Tight (Double) Exponential Lower Bounds for Identification Problems

@ ISAAC, 2024

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Prafullkumar Tale

Joint work with

D. Chakraborty, F. Foucaud, and D. Majumdar

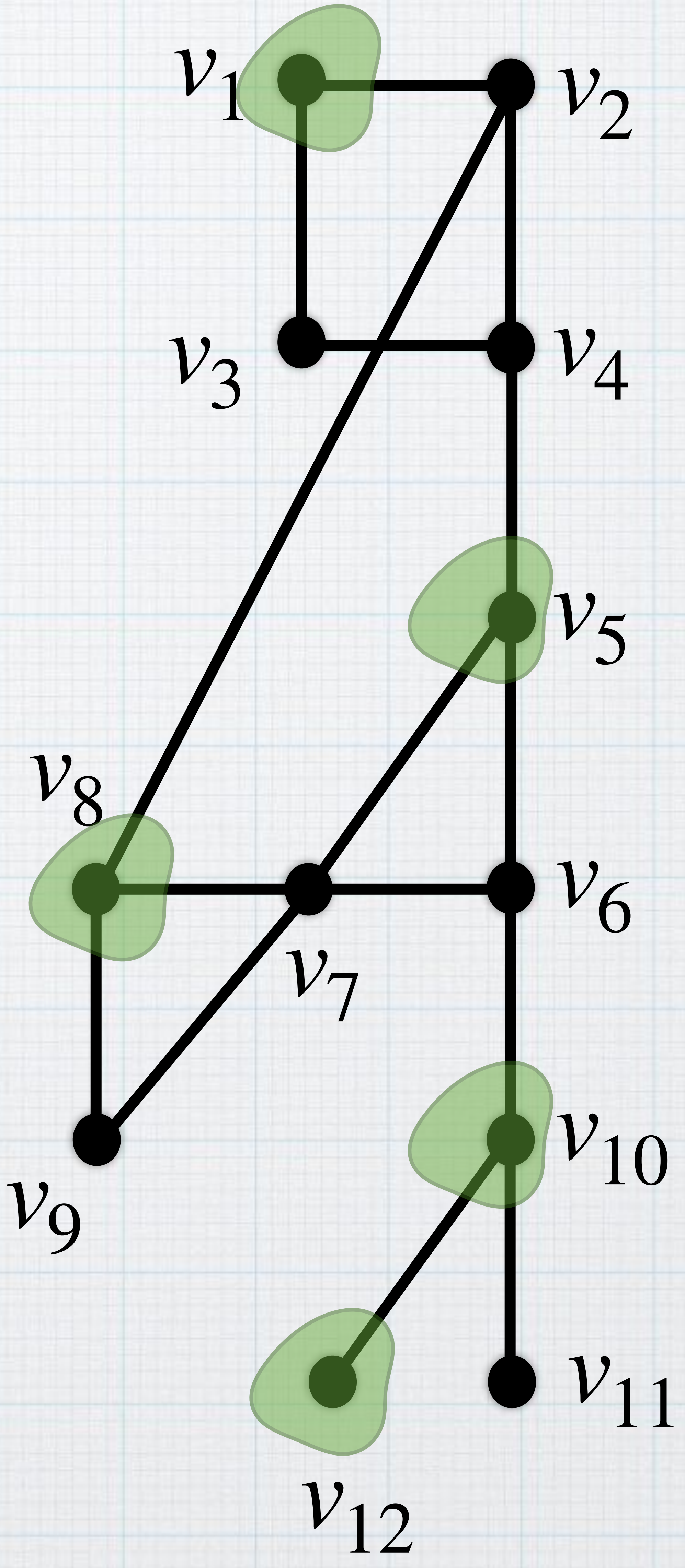
Locating Dominating Set

Input: Graph G, int k

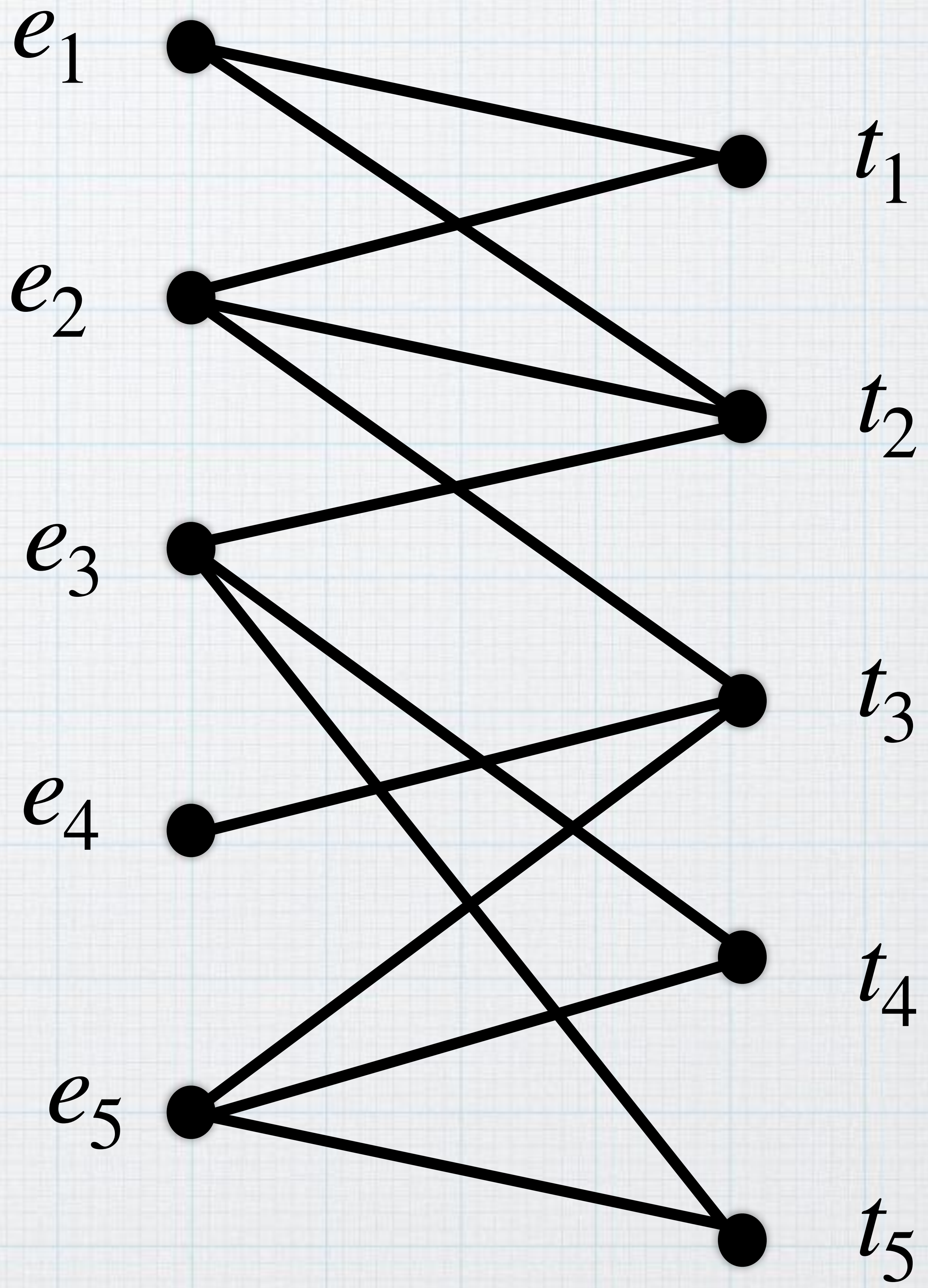
Output: $\exists$ a subset S of size k such that

(i) for any vertex $u \in V(G) \backslash S$, at least one of its neighbour is in S, and

(ii) for any two vertices $u \neq v \in V(G) \backslash S$, their neighbourhood in S are different, i.e., $N(u) \cap S \neq N(v) \cap S$.



Test Cover



Test Cover

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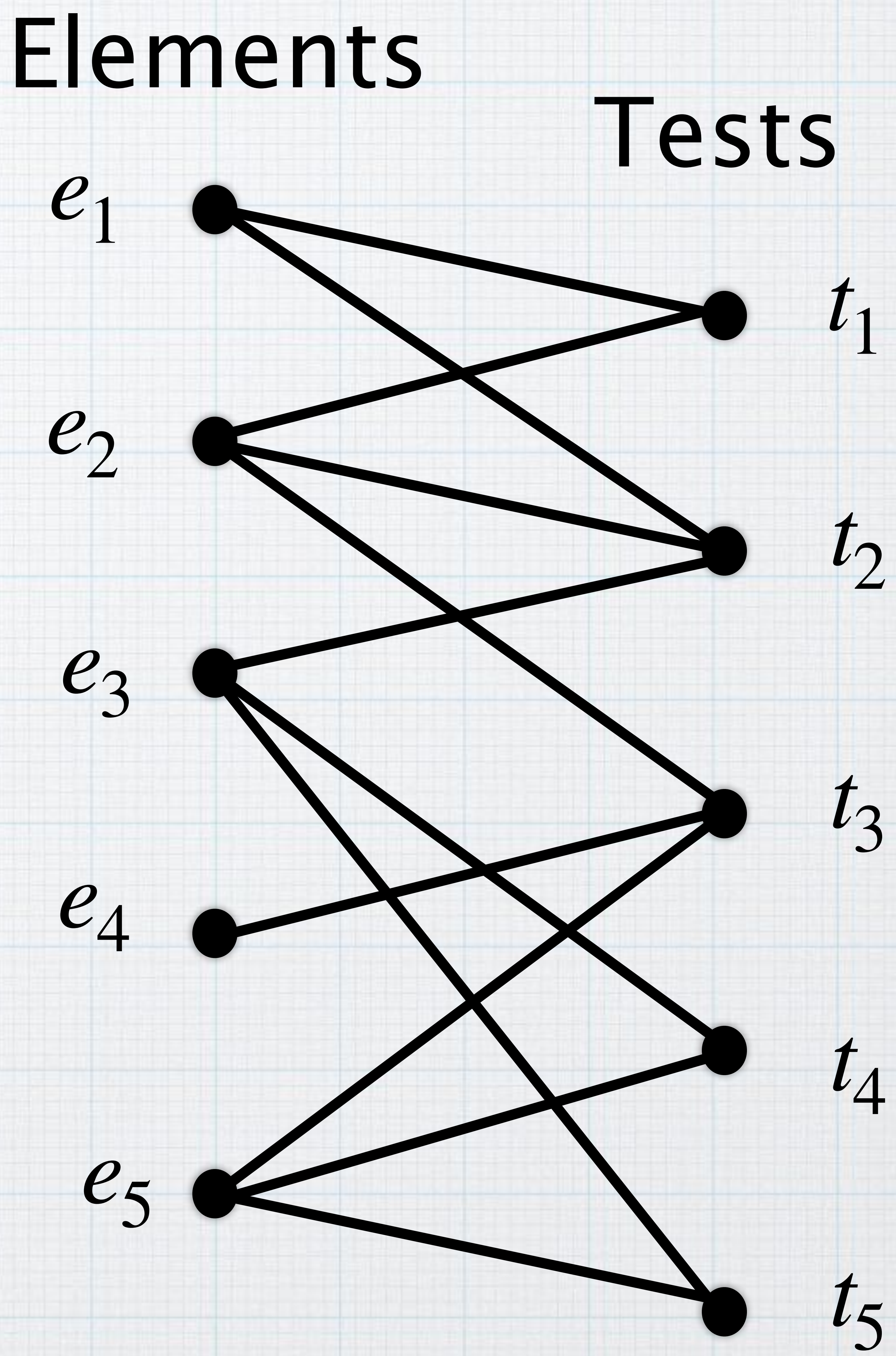

