Domination and cut problems on chordal graphs with bounded leafage

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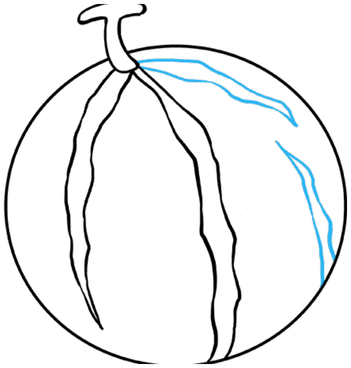
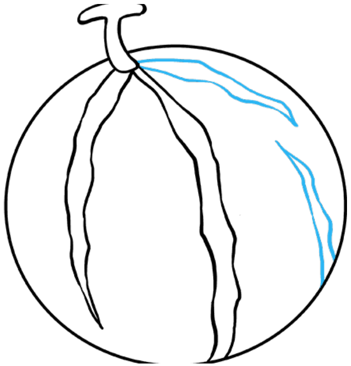
International Symposium on Parameterized and Exact Computation 2022

Chordal Graphs



No induced cycle of length four or more.

Chordal Graphs



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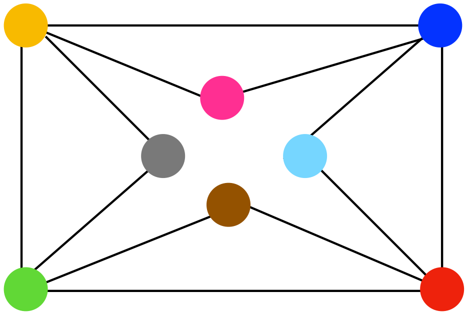

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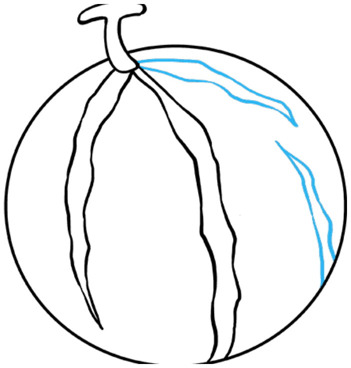

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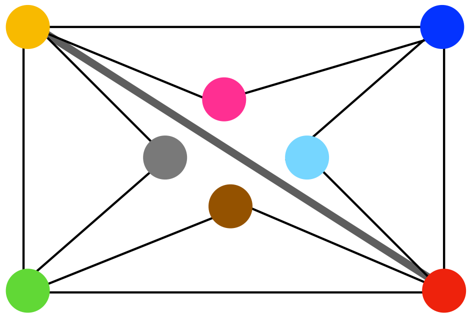

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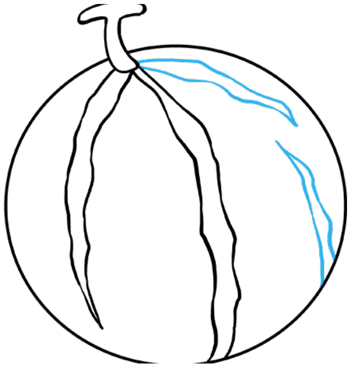

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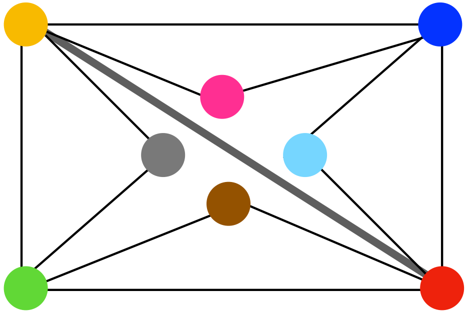

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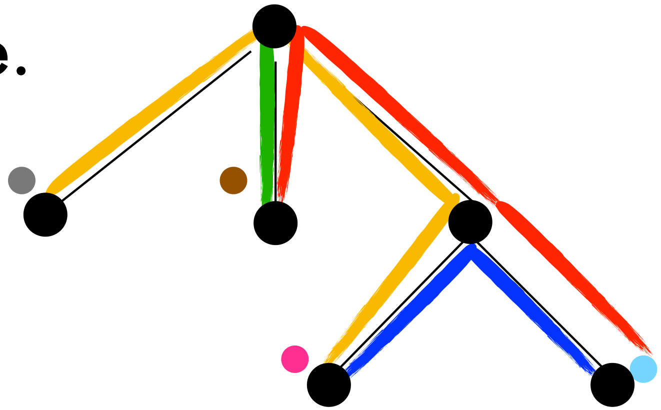
Intersection graph of sub-trees of a tree.



Admit tree decomposition where every bag is a clique.

Dominating Set

Input: A graph G, integer k

Question: Does there exist a set S of size at most k such that N[S] =V(G)?

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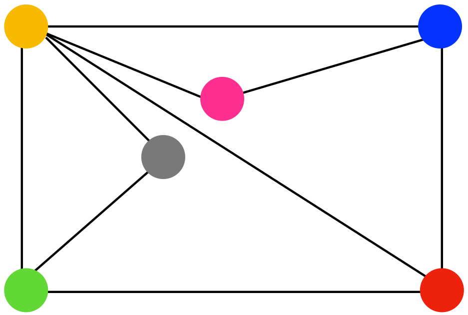
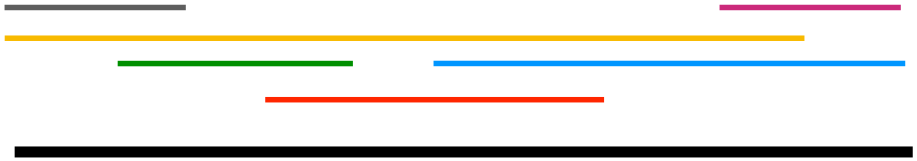
Interval graphs

Split graphs

Interval graphs

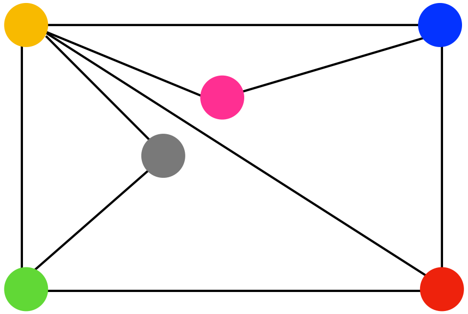


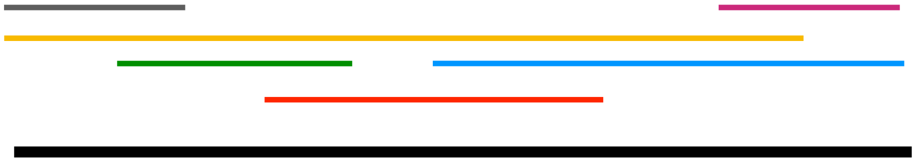


Intersection graph of intervals of a path

Split graphs

Interval graphs





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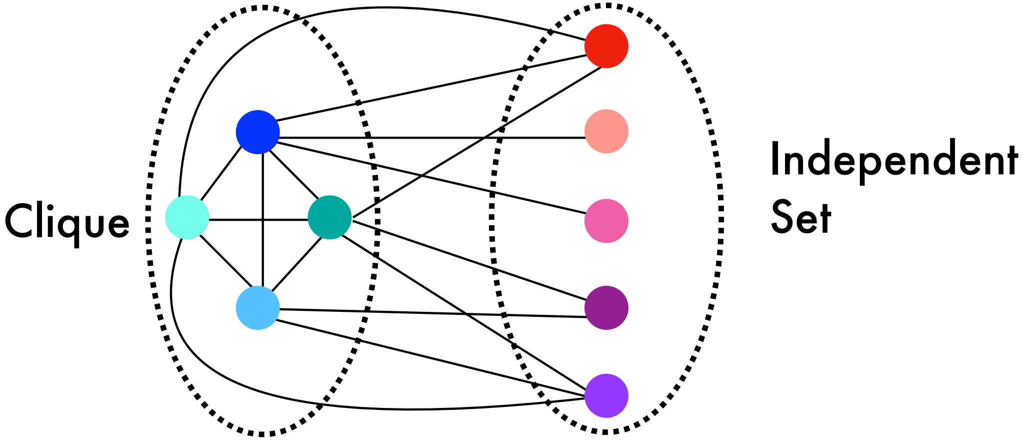

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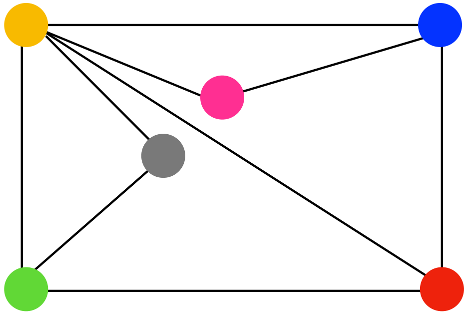

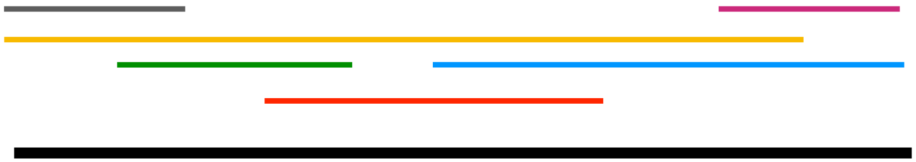

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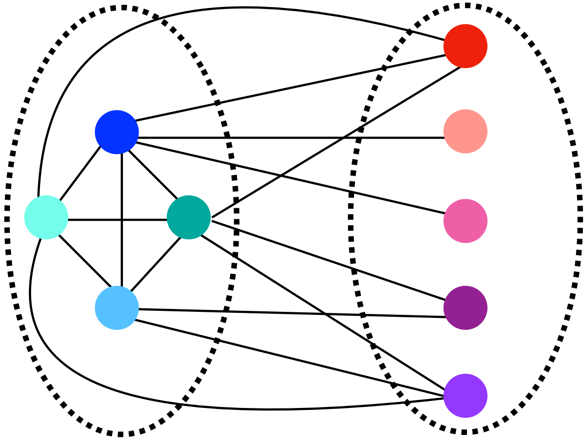

