Parameterized Complexity of

Maximum Degree Contraction

IPEC 2020

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Short Version

Maximum Degree Contraction

Input: Graph G, ints k, d

Question: Can we contract at most k edges in G s.t. the maximum degree in the resulting graph is at most d?

Para: k + d

Parameterized complexity of three edge contraction problems with degree constraints. — Belmonte et. al. *Acta Informatica*, (2014).

Thm 1: Algo running in time (k + d)^O(k).

Thm 2: When d = 2, linear kernel => 2^O(k) algo.

Q: Poly kernel k + d?

R1: No algo running in time $n^{o(k)}$ unless ETH fails.

No algo running in time $(k + d)^{o(k)}$ unless ETH fails.

Thm 1: Algo running in time $(k + d)^{O(k)}$.

(Optimum when k, d both part of input)

R2: Algo running in time $2^{O(kd)}$.

Thm 2: When d = 2, linear kernel => $2^{O(k)}$ algo.

Q: Poly kernel k + d? R3: No, unless NP $\subseteq$ coNP/poly

R1: No algo running in time $n^{o(k)}$ unless ETH fails.

(k x k)-MultiColored-Clique

R2: Algo running in time $2^{O(kd)}$.

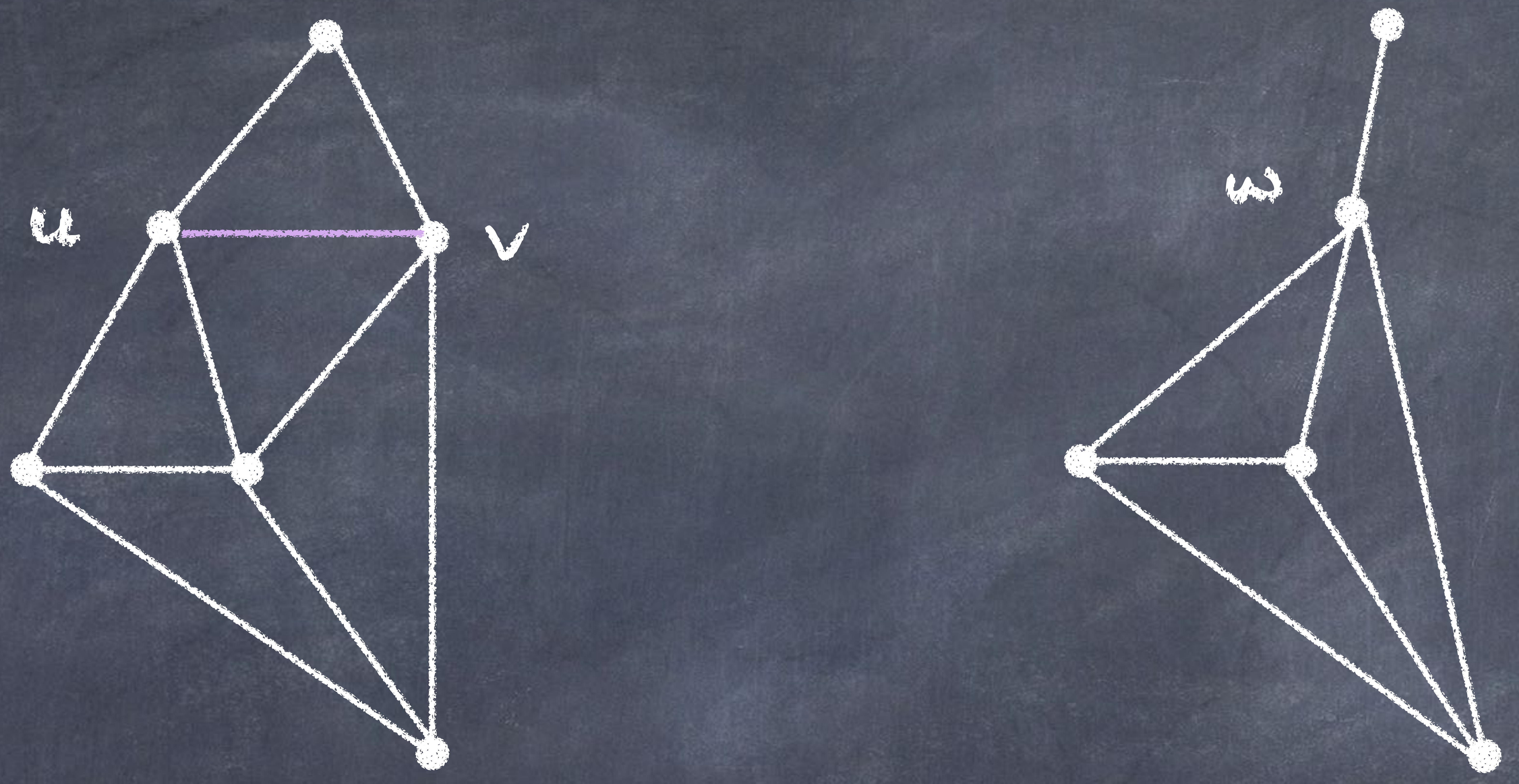
Universal Sets + Branching

Q: Poly kernel k + d?

R3: No, unless NP ⊆ coNP/poly

RBDS para |B| + log |R|

Longer Version



Edge contraction

No self-loop
No parallel edges

Known Results

Our Contributions

FPT Algorithm

Open Questions

$\mathcal{G}$ – Contraction

Input: Graph G, int k

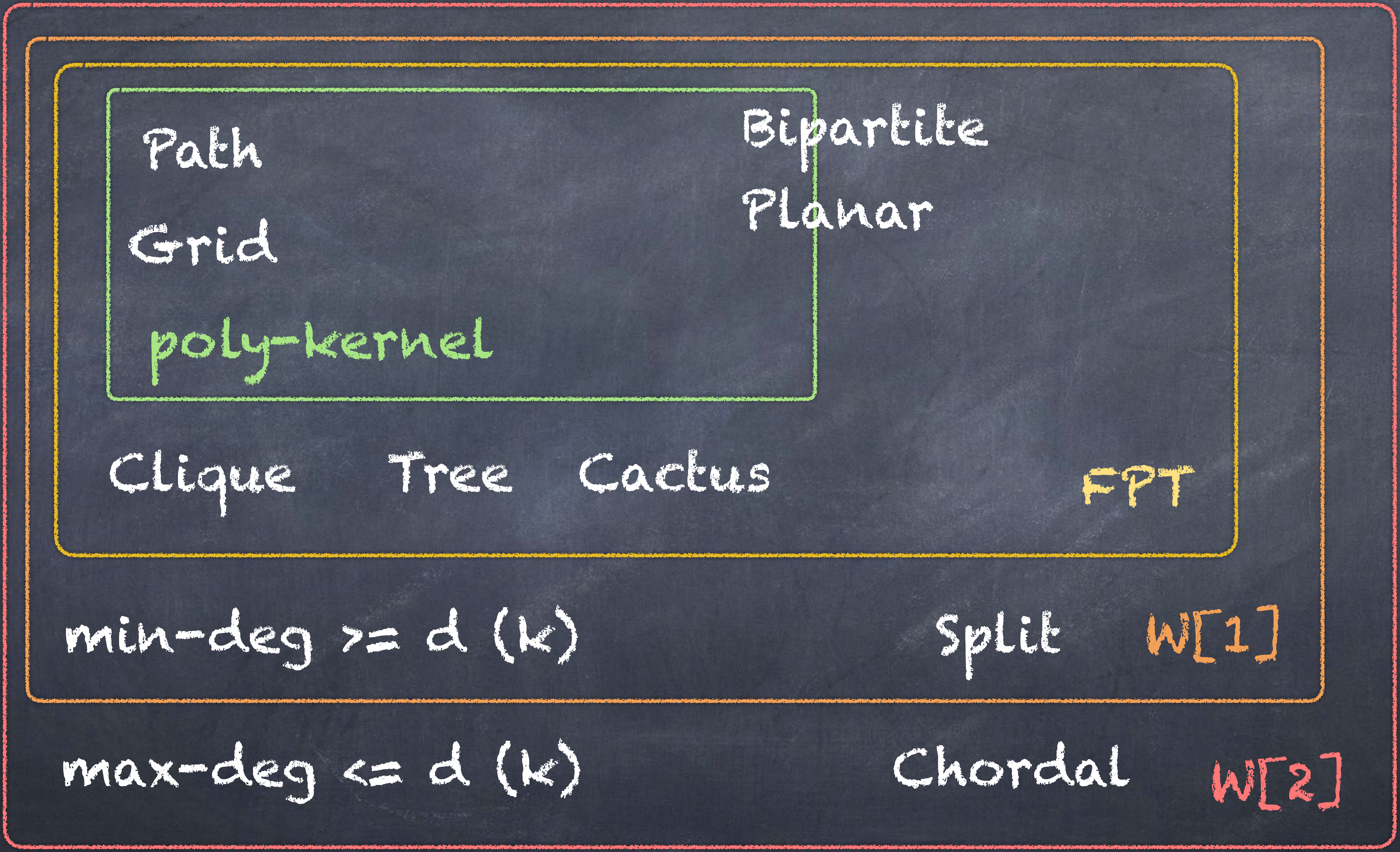
Question: Can we contract at most k edges in G s.t. resulting graph is in $\mathcal{G}$?

