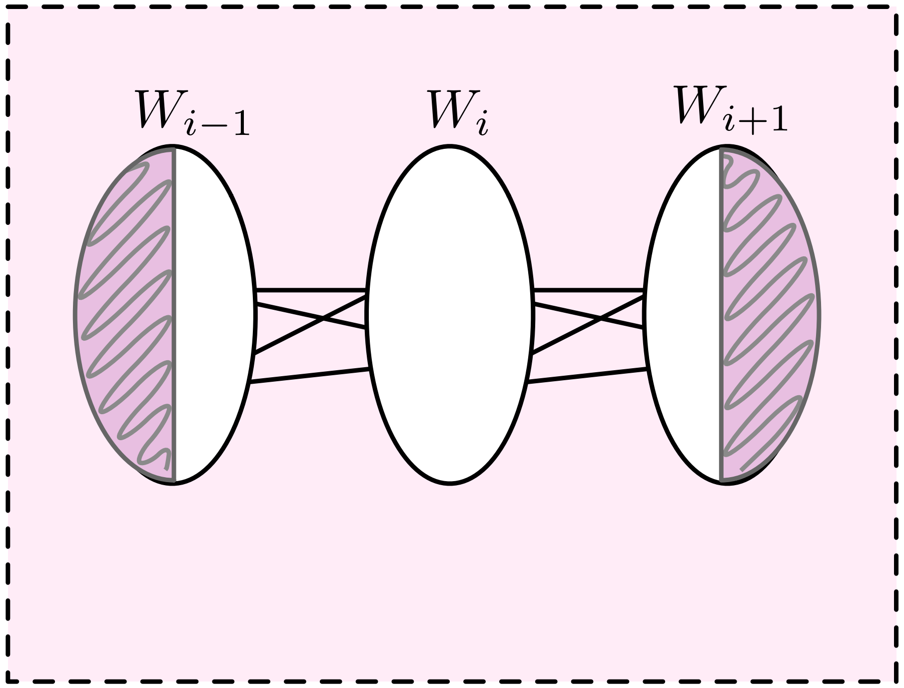
NEAR SMALL ODD/EVEN PC

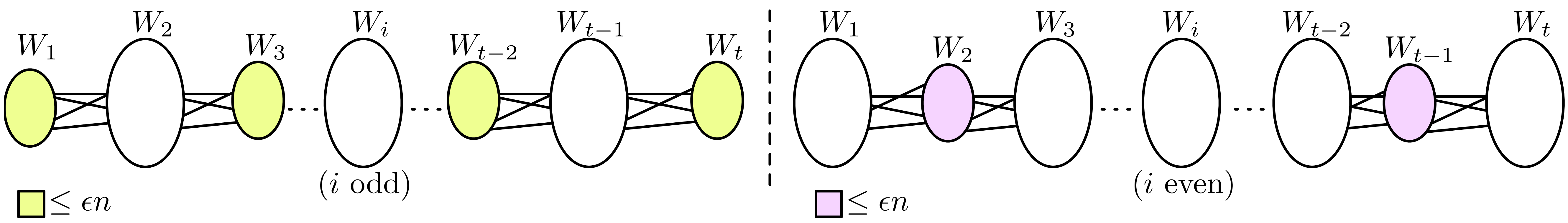


- Find the yellow set, by trying all possibilities.



Large!

NEAR SMALL ODD/EVEN PC

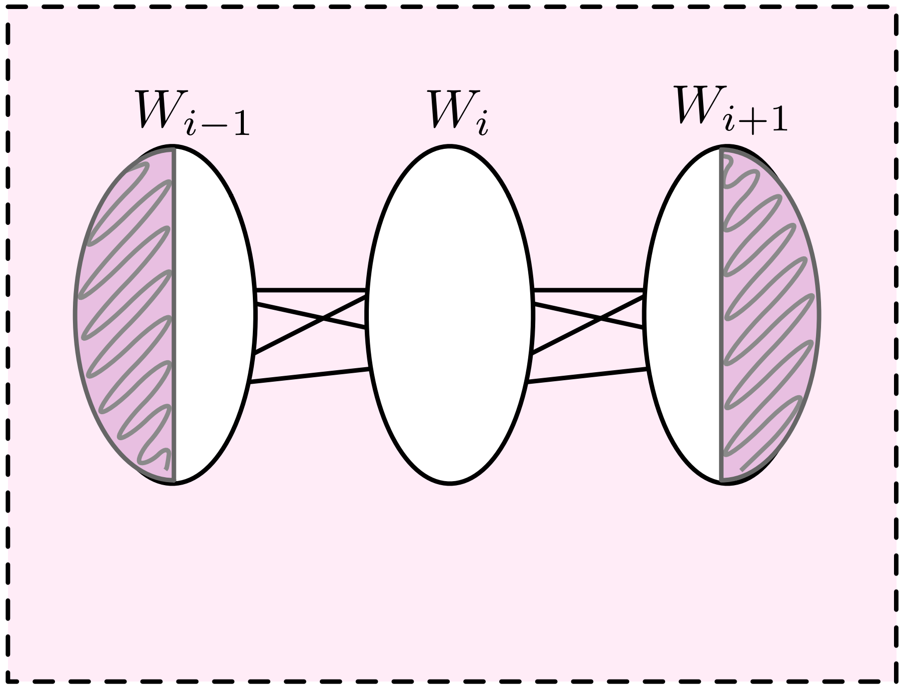


- Find the yellow set, by trying all possibilities.

3-DISJOINT CONNECTED SUBGRAPHS

$O^*(1.88^n)$

[This paper]



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CONCLUSION

- We obtained an algorithm that breaks the 2^n barrier for PATH CONTRACTION, where the improvement is very marginal.
- We gave an $O^*(1.88^n)$ algorithm for 3-DISJOINT CONNECTED SUBGRAPHS, which is a generalisation of 2-DISJOINT CONNECTED SUBGRAPHS.
- Can we obtain substantially better algorithm for PATH CONTRACTION?

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Thanks!