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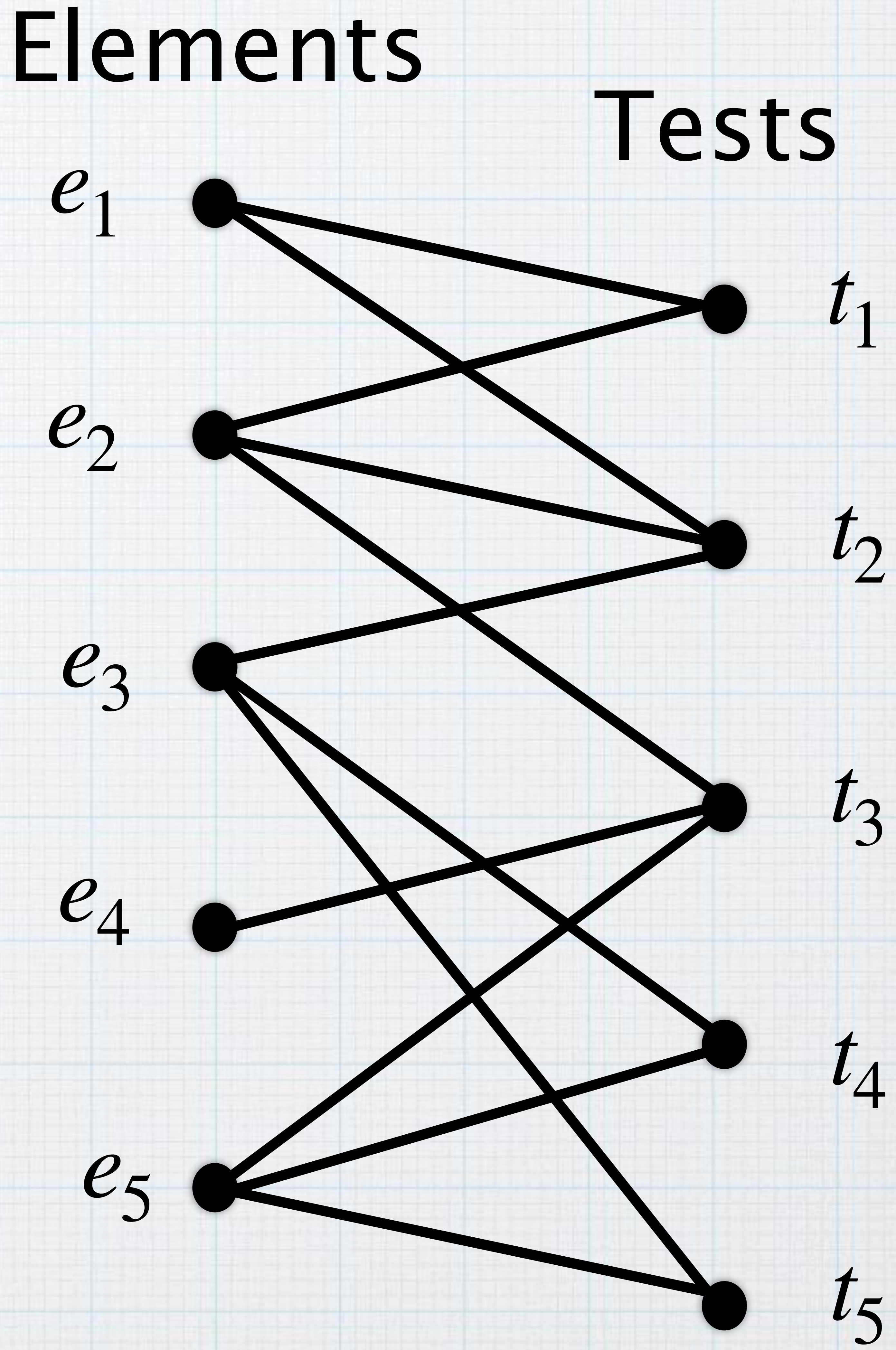

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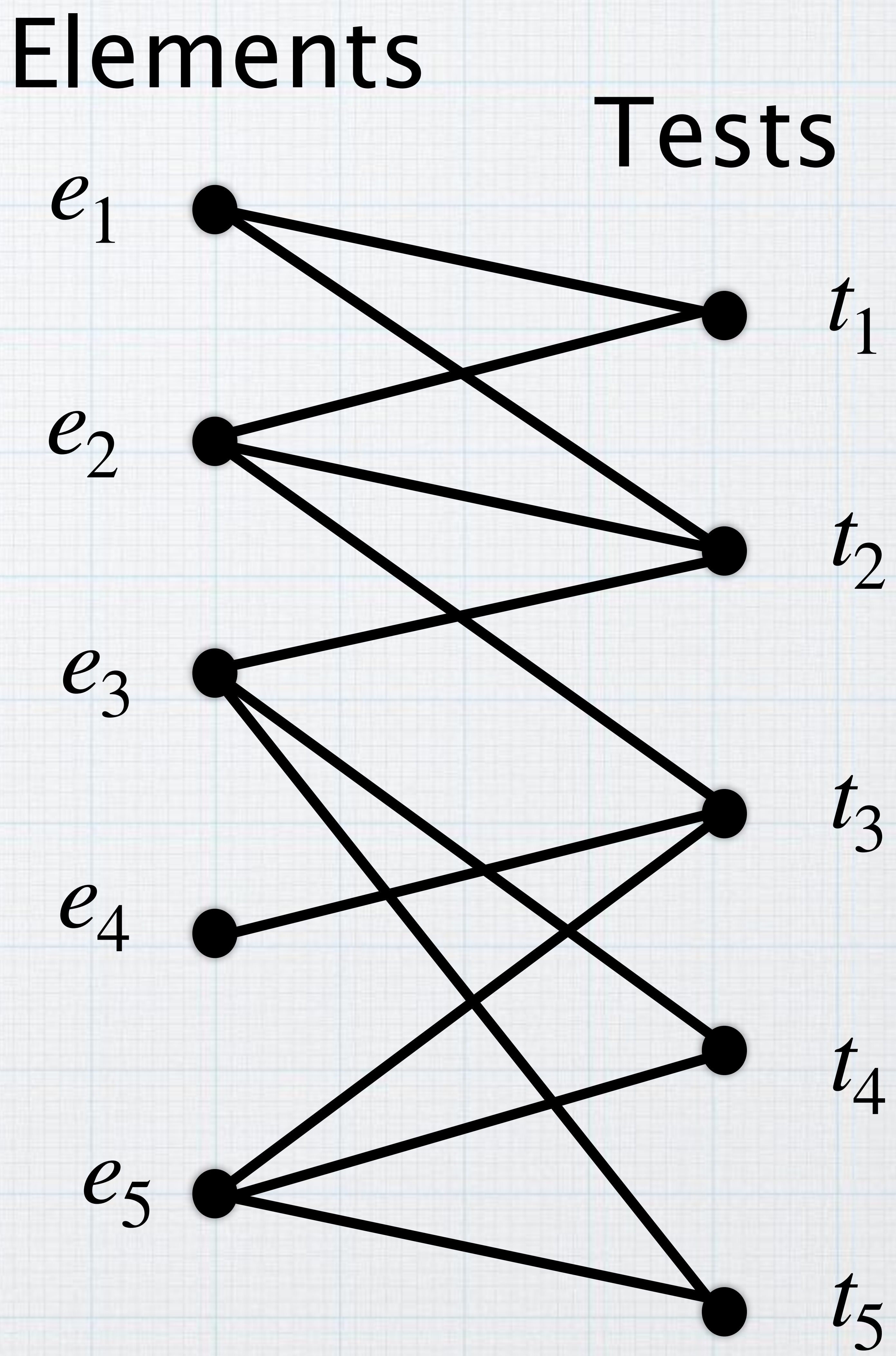
– Each test is positive for one or more elements
– Determine the smallest set of tests that need to be performed to uniquely identify the element



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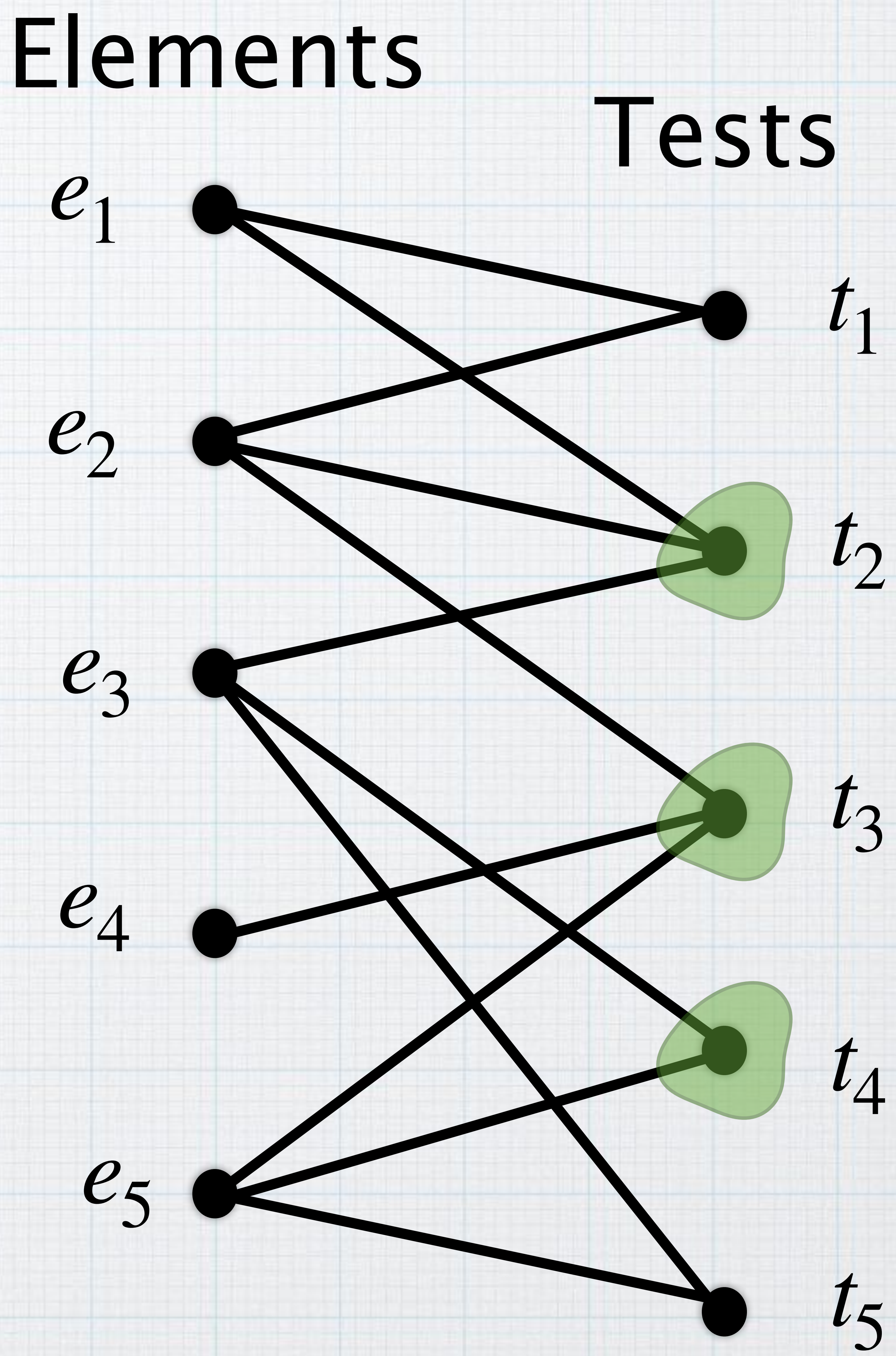