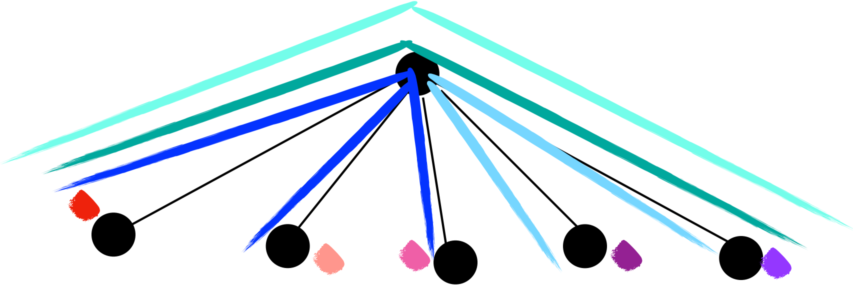

Intersection graph of sub-stars of a star

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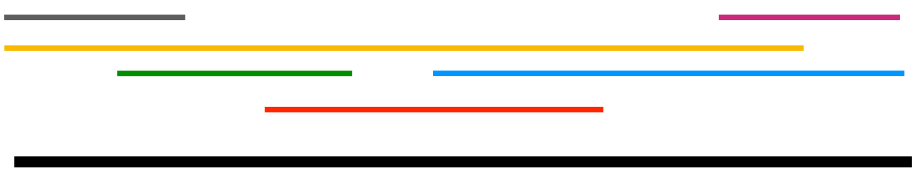
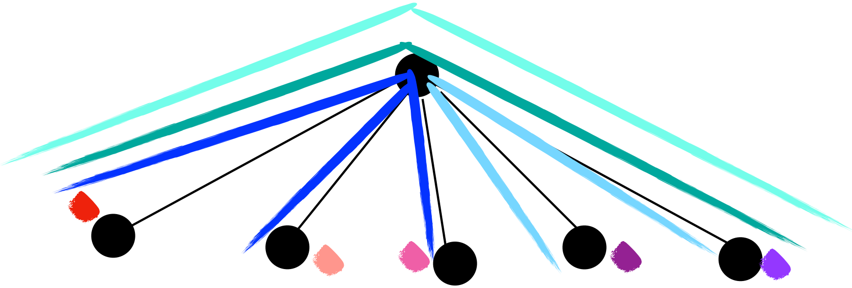
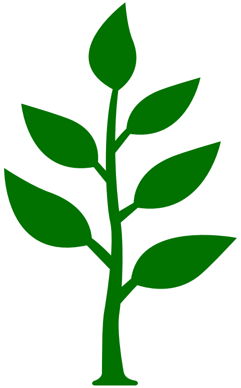
How close is a chordal graph to an interval graph?





Polynomial

NP-hard



Leafage: minimum ℓ
such that a graph is an intersection graph
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Parameterized Complexity on chordal graphs
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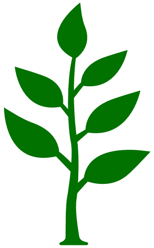
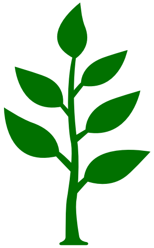
A tree representation of minimum leafage
can be computed in polynomial time
with linear in n nodes [Habib, Stacho ESA 2009].



Parameter: leafage ℓ



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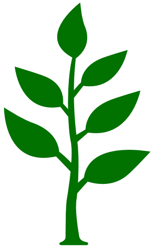
DOMINATING SET



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Dominating Set

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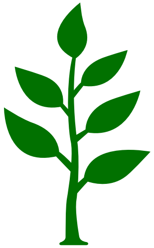

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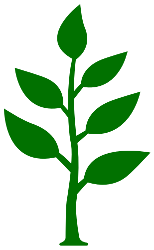

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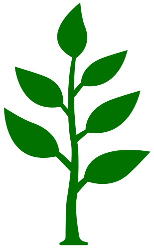
$2^{\mathcal{O}(\ell)}$ [Our Result]

no $2^{o(\ell)}$
under ETH



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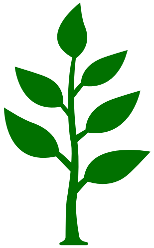
CONNECTED DOMINATING SET

STEINER TREE



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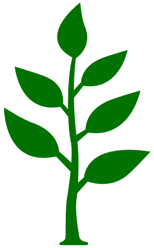

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Multicut

Input: A graph G, terminal pairs (s_1,t_1), ..., (s_p,t_p)

Question: Find a minimum set of non-terminal vertices S such that G-S has no s_i-t_i path.

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Parameter: solution size 2^k FPT [MPR**S**S MFCS 2019]

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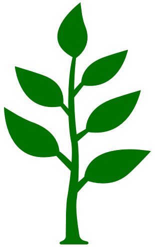

Parameter: leafage ℓ W[1]-complete $n^{\mathcal{O}(\ell)}$ no $n^{o(\ell)}$ under ETH

[Our Result]

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Polynomial-time
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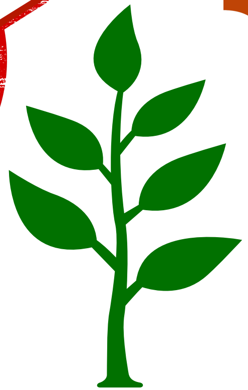


DOMINATING SET

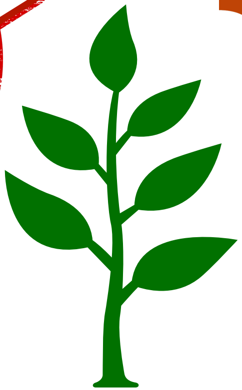
$2^{\mathcal{O}(\ell)}$

MULTICUT

W[1]-complete

MULTIWAY CUT

Polynomial time
(on chordal)

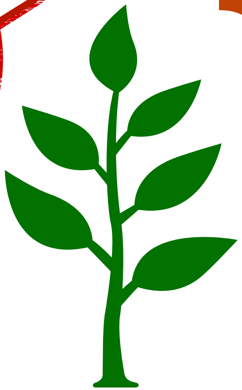


Dominating Set	Multicut	Multiway Cut
$2^{\mathcal{O}(\ell)}$	W[1]-complete	Polynomial time (on chordal)