Para: k

Path Tree Cactus Planar Bipartite Grid

Clique Split Chordal

max-deg <= d (k) min-deg >= d (k)


Path
Bipartite
Planar
Grid
poly-kernel
Clique
Tree
Cactus
FPT
min-deg >= d (k)
Split
W[1]
max-deg <= d (k)
Chordal
W[2]

Known Results

Our Contributions

FPT Algorithm

Open Questions

Theorem 1. *Unless ETH fails, there is no algorithm that given any instance* (G, k, d) *of* Maximum Degree Contraction *runs in time* $n^{o(k)}$ *and correctly determines whether it is a* Yes *instance.*

Brute force algo: n^O(k)

Theorem 2. *There is an algorithm that given an instance* (G, k, d) *of* Maximum Degree Contraction *runs in time* $2^{O(dk)} \cdot n^{O(1)}$ *and correctly determines whether it is a* Yes *instance.*

Lower bound: 2^o(kd)

Theorem 3. *Unless* NP ⊆ coNP/poly, Maximum Degree Contraction, *parameterized by* k + d, *does not admit a polynomial compression.*

Known Results

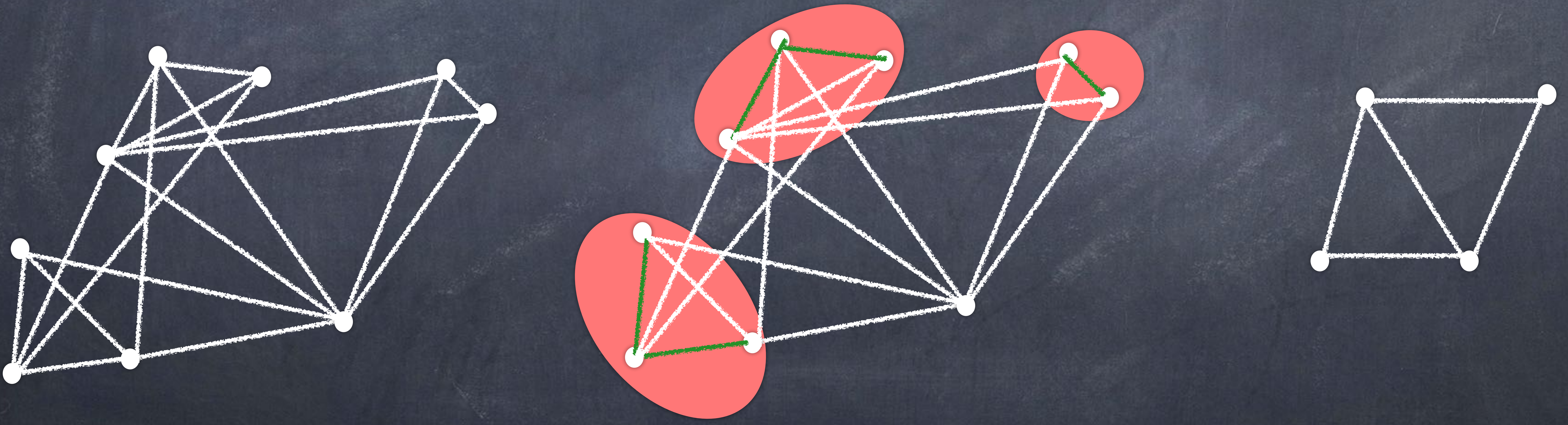
Our Contributions

FPT Algorithm

Open Questions

Max-Deg Contra:: Partition of V(G) s.t.

(a) Each part (called witness set) is connected
(b) Any witness set is adj with at most d other



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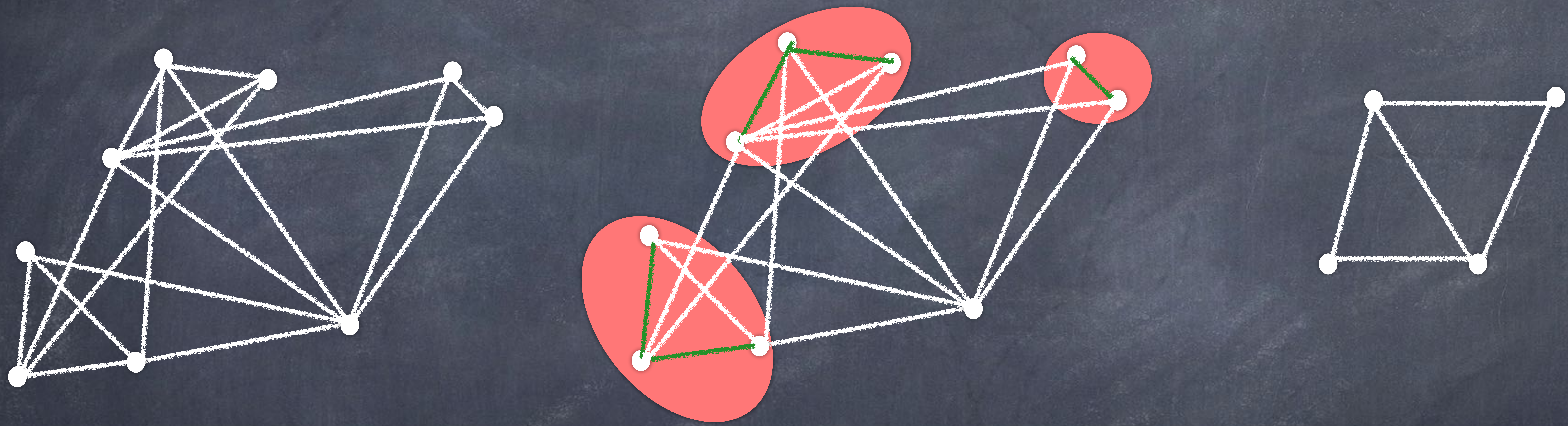