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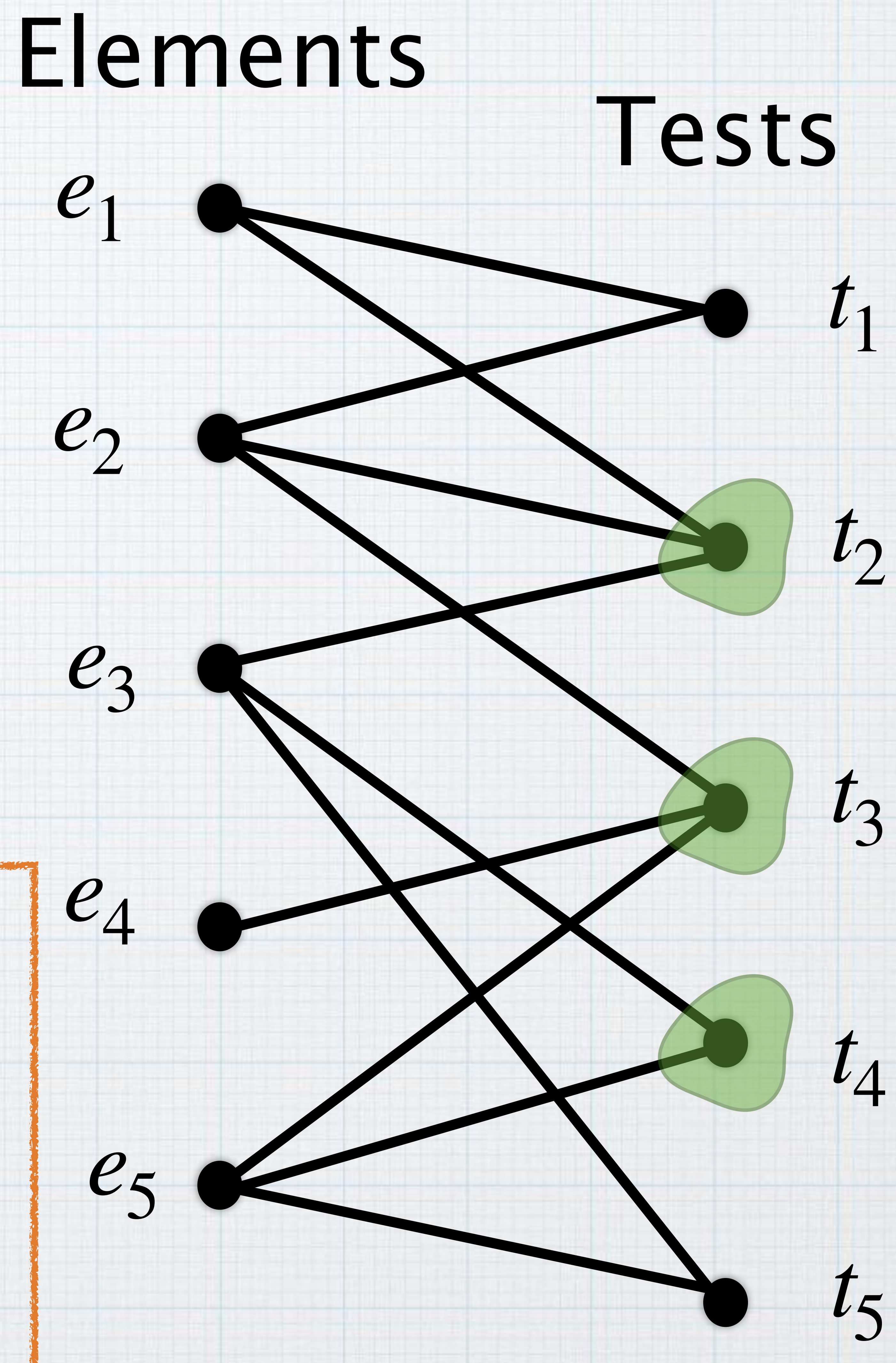

Test Cover

Input: Set of elements, collection of tests, int k

Output: Does there exist a collection of k tests s.t. for each pair of elements, there is a test that is positive for exactly one of them?

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- Π is fixed-parameter tractable (FPT) parameterized by k if there is an algo that solves it in $f(k) \cdot poly(n)$ time.
- Π is $W[1]$-hard when parameterized by k if there is no algo that solves it in $f(k) \cdot poly(n)$ time.

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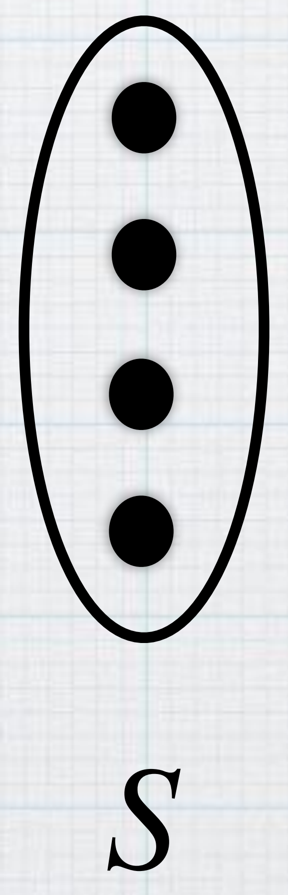

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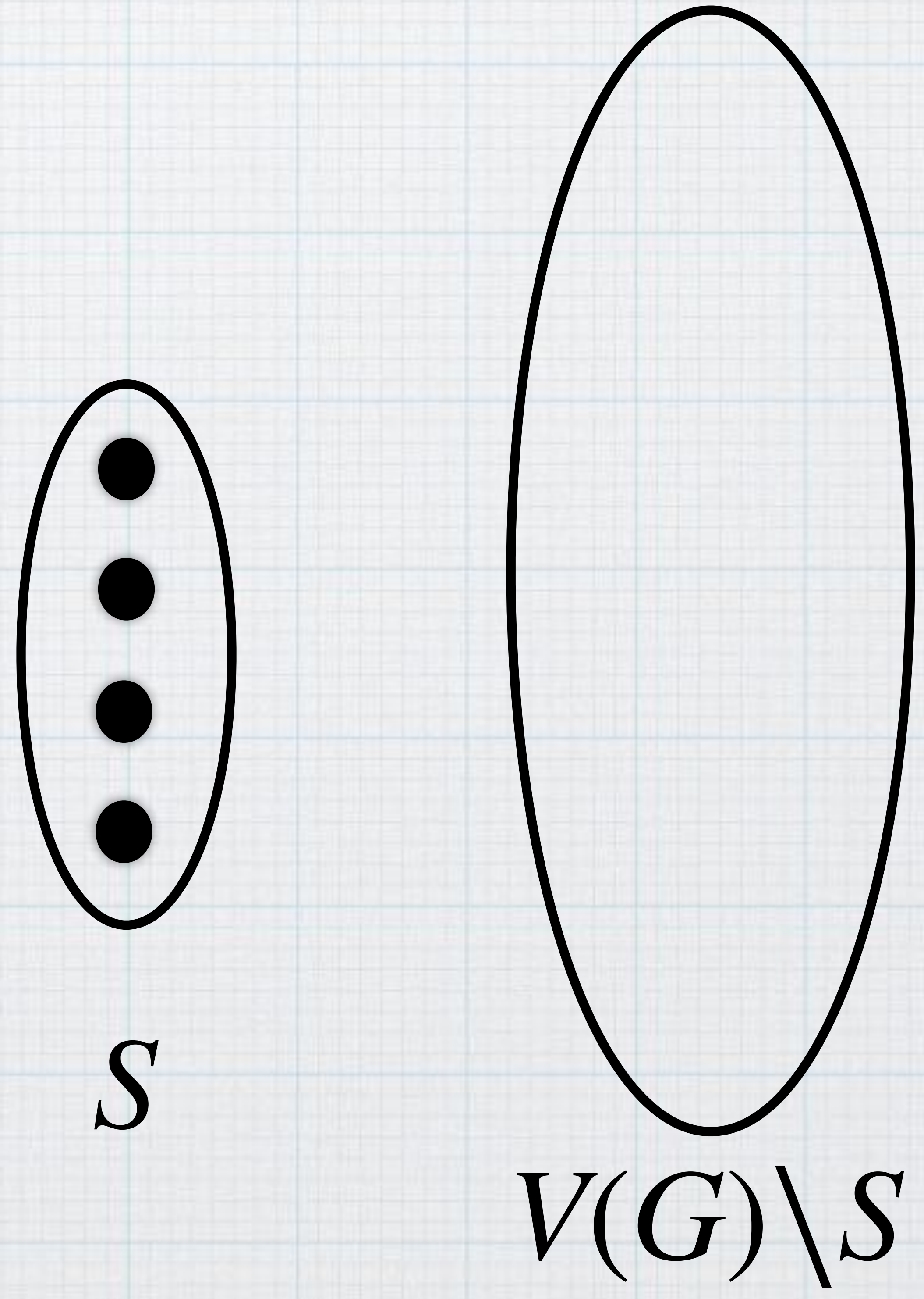