Branching

Greedy

Hitting Set and Set Cover/ |U|




DOMINATING SET

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MULTIWAY CUT

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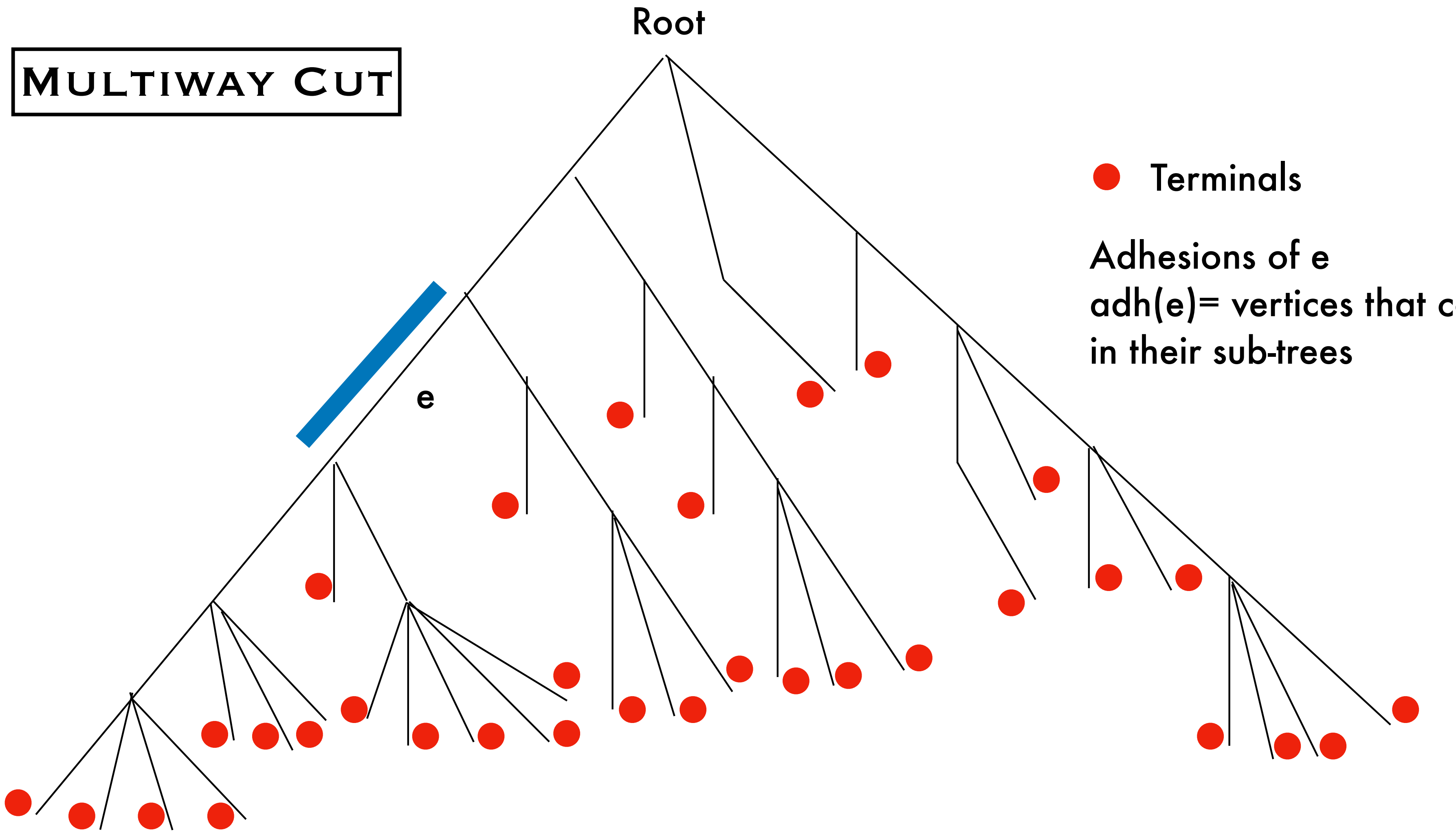
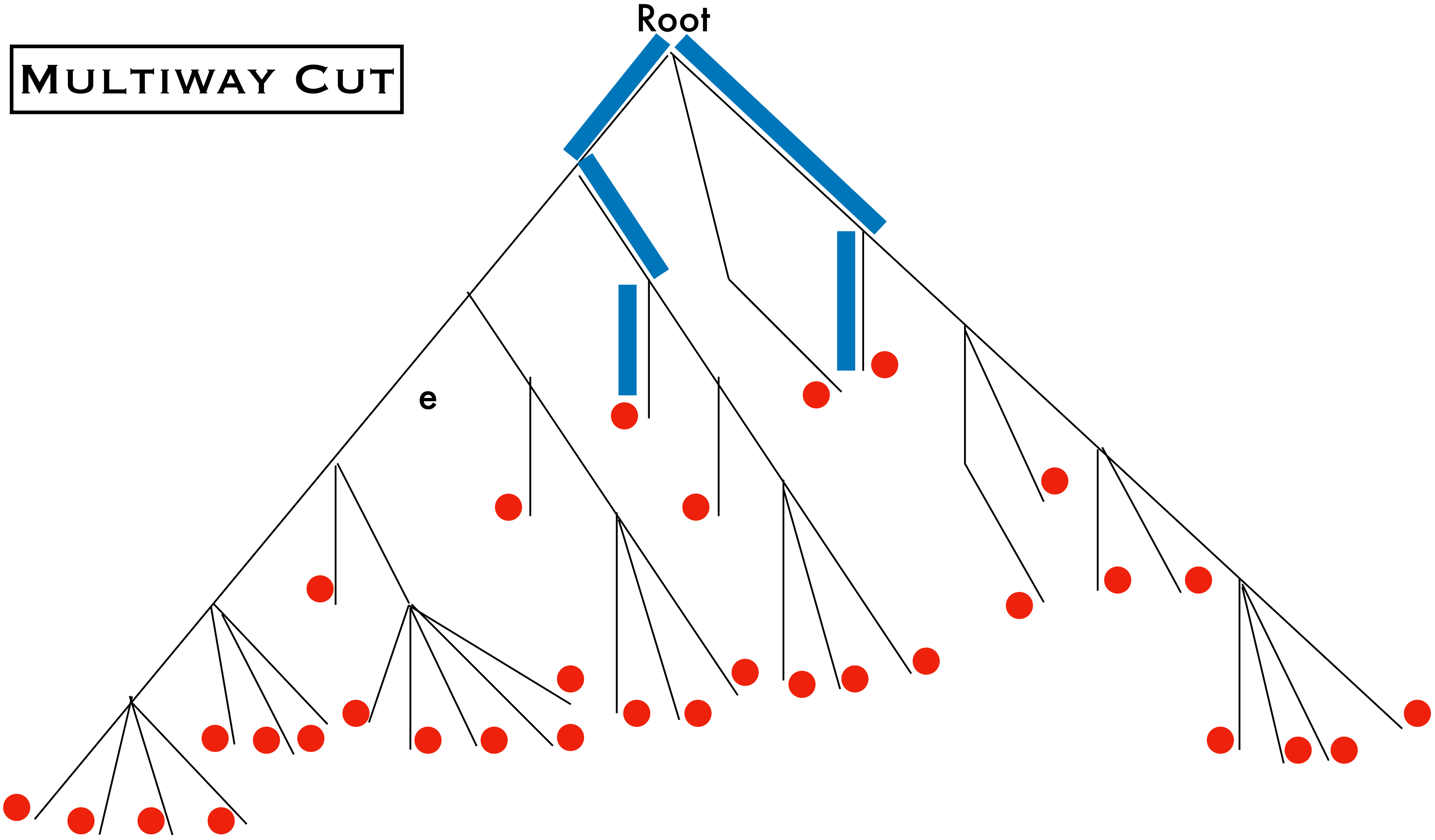
Multiway Cut



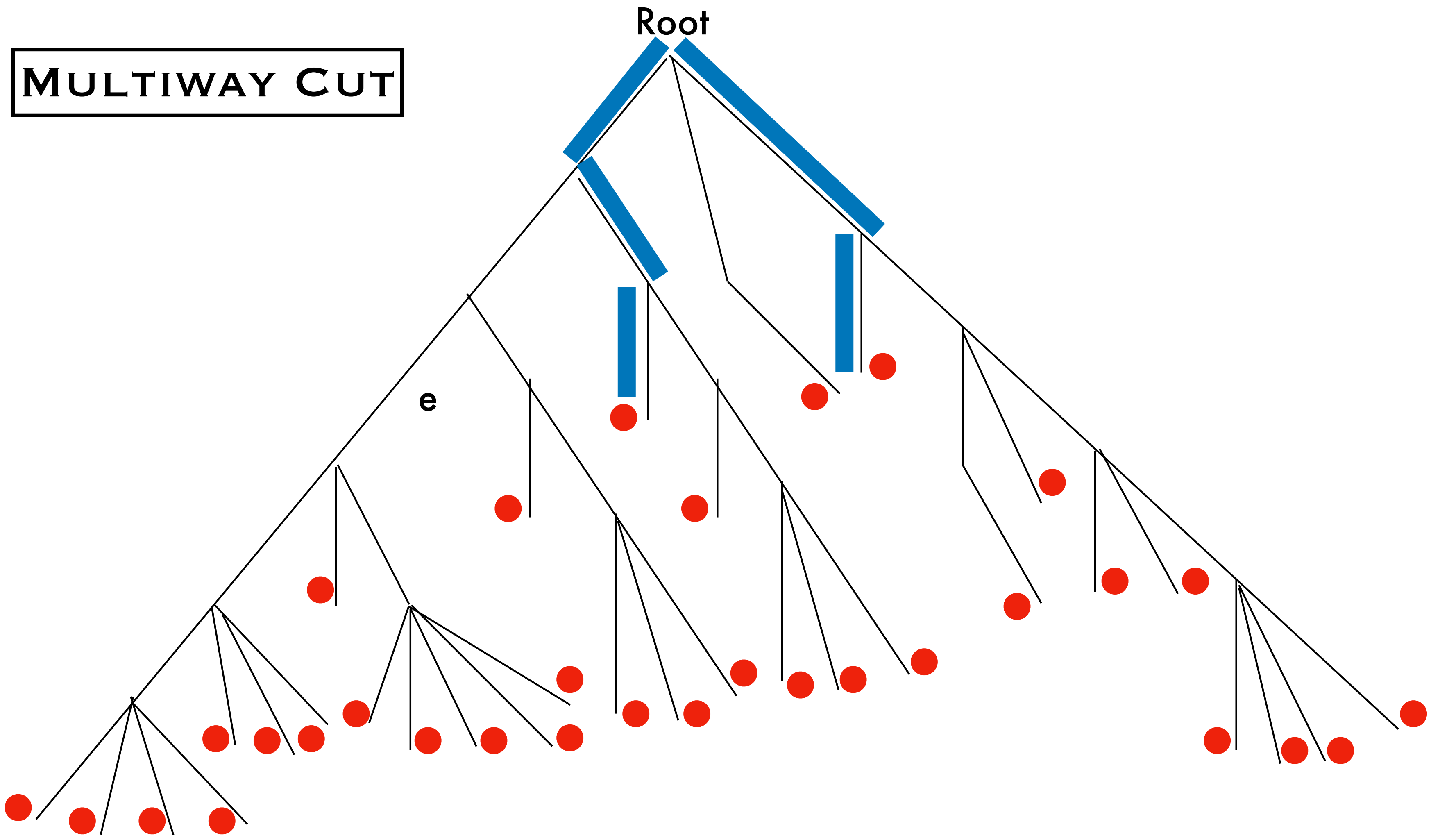
Adhesions of e
adh(e)= vertices that contain e in their sub-trees

Multiway Cut



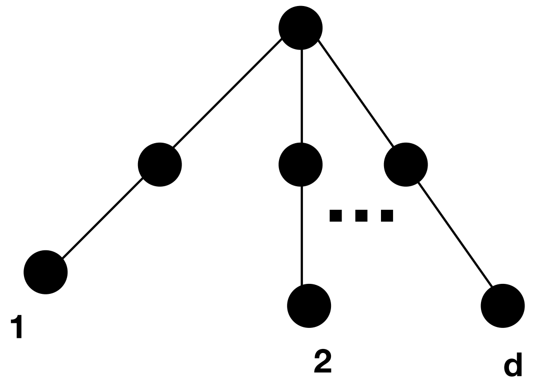
Observation: For each root to leaf path, except at most 1, any solution contains some adhesion of this path.

MULTIWAY CUT



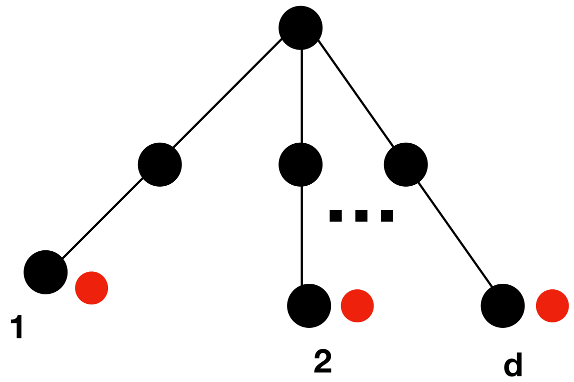
Assume: There exists a solution that contains some adhesion of every root to leaf path.