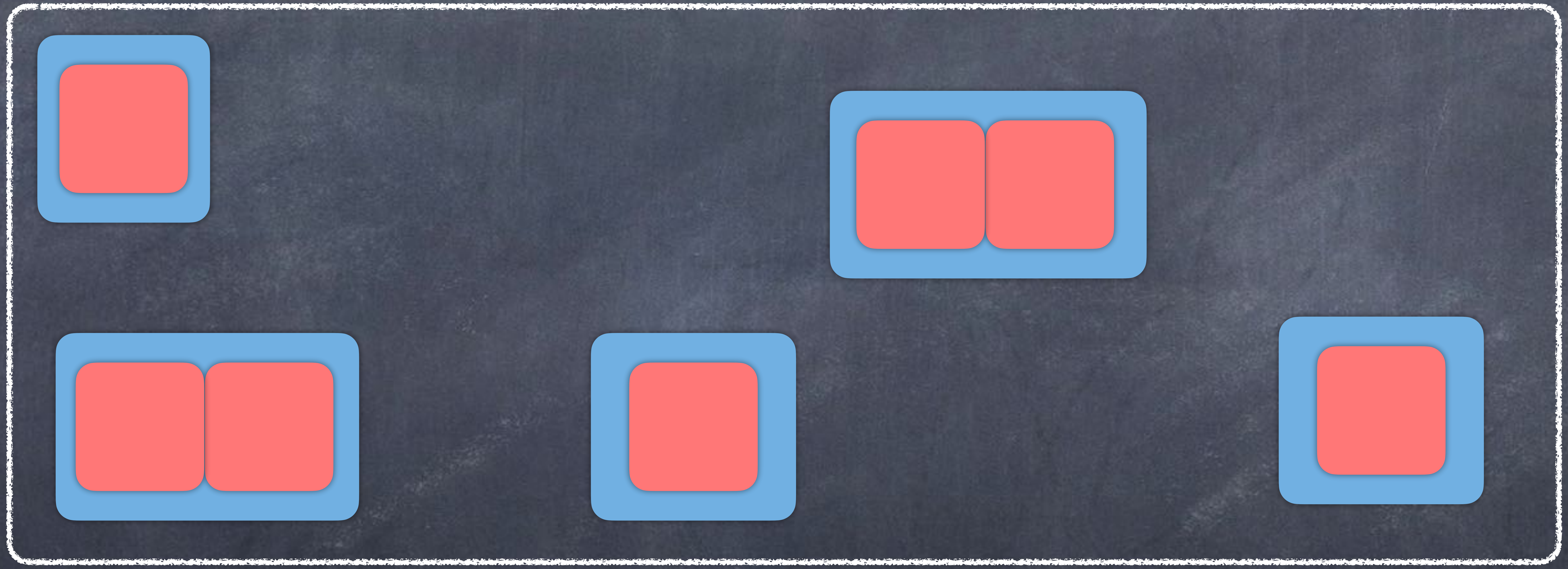
(a) + Solution size => 2k vert in big witness sets