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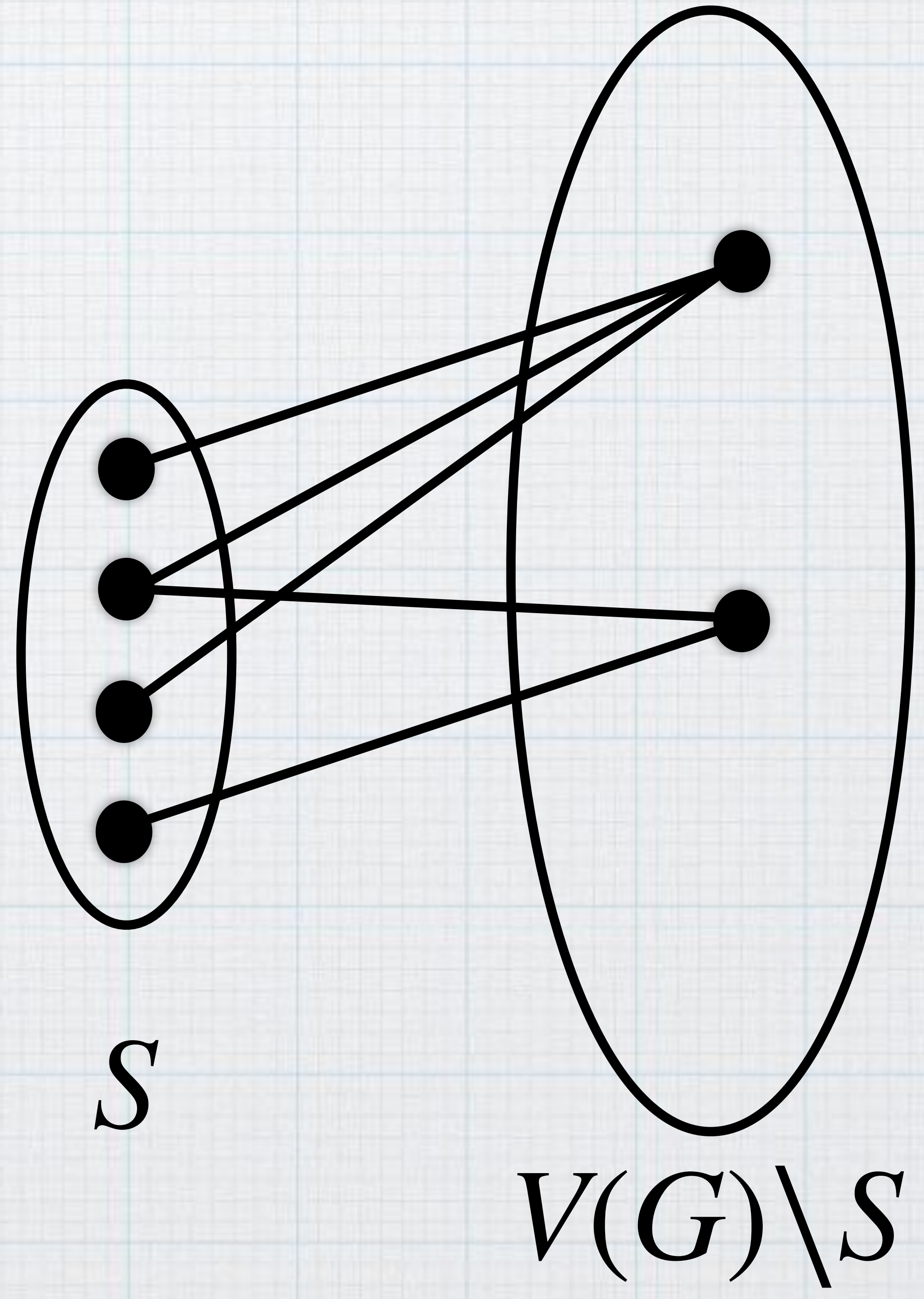

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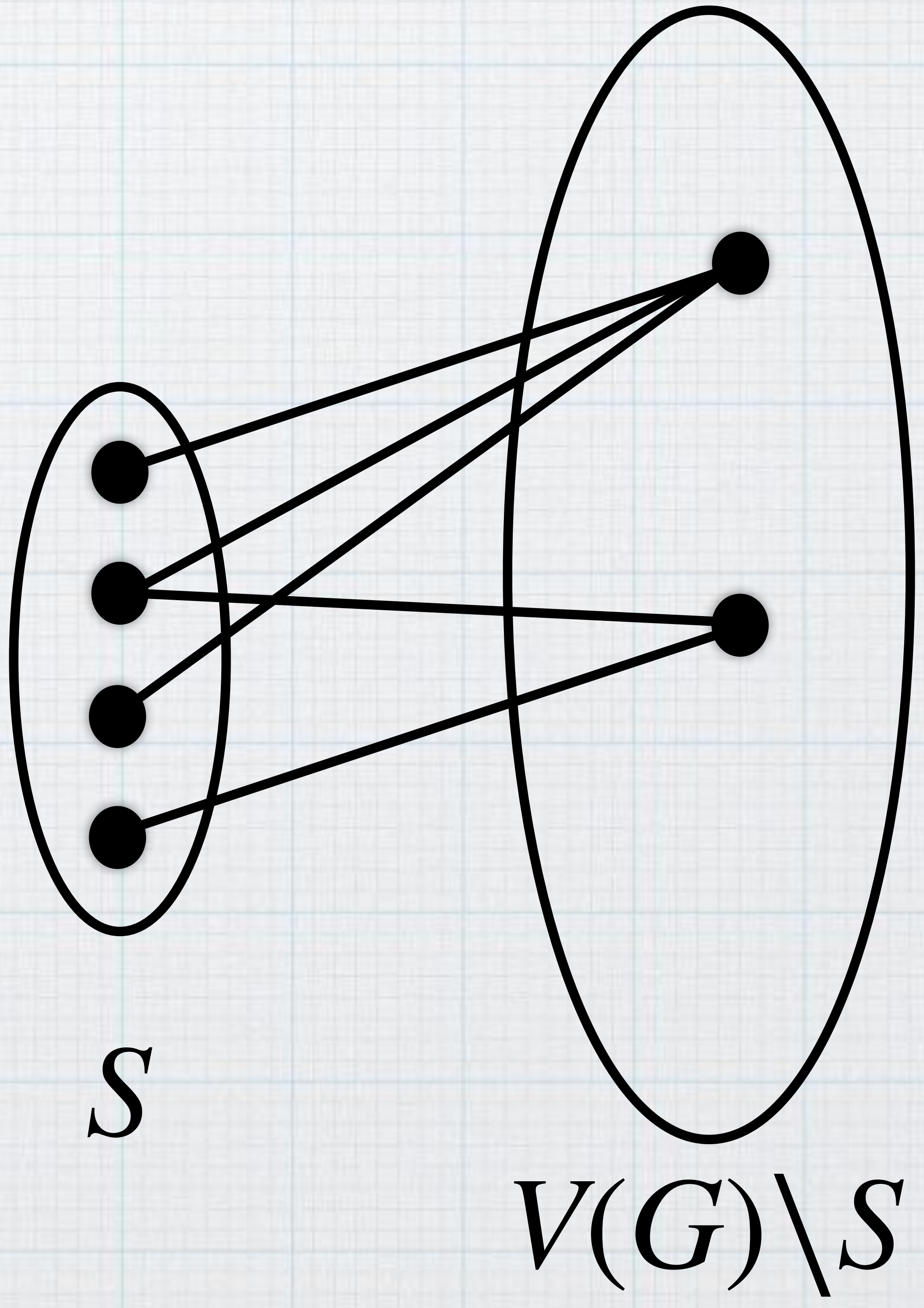

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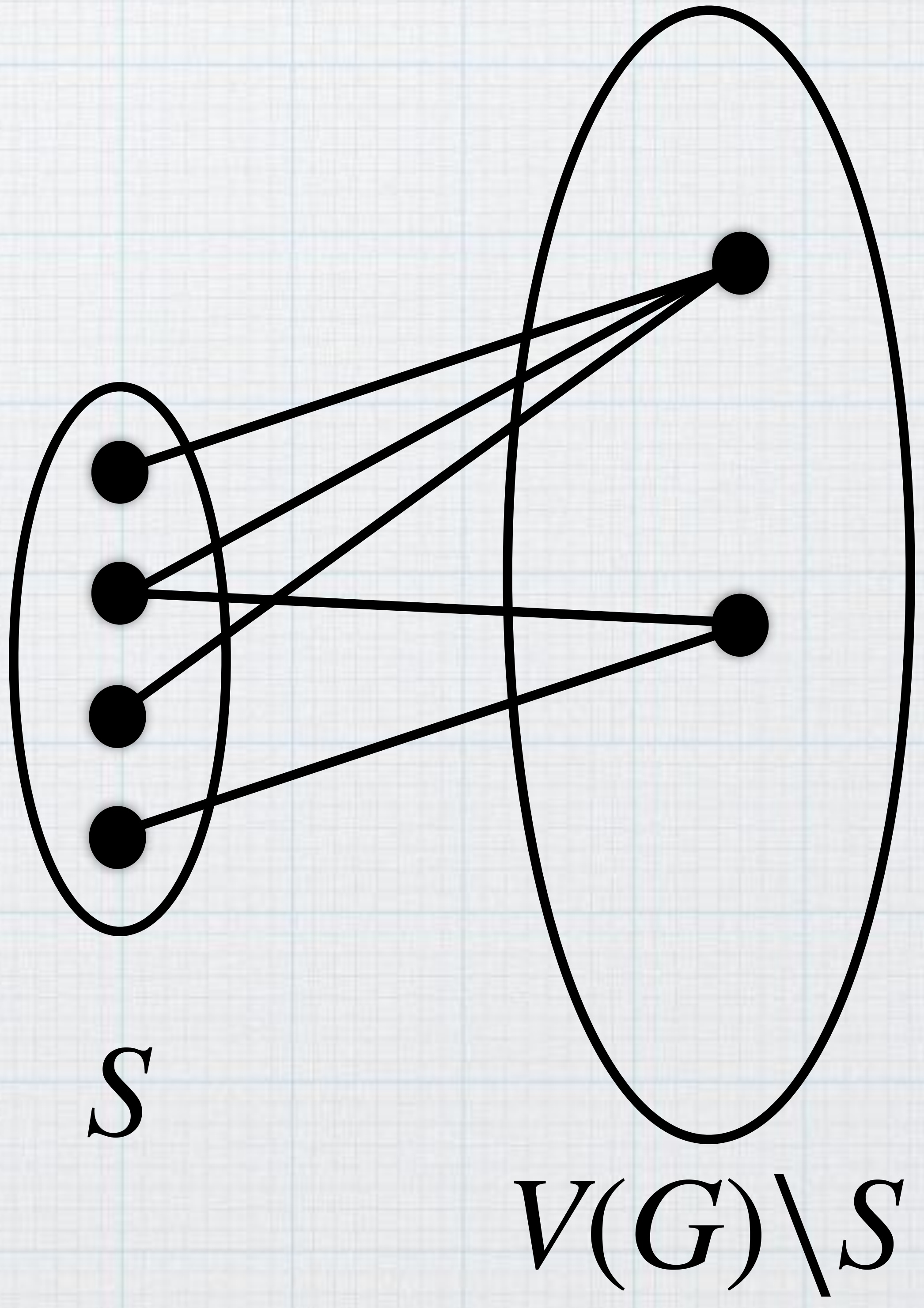
Claim: Locating Dominating Set admits algo running in time $\binom{2^k}{k} \cdot n^{\mathcal{O}(1)} = 2^{\mathcal{O}(k^2)} \cdot n^{\mathcal{O}(1)}$.