Multiway Cut* → Weighted Vertex Cover in bipartite graphs



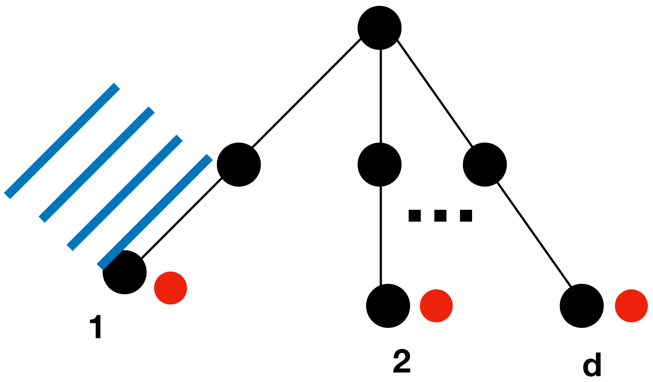
MULTIWAY CUT*

WEIGHTED VERTEX COVER in bipartite graphs



Terminals at leaves

MULTIWAY CUT* → WEIGHTED VERTEX COVER in bipartite graphs



Terminals at leaves

Vertices without branching node

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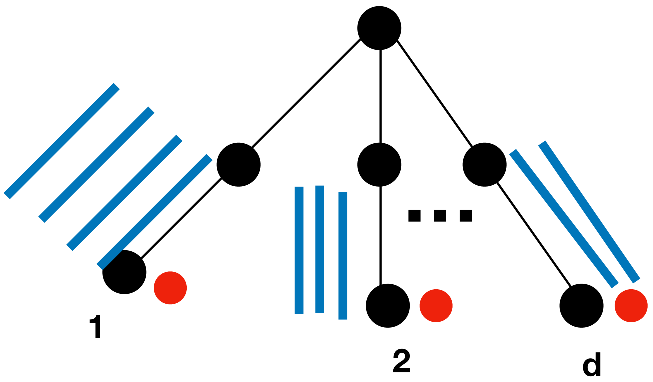

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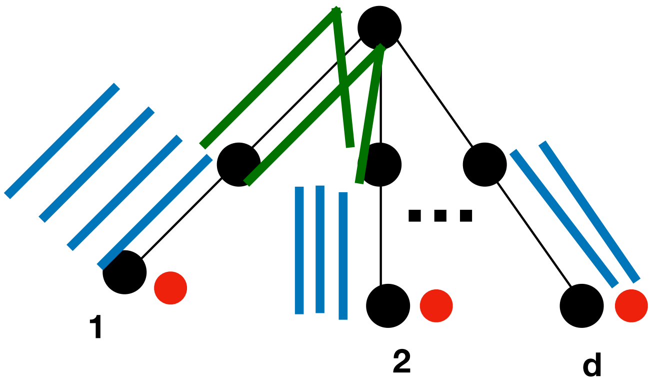
MULTIWAY CUT* → **WEIGHTED VERTEX COVER in bipartite graphs**



- Terminals at leaves
- Vertices without branching node
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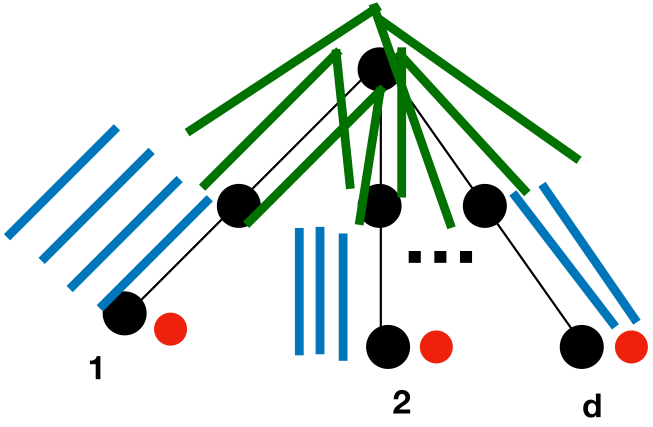
Multiway Cut* → Weighted Vertex Cover in bipartite graphs



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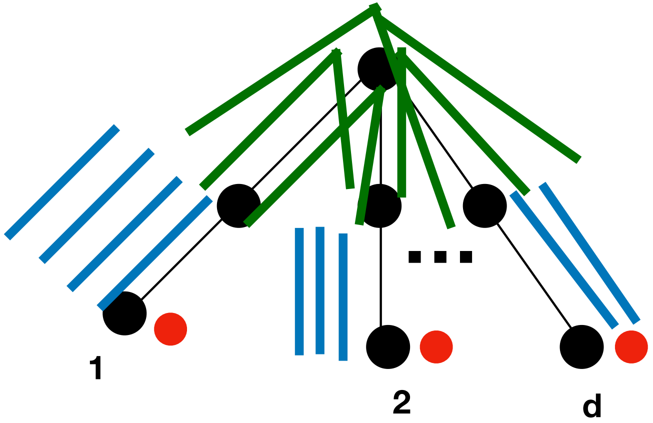

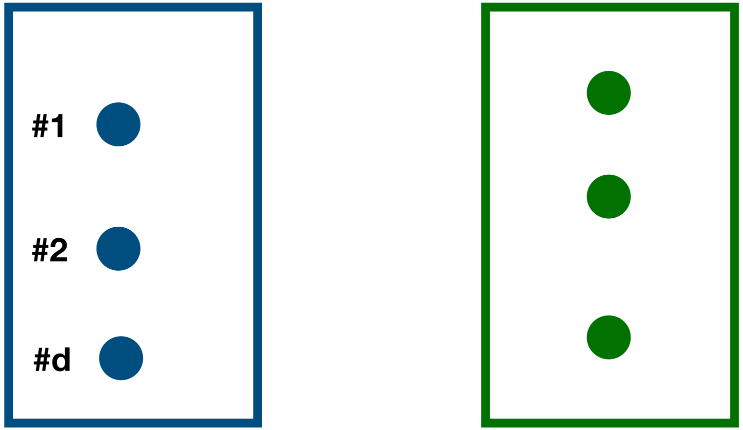

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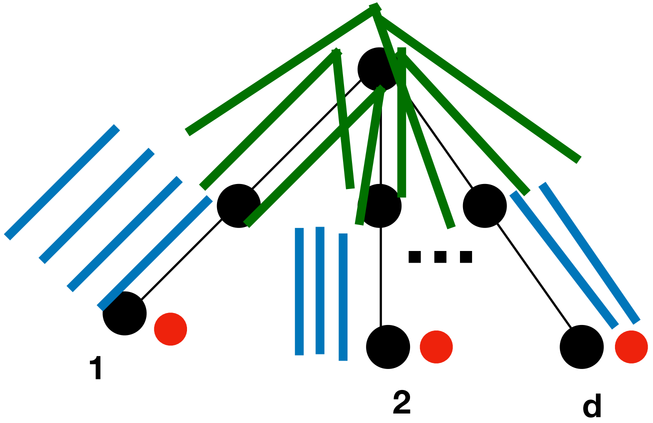

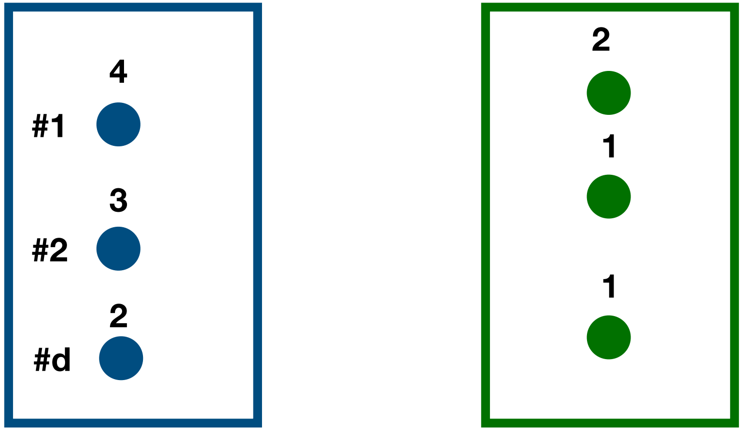

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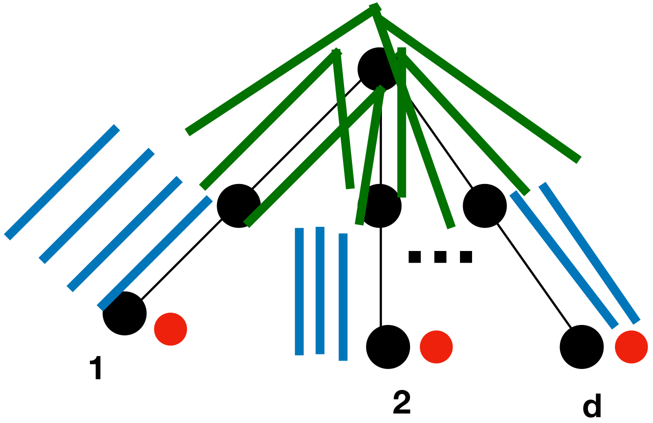