(b) + Solution size => kd vert adj to big witness sets

A 2-coloring of V(G) that colors

2k vert in big witness sets with red-color

kd vert adj to big witness sets with blue-color



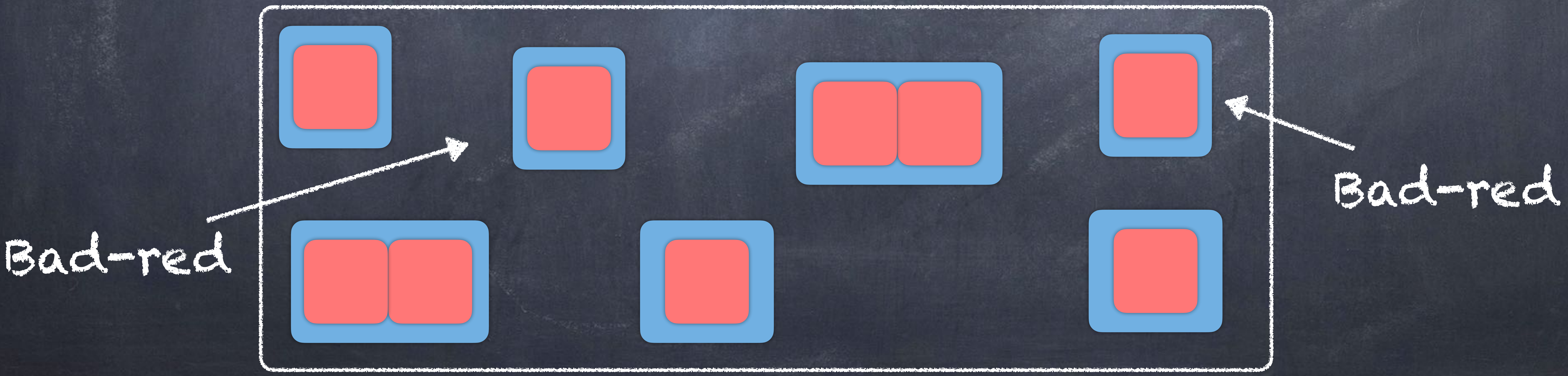
(n, 2k + dk) Universal Sets

(n, 2k + dk) Universal Sets

- Collection of exp(kd) n log n 2-colorings of [n]
- For any set X of size 2k + dk, and its partition (X1, X2), there is a coloring 'compatible' with it.

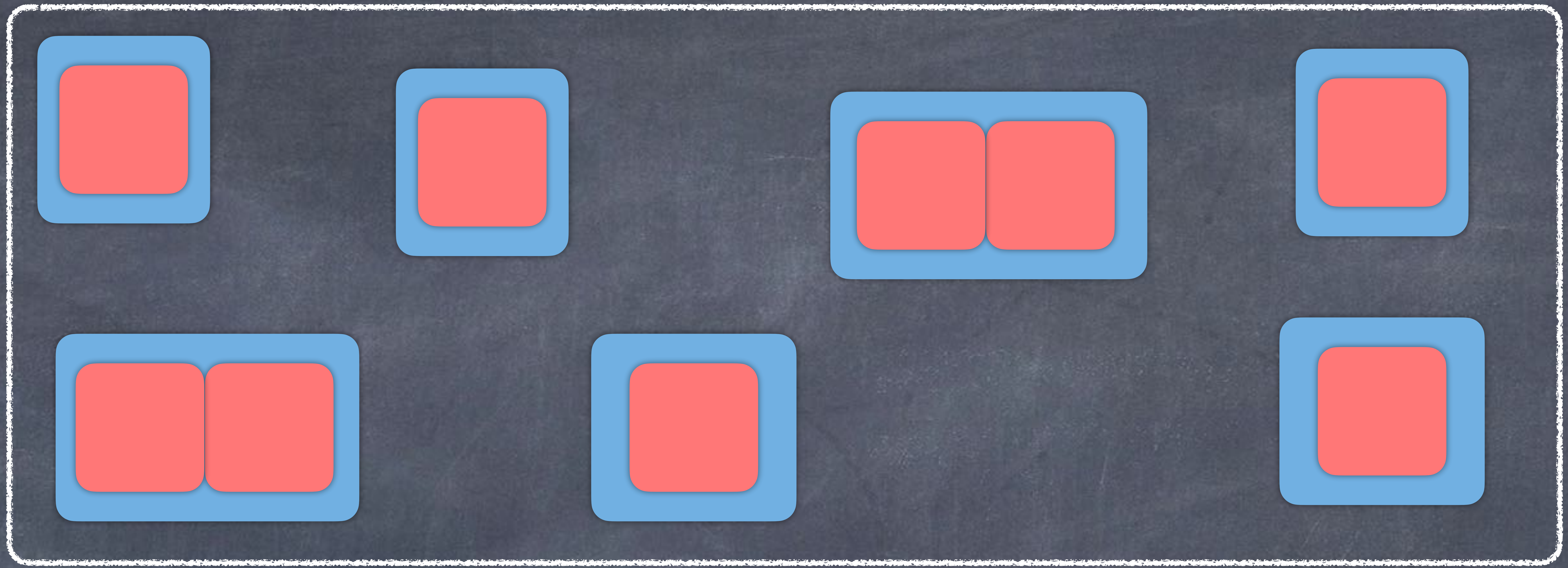
X1 - vert in big witness sets

X2 - vert adj to big witness sets



Bad-red

(Does not contain a big witness set)