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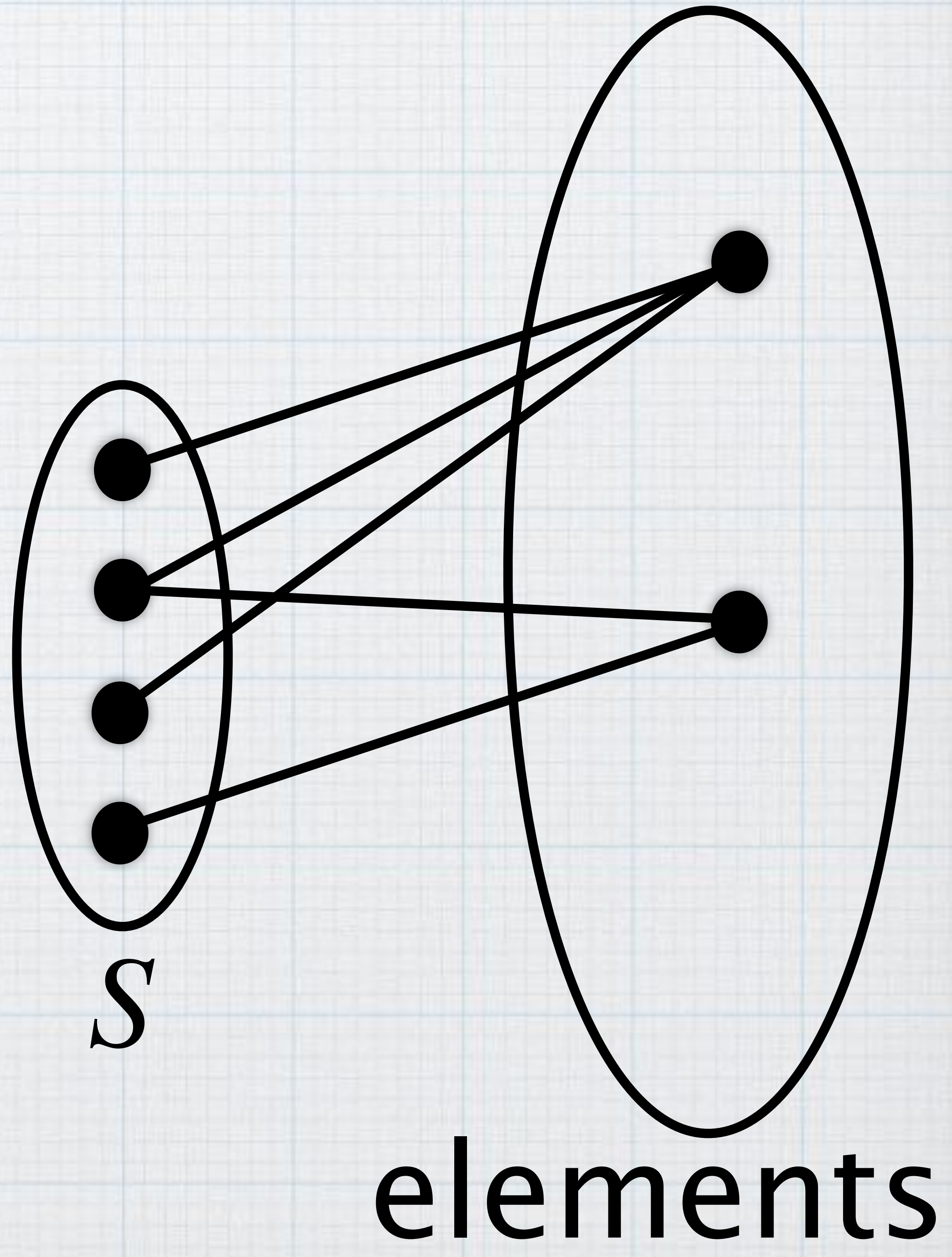

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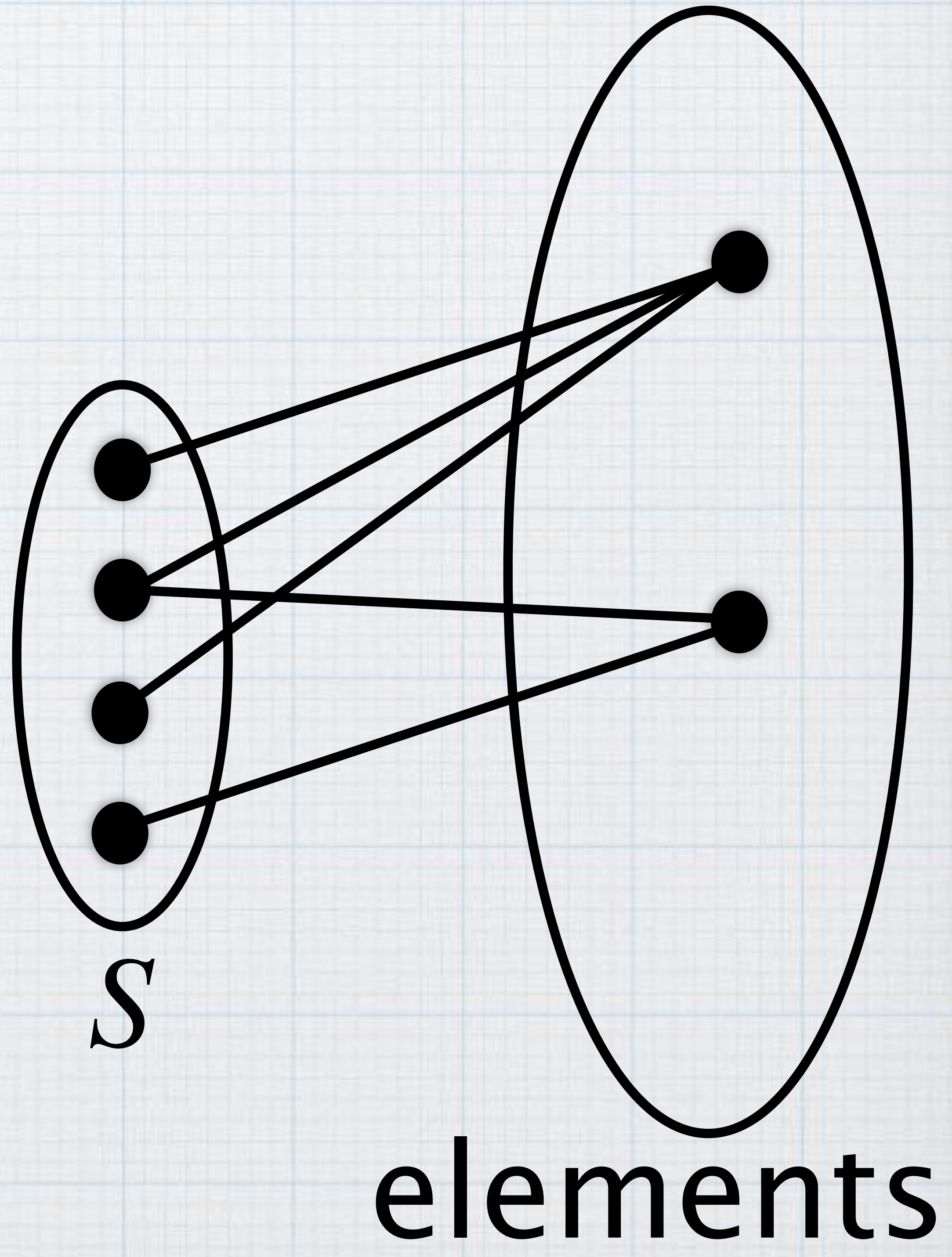

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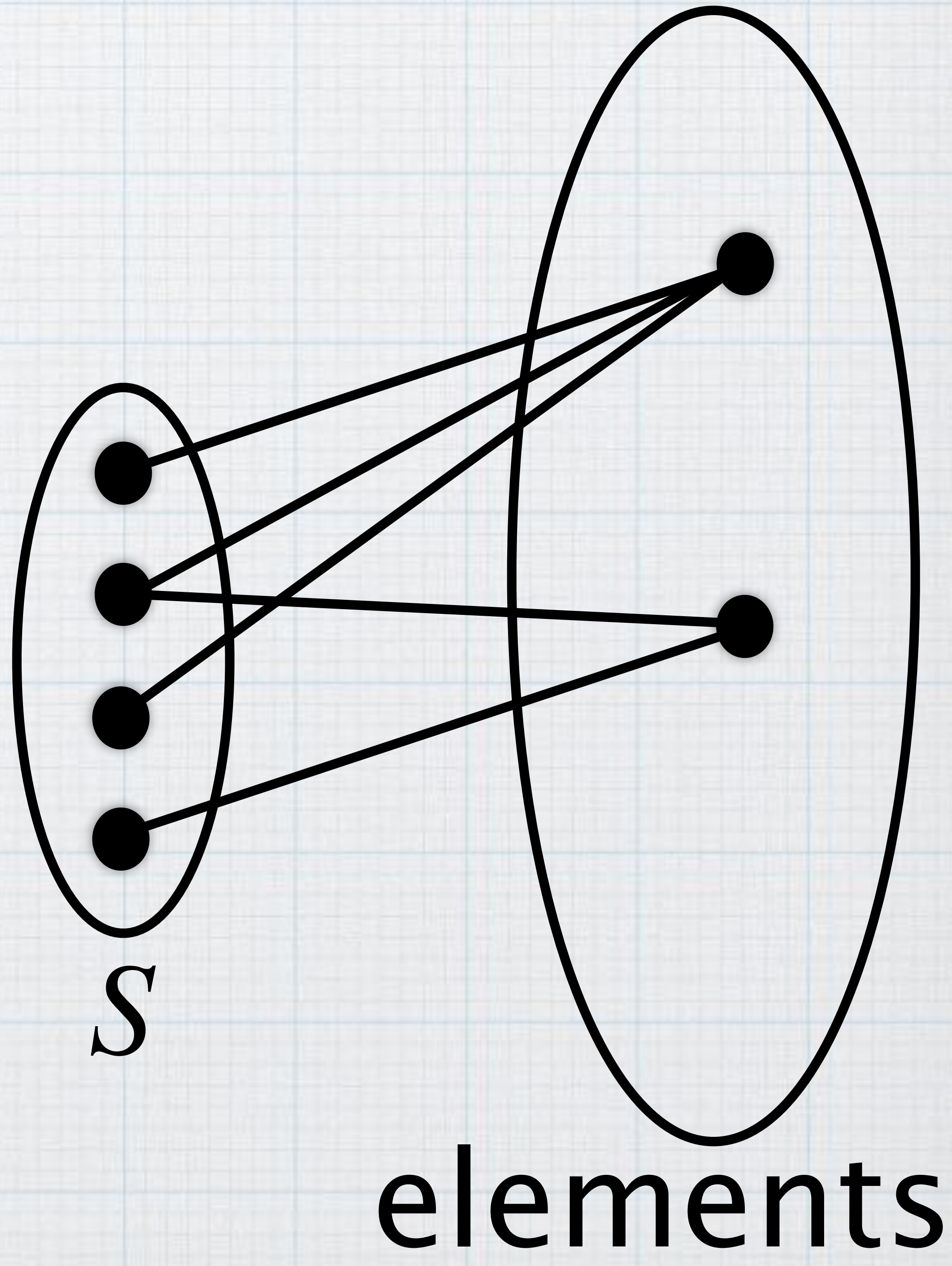