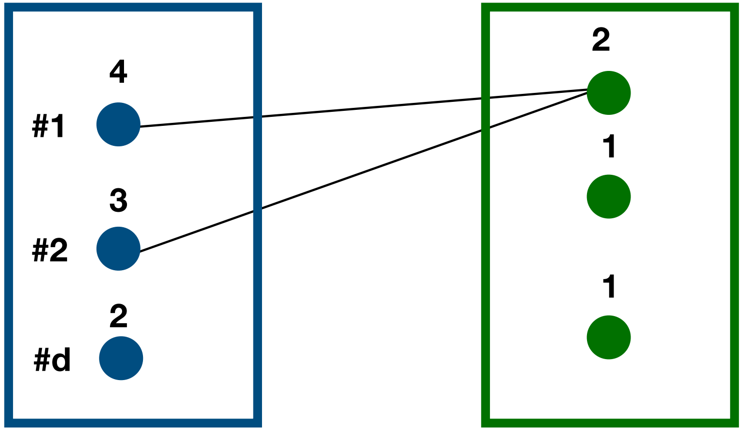

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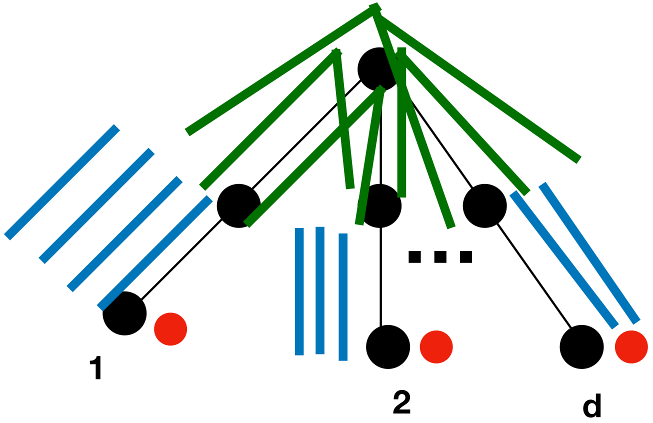

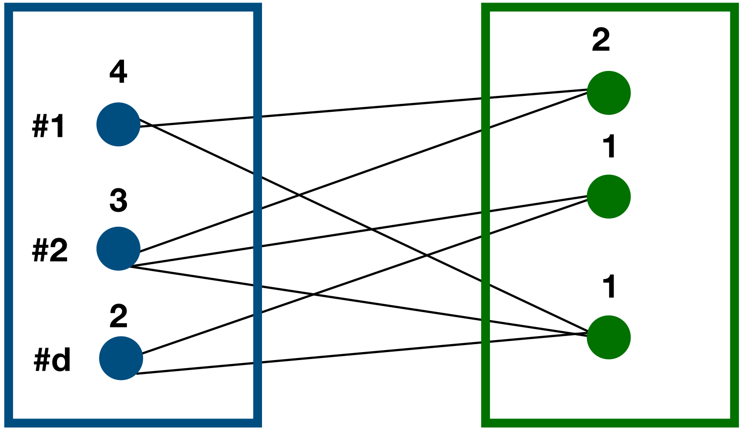

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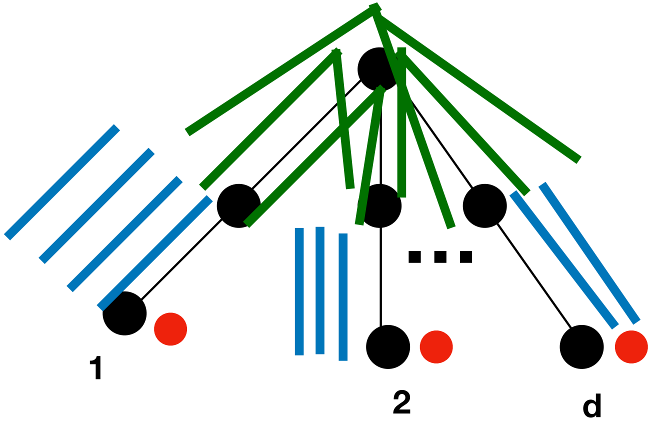

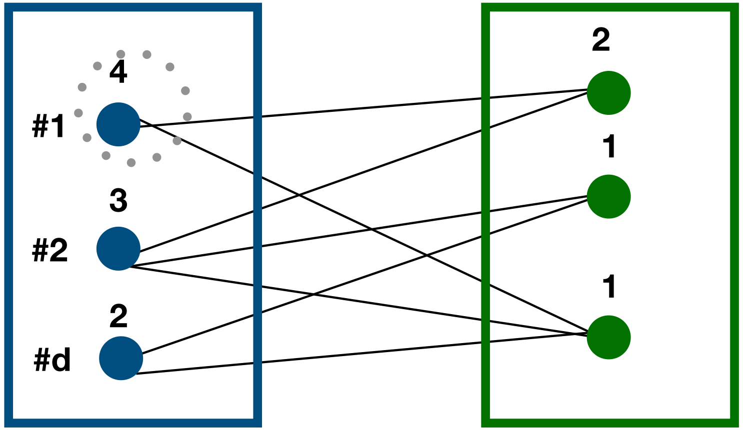

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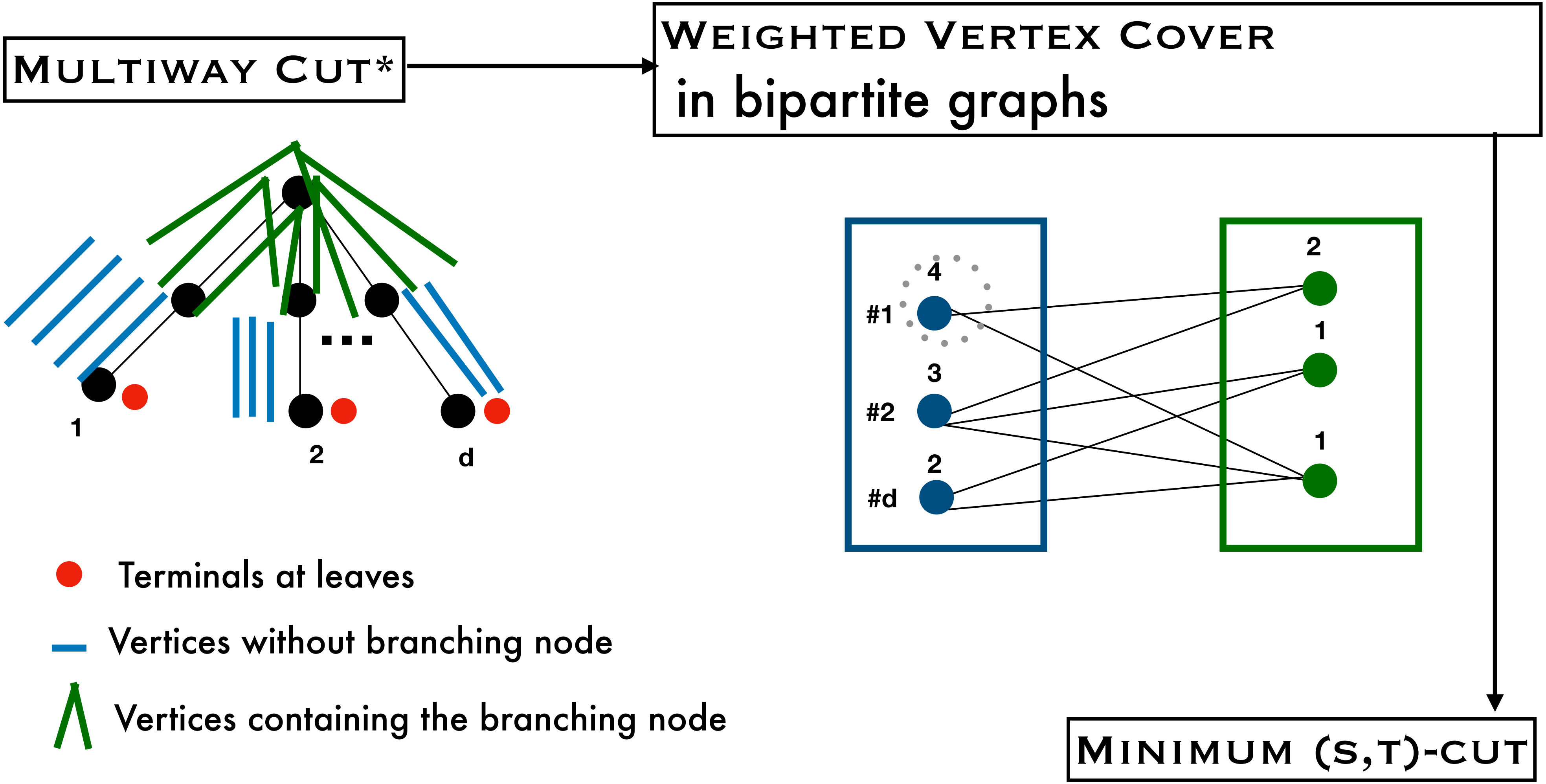
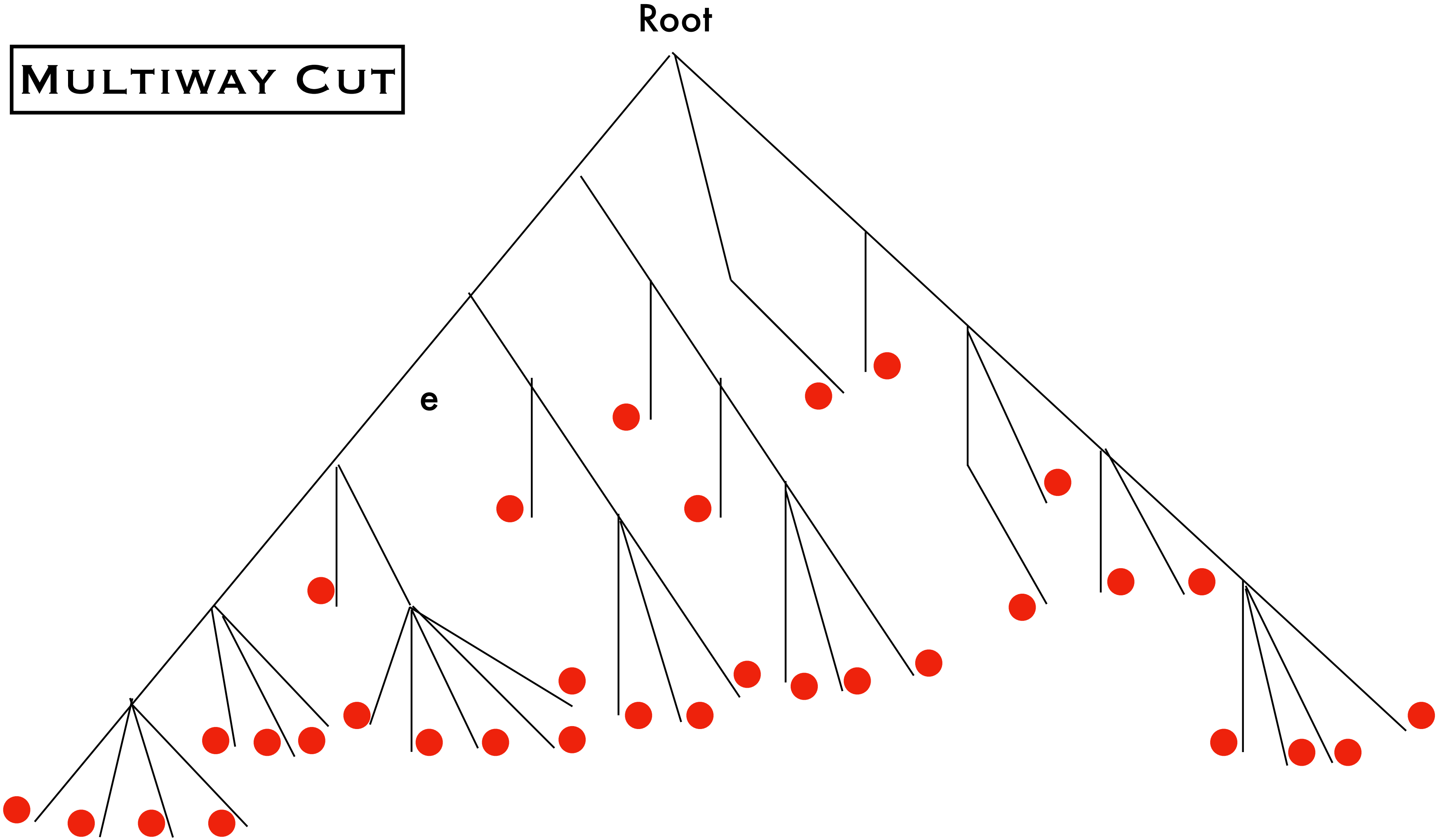