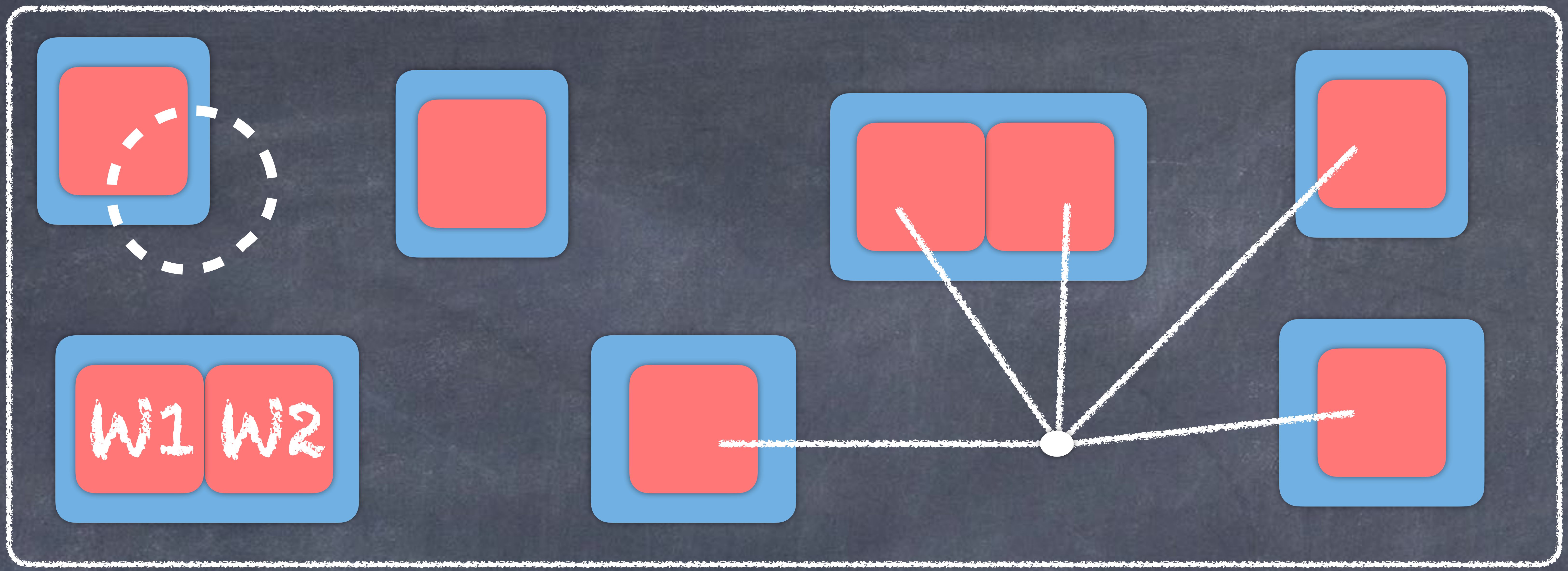
Maximum Degree Contraction <=>

Given a 'compatible' 2-coloring: good-red vs bad-red

Fix a hypothetical solution F. X1 = V(F), X2 = N(V(F))

|F| <= k => |X1| <= 2k; |X2| <= kd

One of 2-colouring in (n, 2k + dk)-Universal Sets is compatible with (X1, X2).



Given a 'compatible' 2-coloring: good-red vs bad-red

- Any witness set is completely in red-part
- Any red-part is either union of witness sets or doesn't intersect any witness set
- Any vertex in blue-part can see at most d red-parts

- Any red-part is either union of witness sets or doesn't intersect any witness set
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```
while( deg(u) >= d + 1 )
   if u in red-part: contract that part
   if u in blue-part: branch over 2^d possibilities
```

Measure (k) drops in each case

Running time = |Universal Sets| * |Branching|
= exp(kd) * exp(kd)

Known Results

Our Contributions

FPT Algorithm

Open Questions

Q1: When $d = 3$, does MAXIMUM DEGREE CONTRACTION admits a poly?

- $d = 2$, kernel of size $O(k)$ on connected graph
- arbitrary d, no poly kernel

Q2: Lossy kernel for MAXIMUM DEGREE CONTRACTION?

Thank you!