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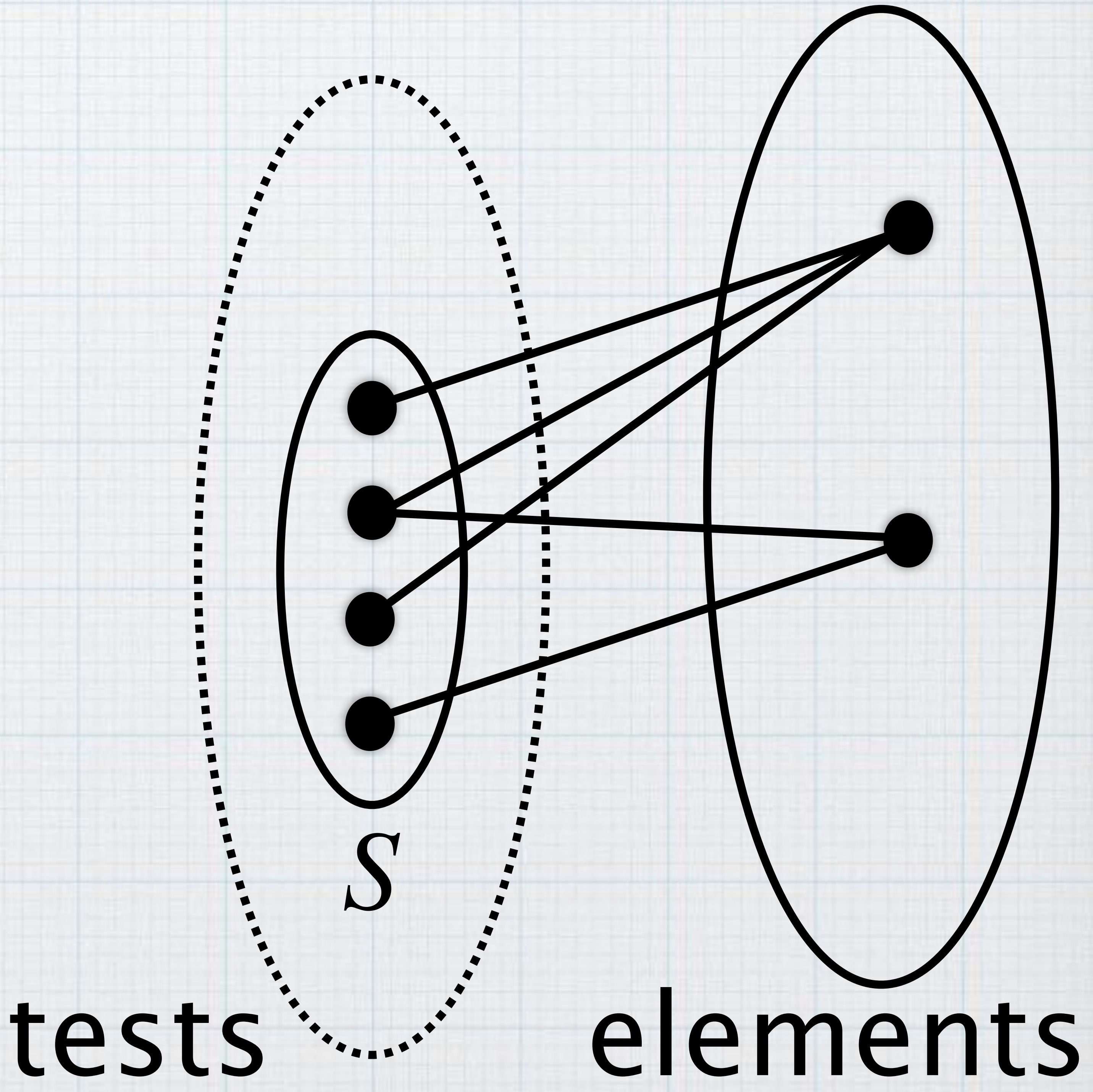

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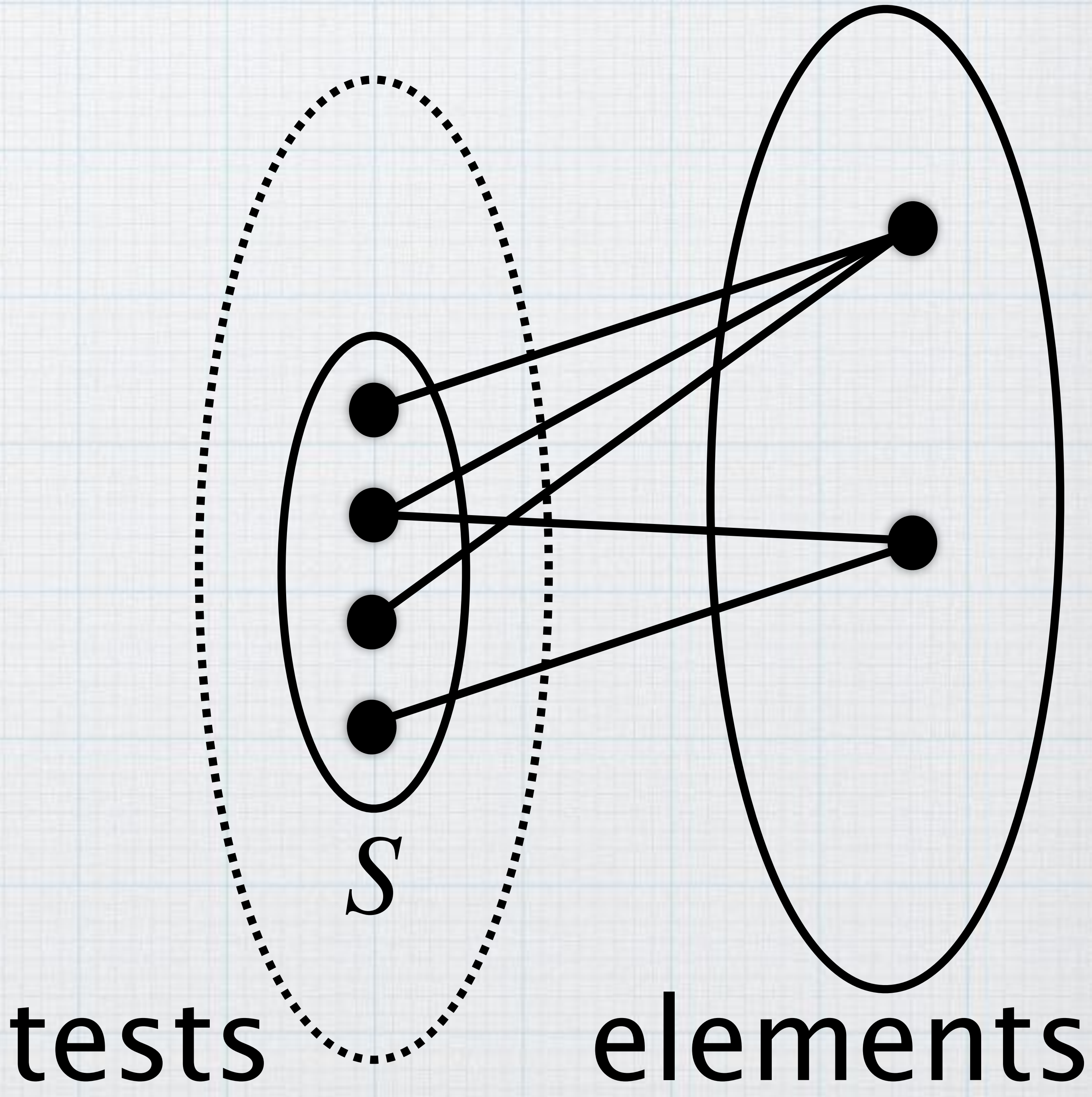