Multiway Cut*
Weighted Vertex Cover
in bipartite graphs
1
2
d
...
#1
#2
#d
4
3
2
2
1
1
Terminals at leaves
Vertices without branching node
Vertices containing the branching node
Minimum (s,t)-cut

Multiway Cut



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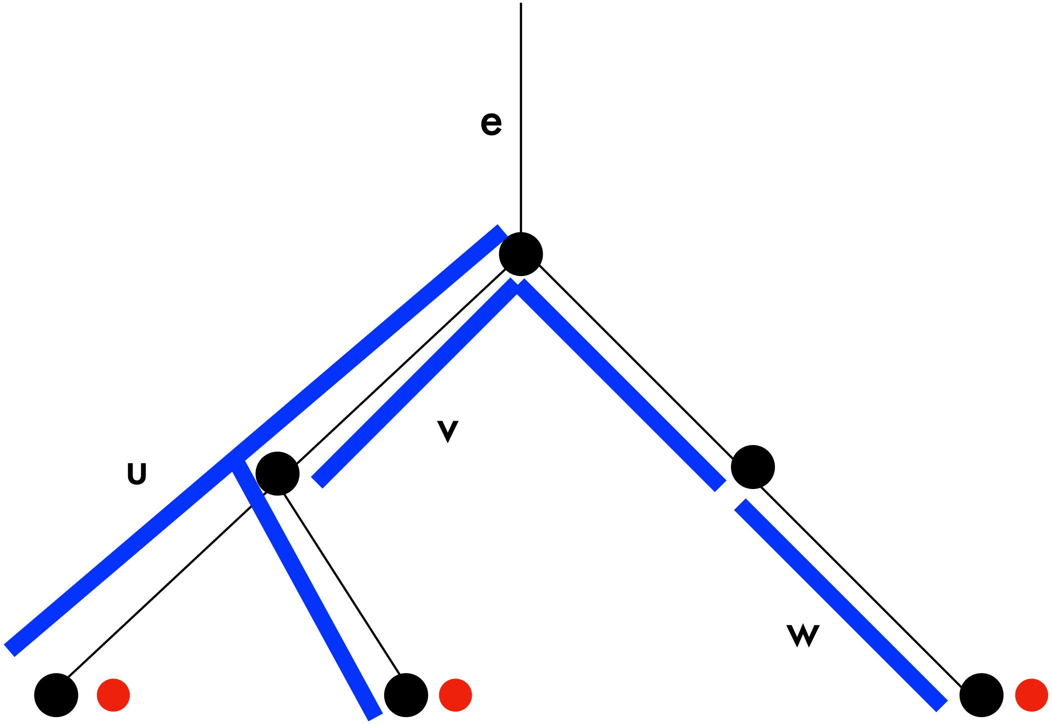

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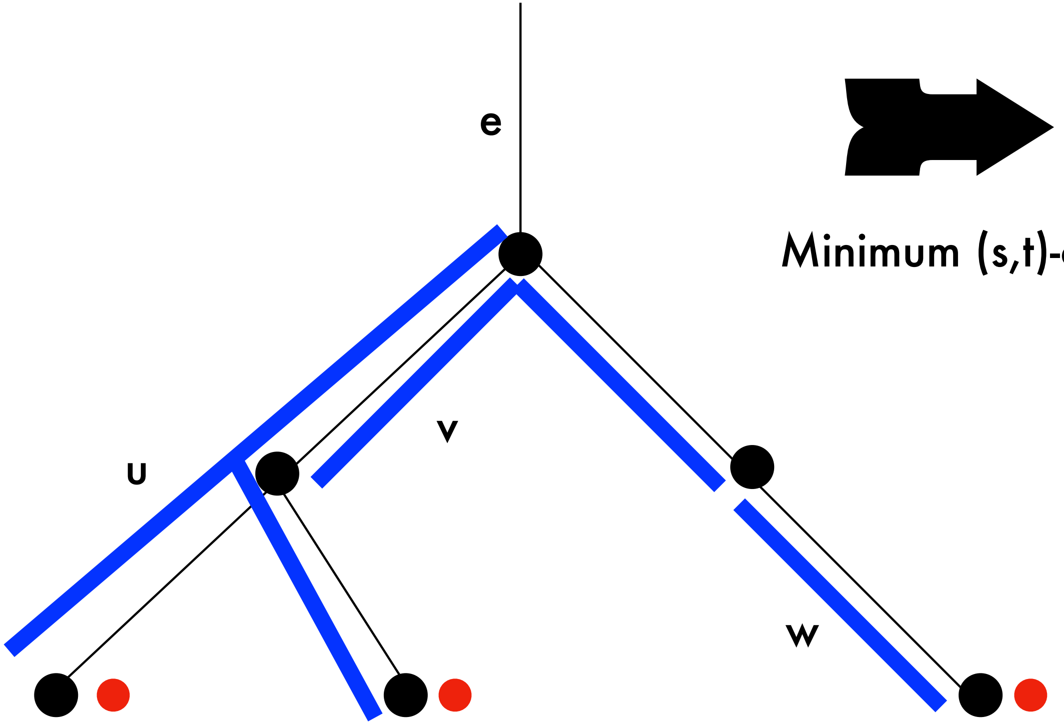

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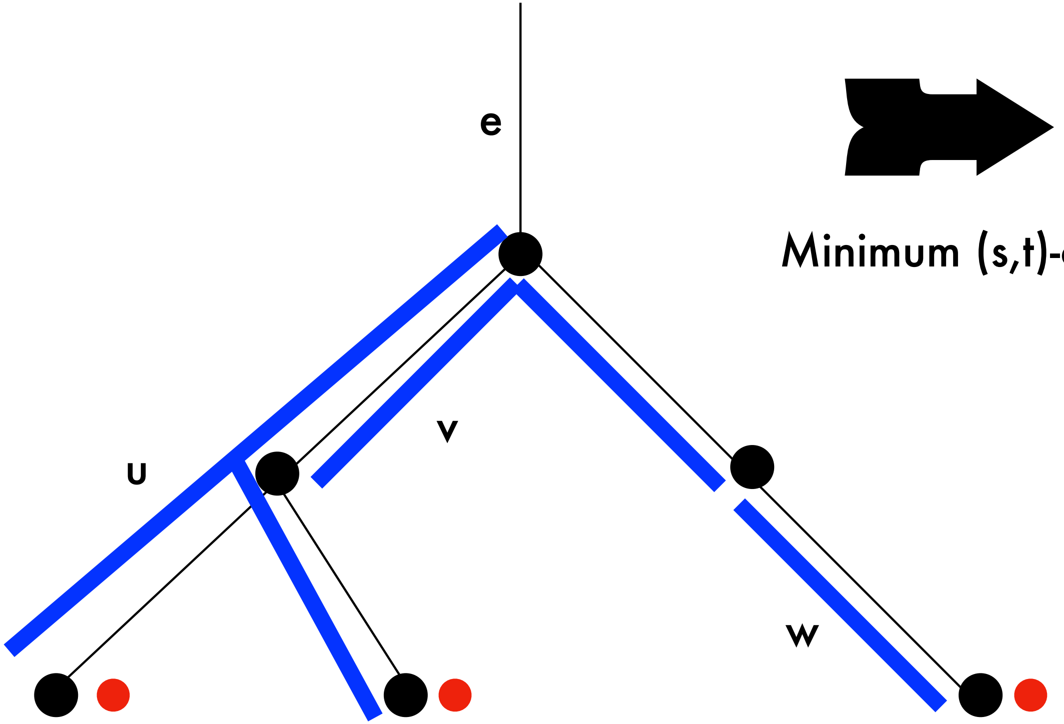