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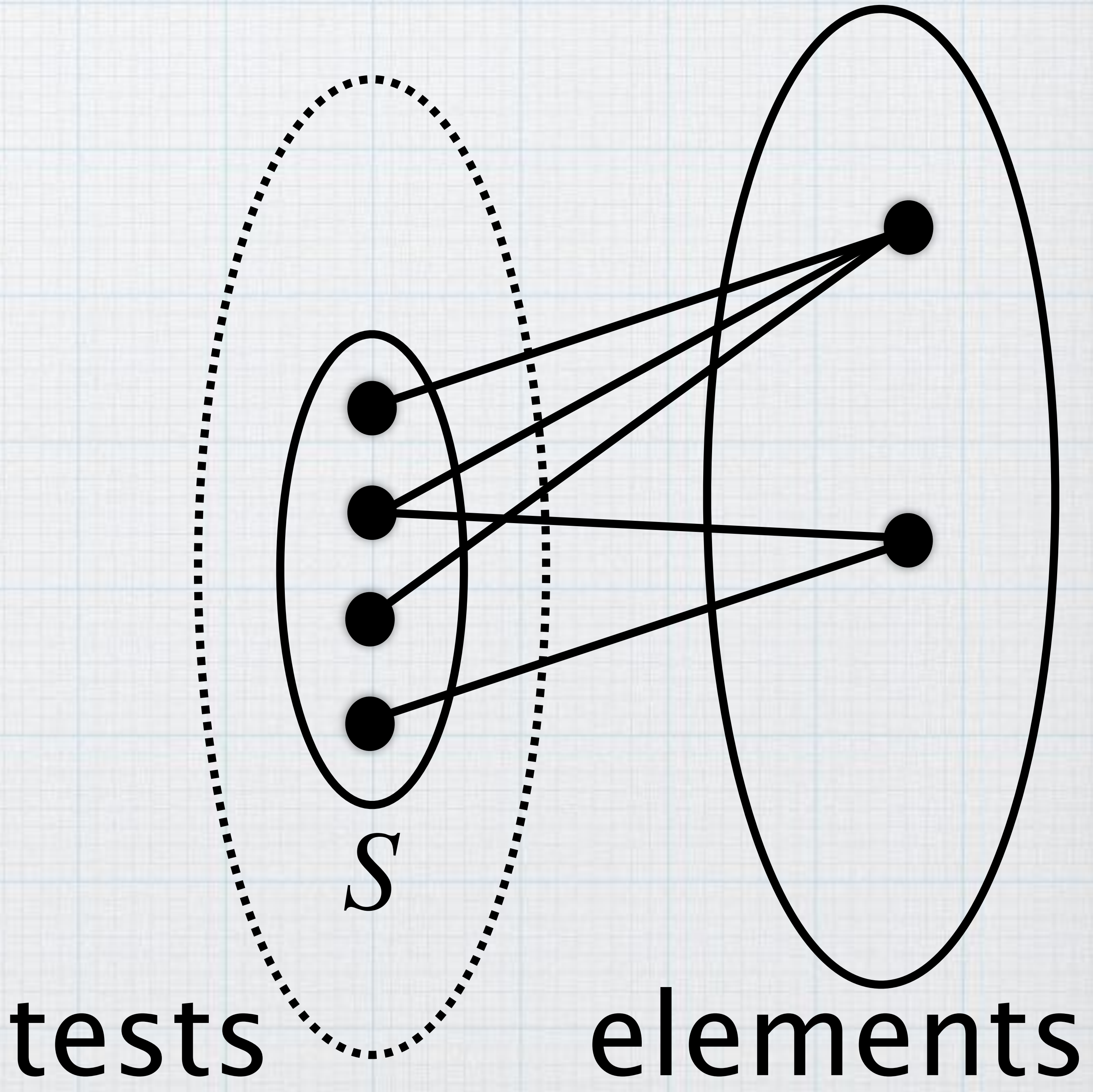

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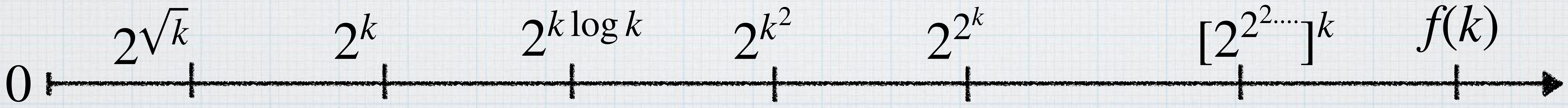

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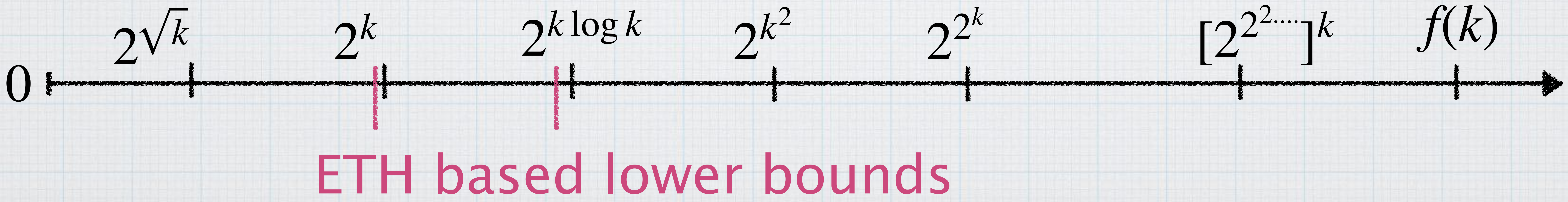

Our results

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Thm. The Locating Dominating Set problem

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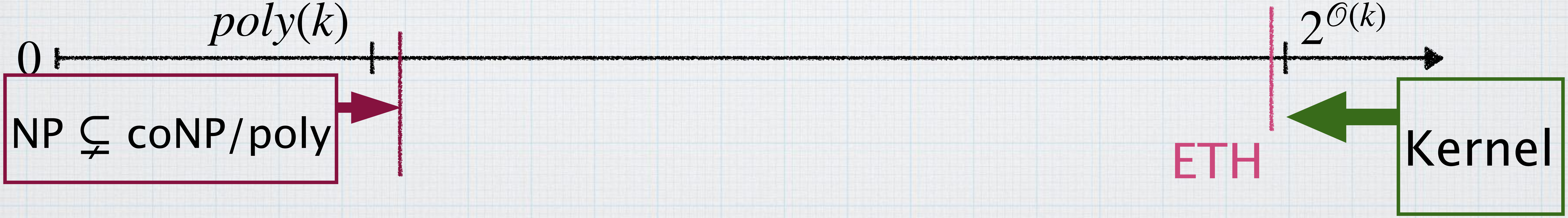

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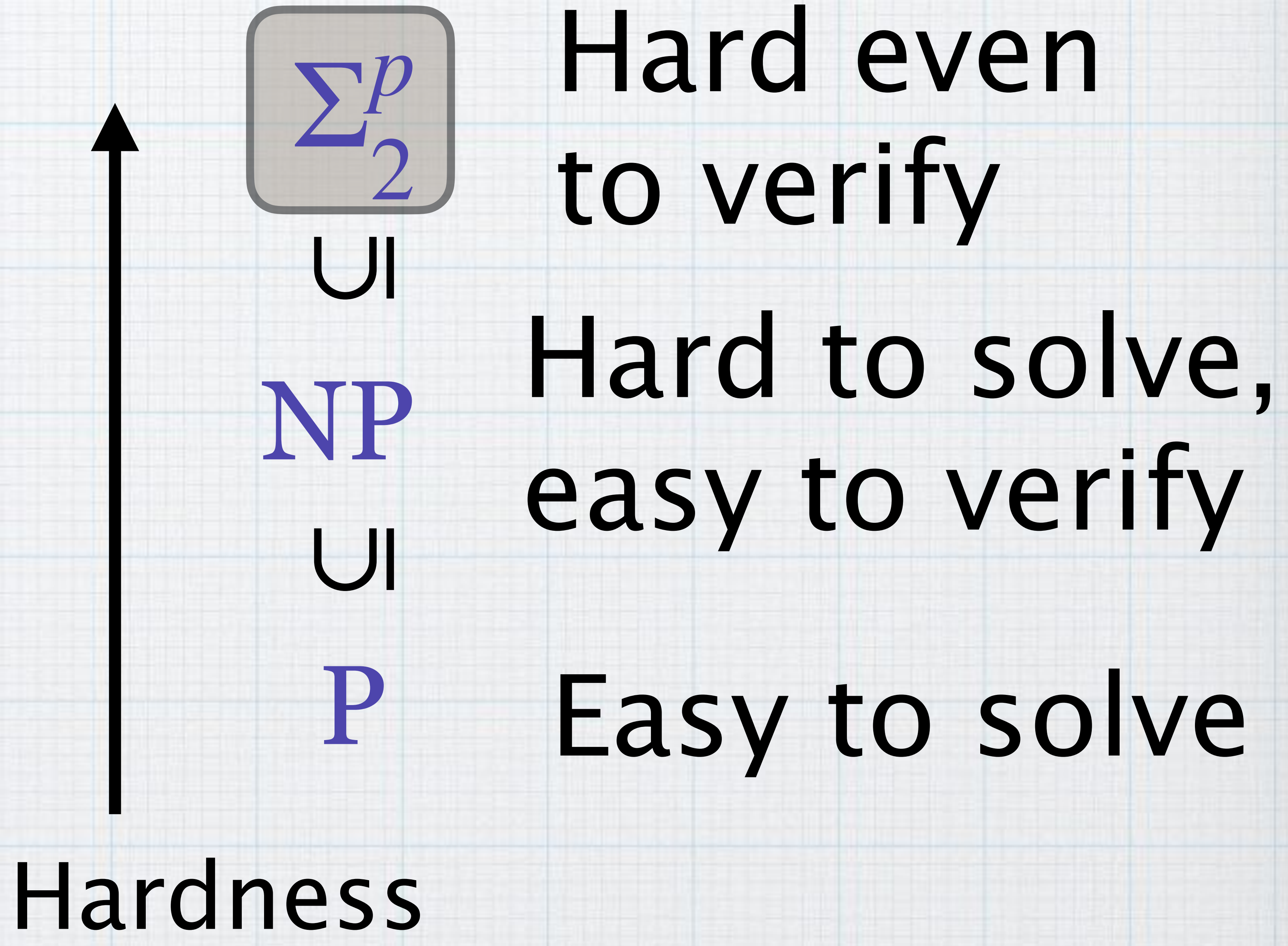