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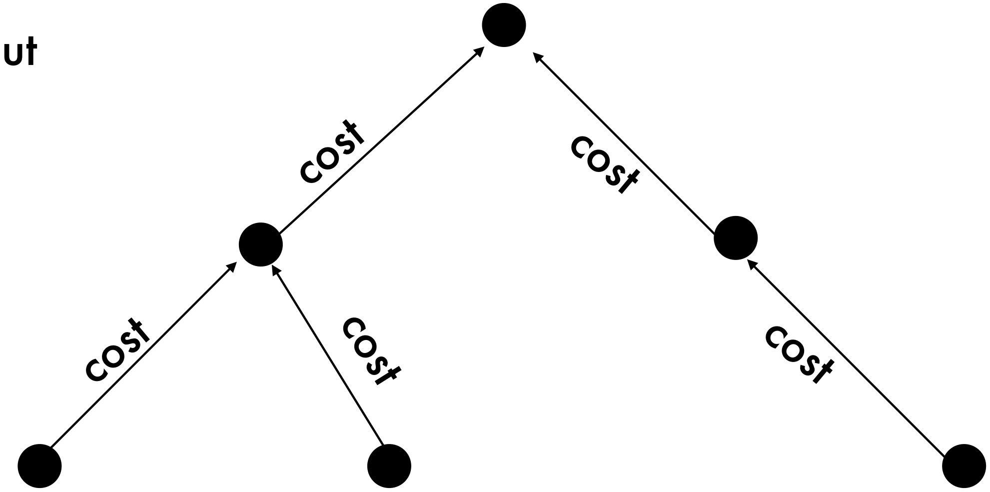

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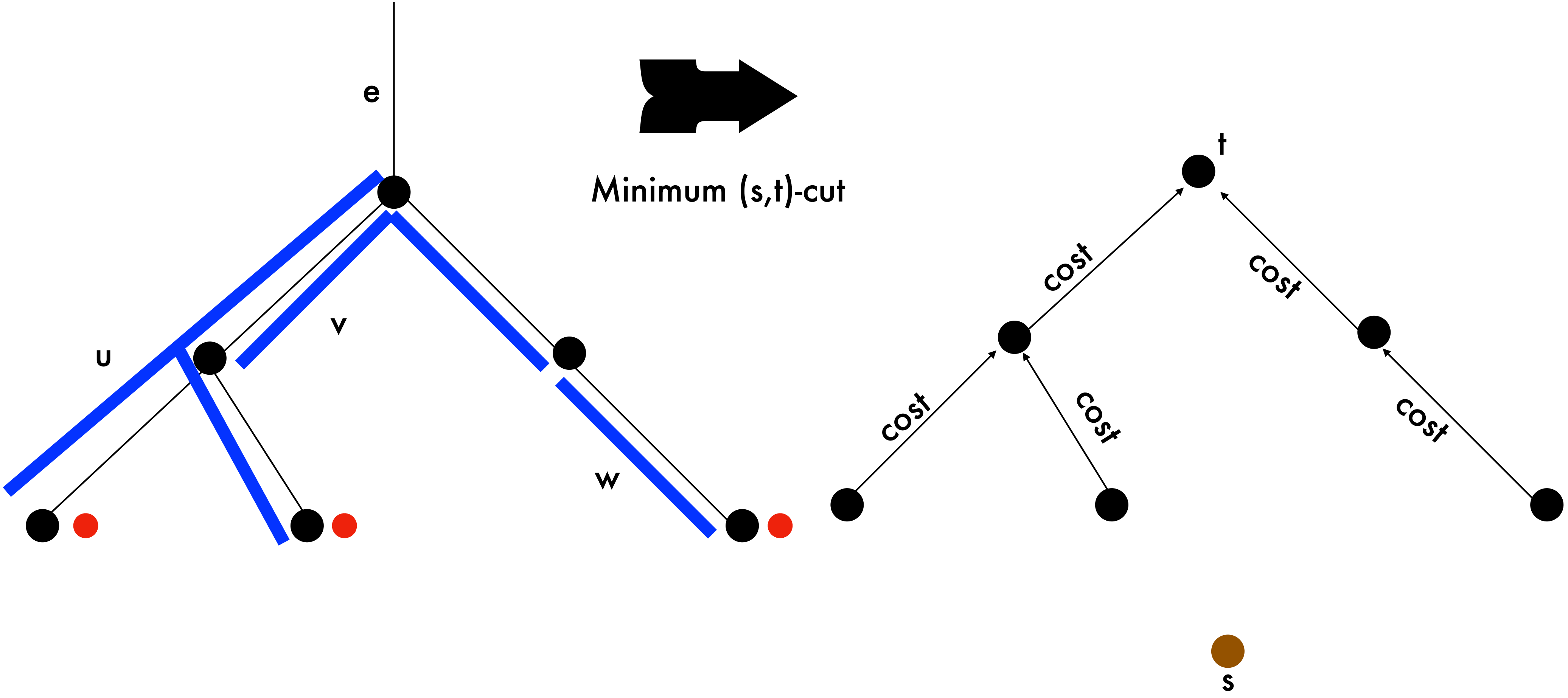

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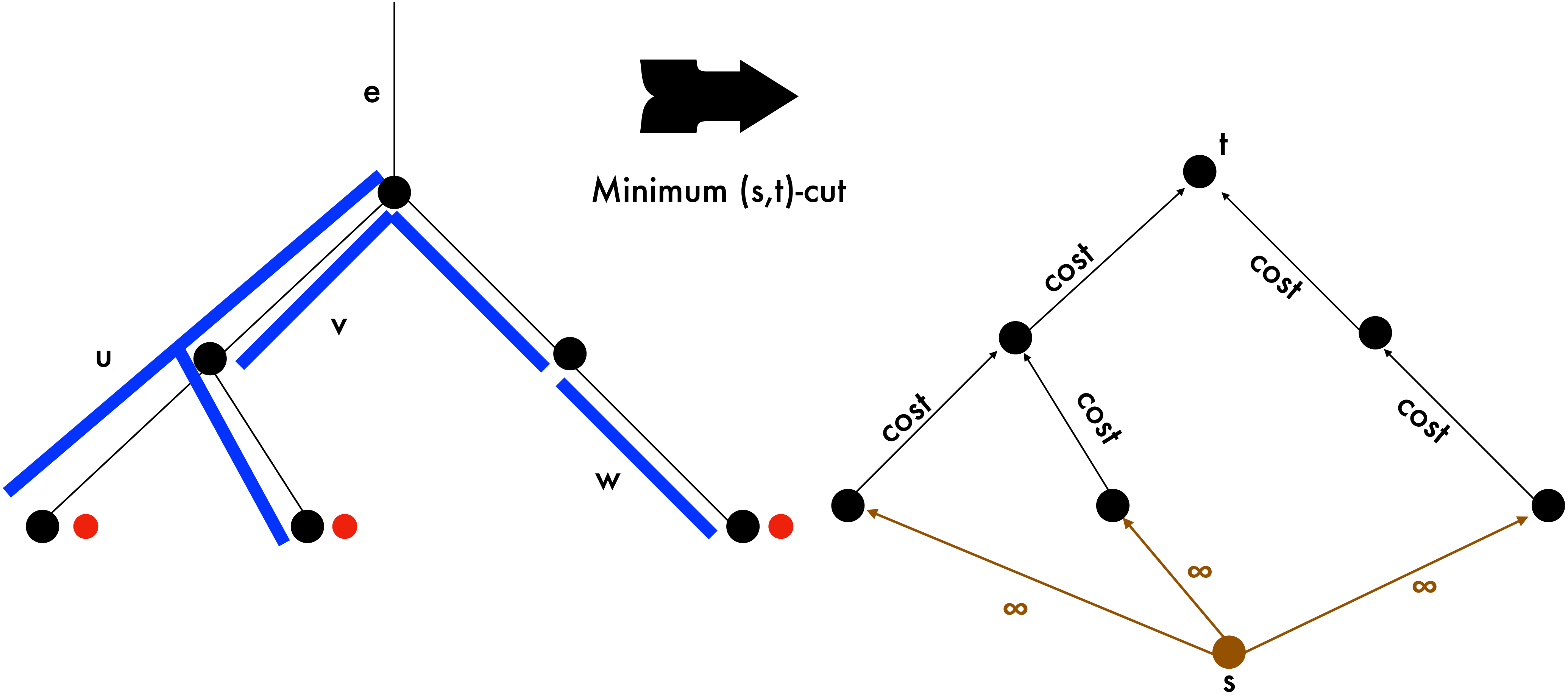

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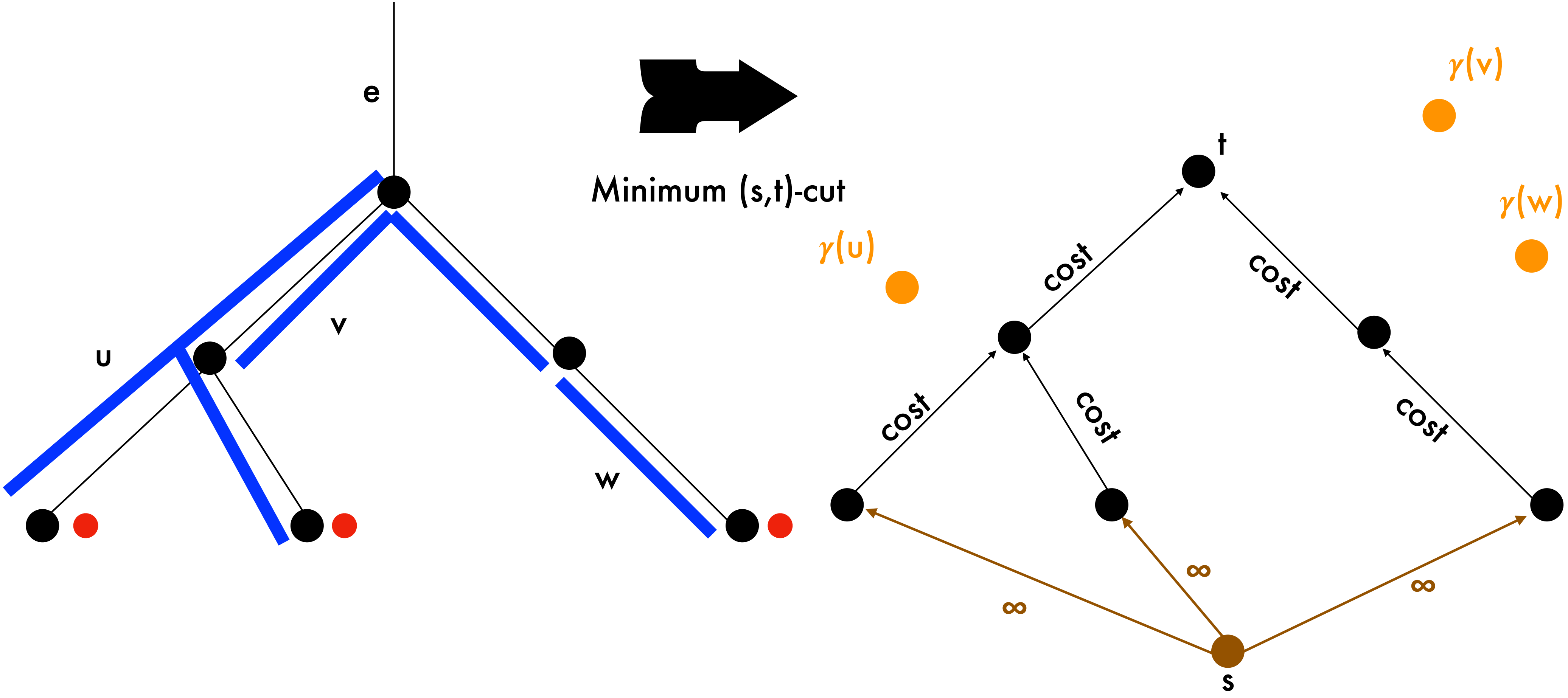

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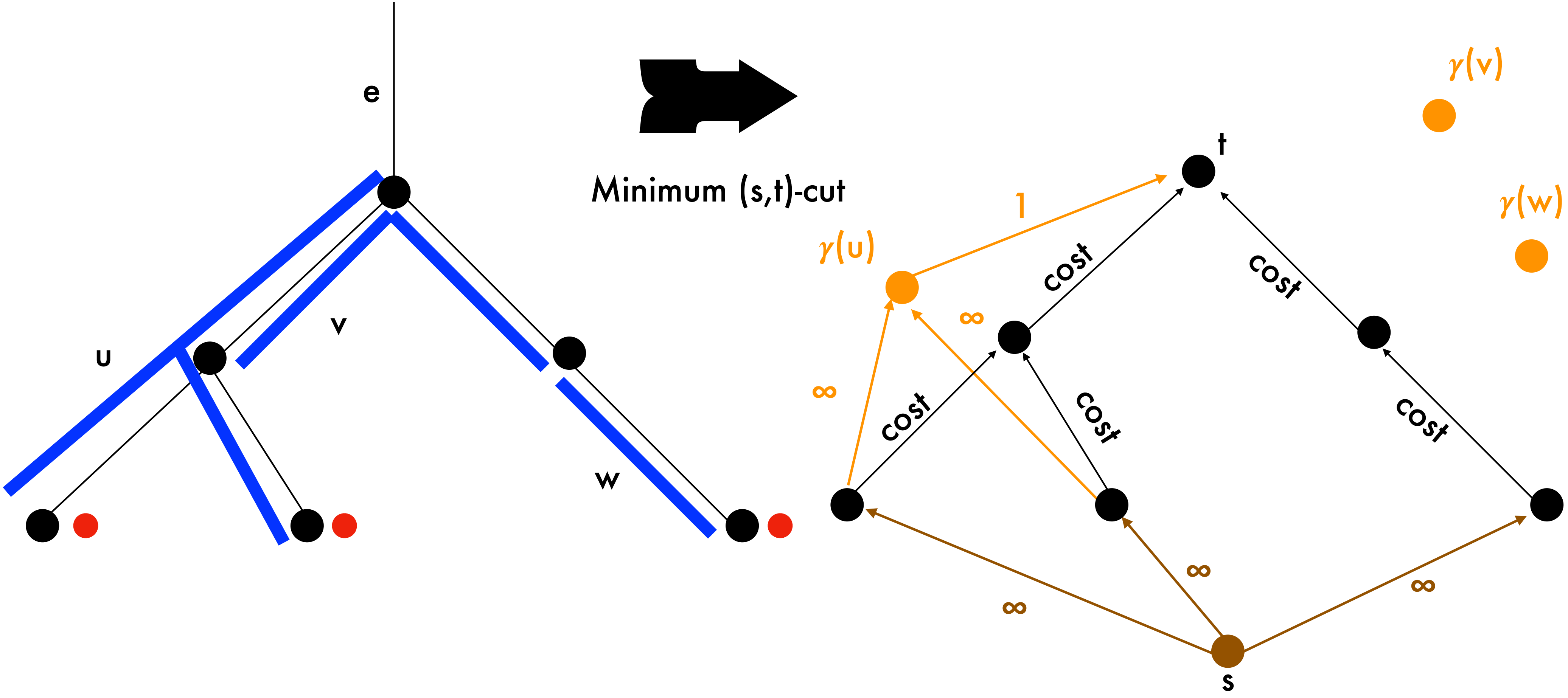

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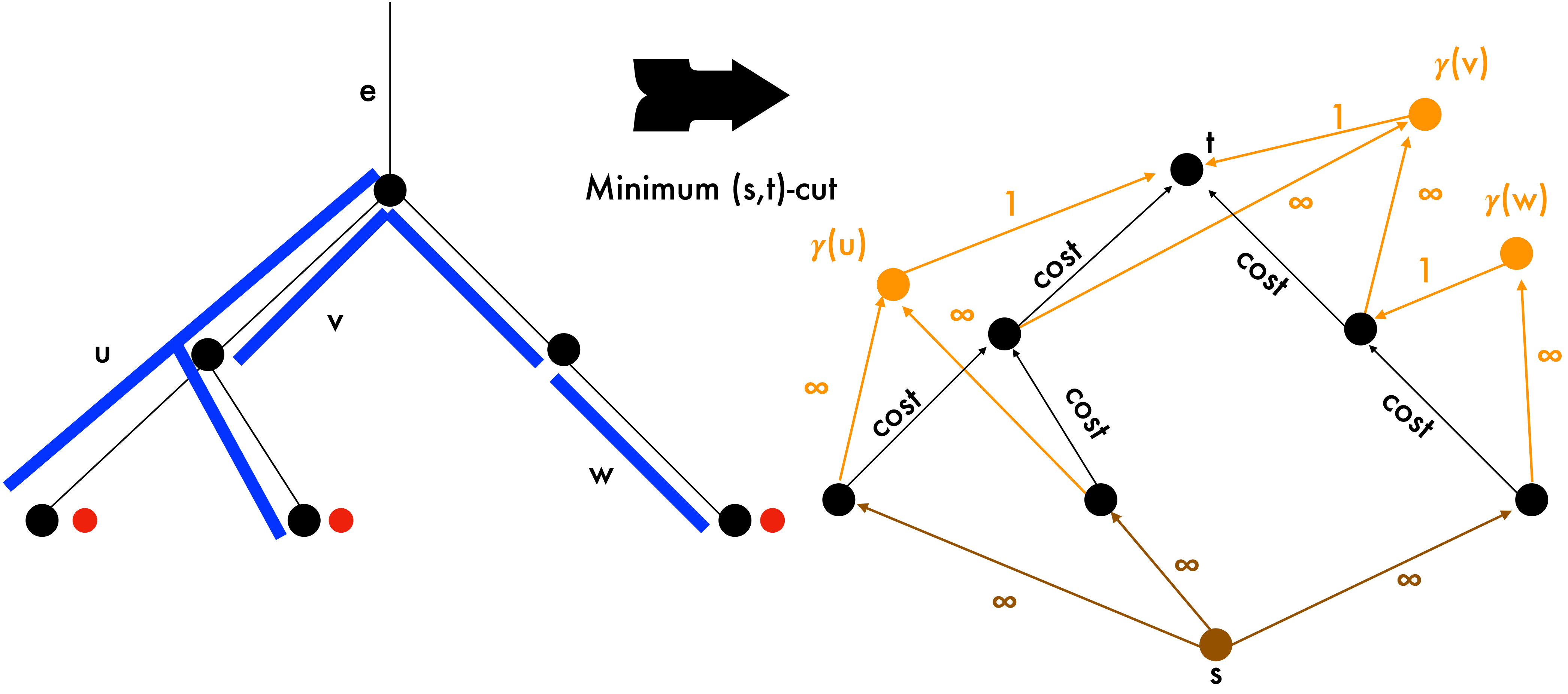

Remarks and Questions!?

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Multiway Cut on chordal graphs require flow-based arguments as Vertex Cover on bipartite graph reduces to it.

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Longest Cycle
Longest Path
Component Order
s-Club Contraction
Independent Set
Bandwidth
Cluster Vertex Deletion

PC wrt to leafage?

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