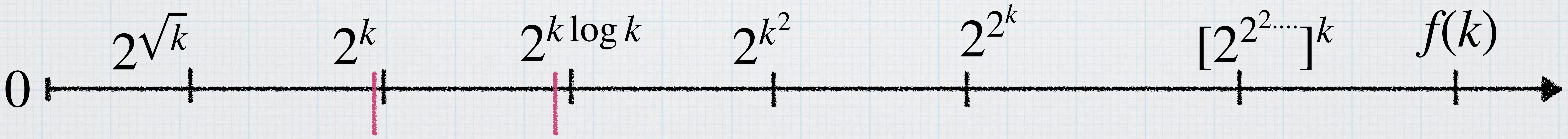

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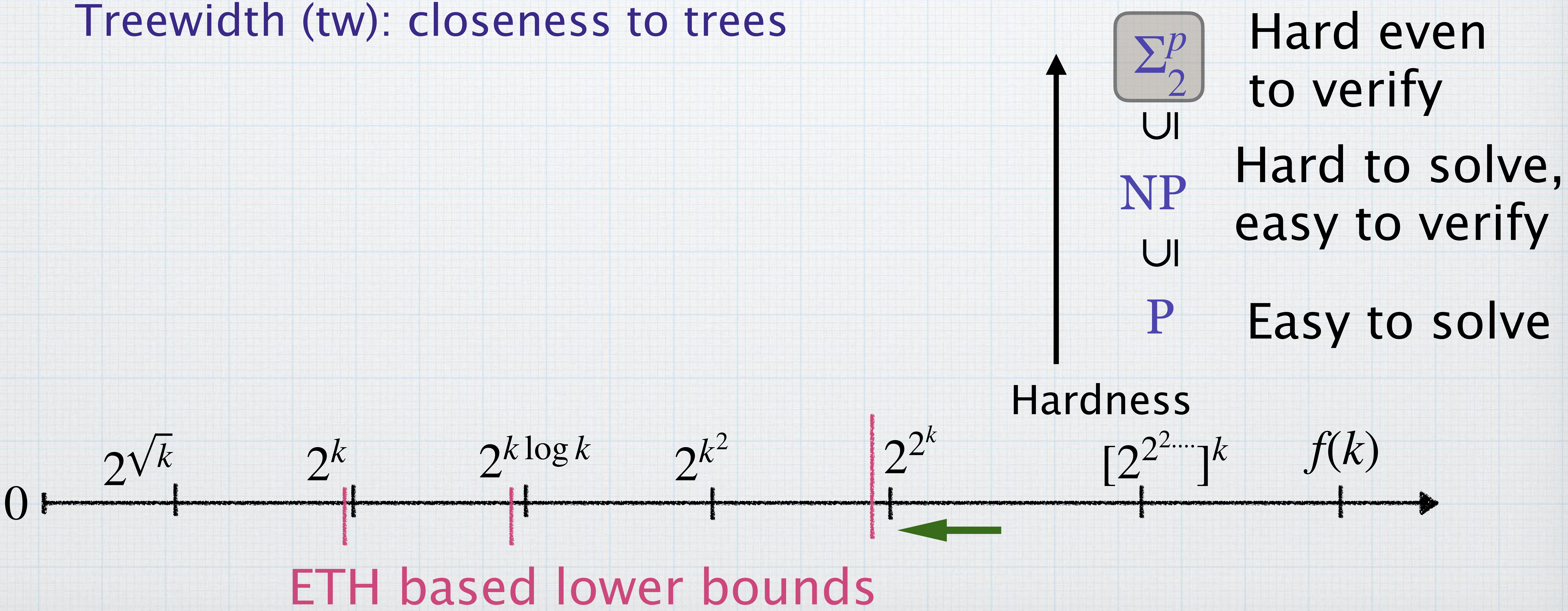

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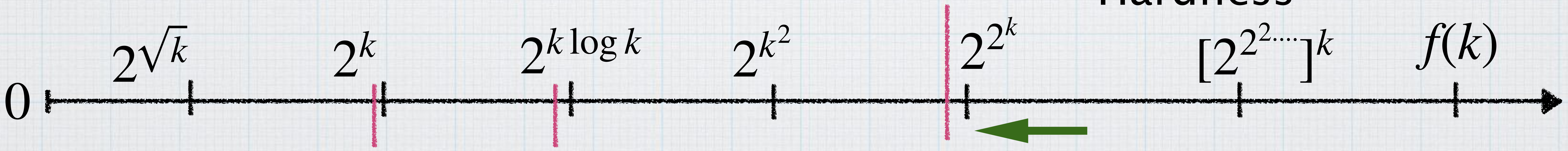

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Our result (in ICALP'24): Not necessary to go higher up in the polynomial hierarchy to achieve double-exponential lower bounds.

Problems in NP can admit double-exponential lower bounds when parameterized by treewidth and vertex cover

by Foucaud, Galby, Khazaliya, Li, Mc Inerney, Sharma, Tale (2023)

Thm [FGKLIST (ICALP'24)]. The Metric Dimension problem on bounded diameter graphs

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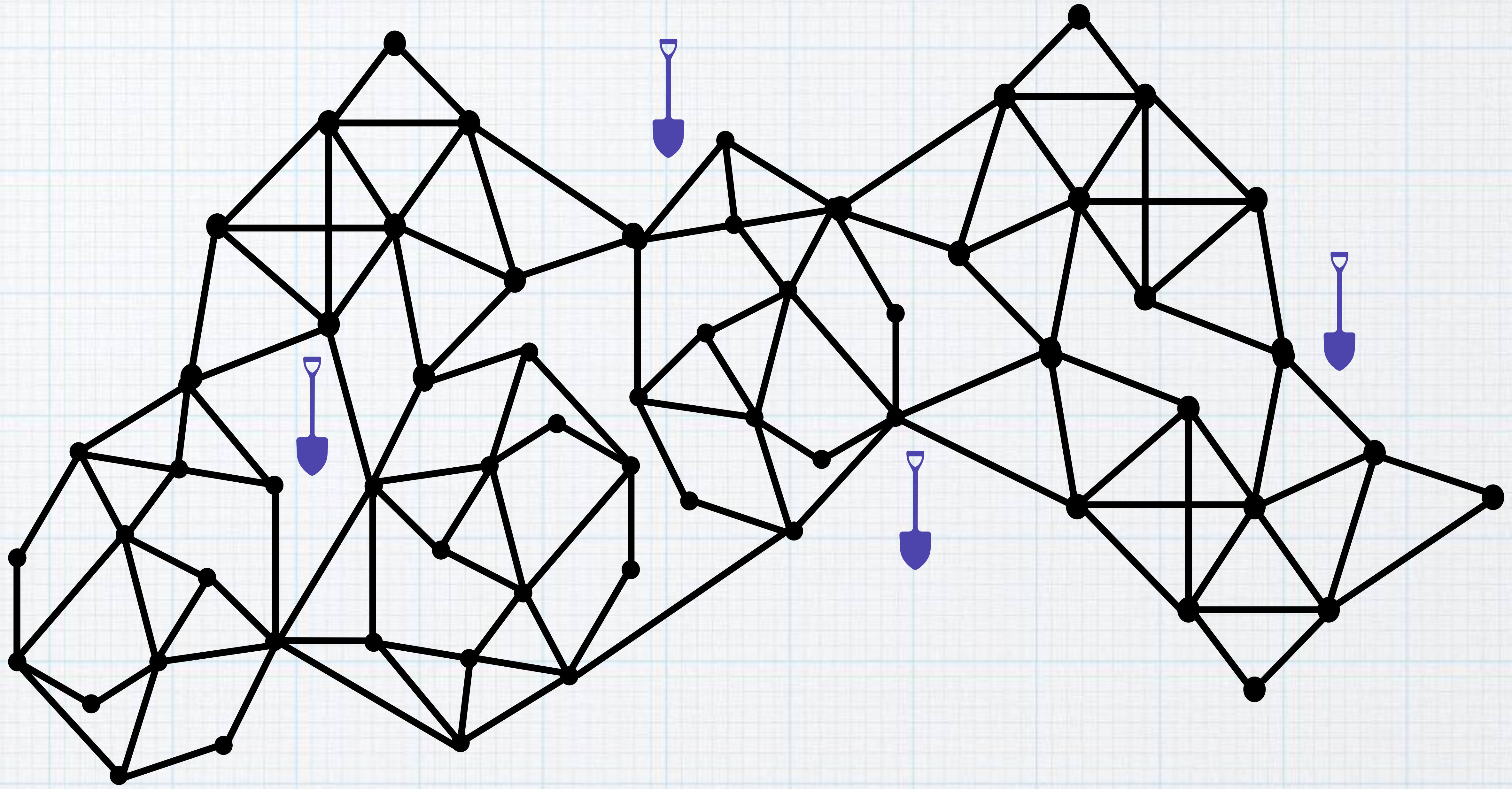
– Metric Dimension (Treasure Hunt)

Digging at a vertex reveals its distance from the treasure.

Can we dig at k vertices to locate the treasure?



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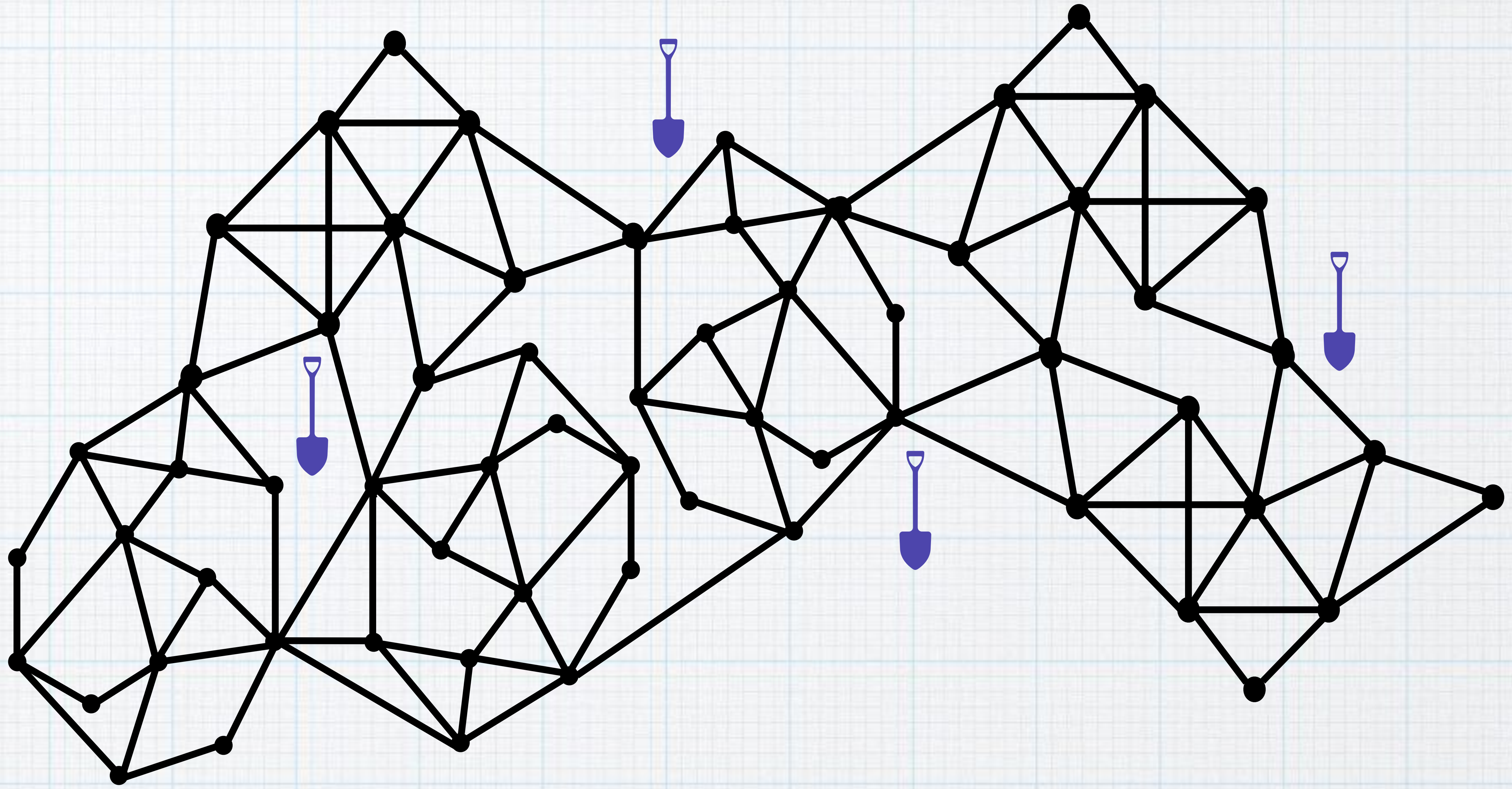

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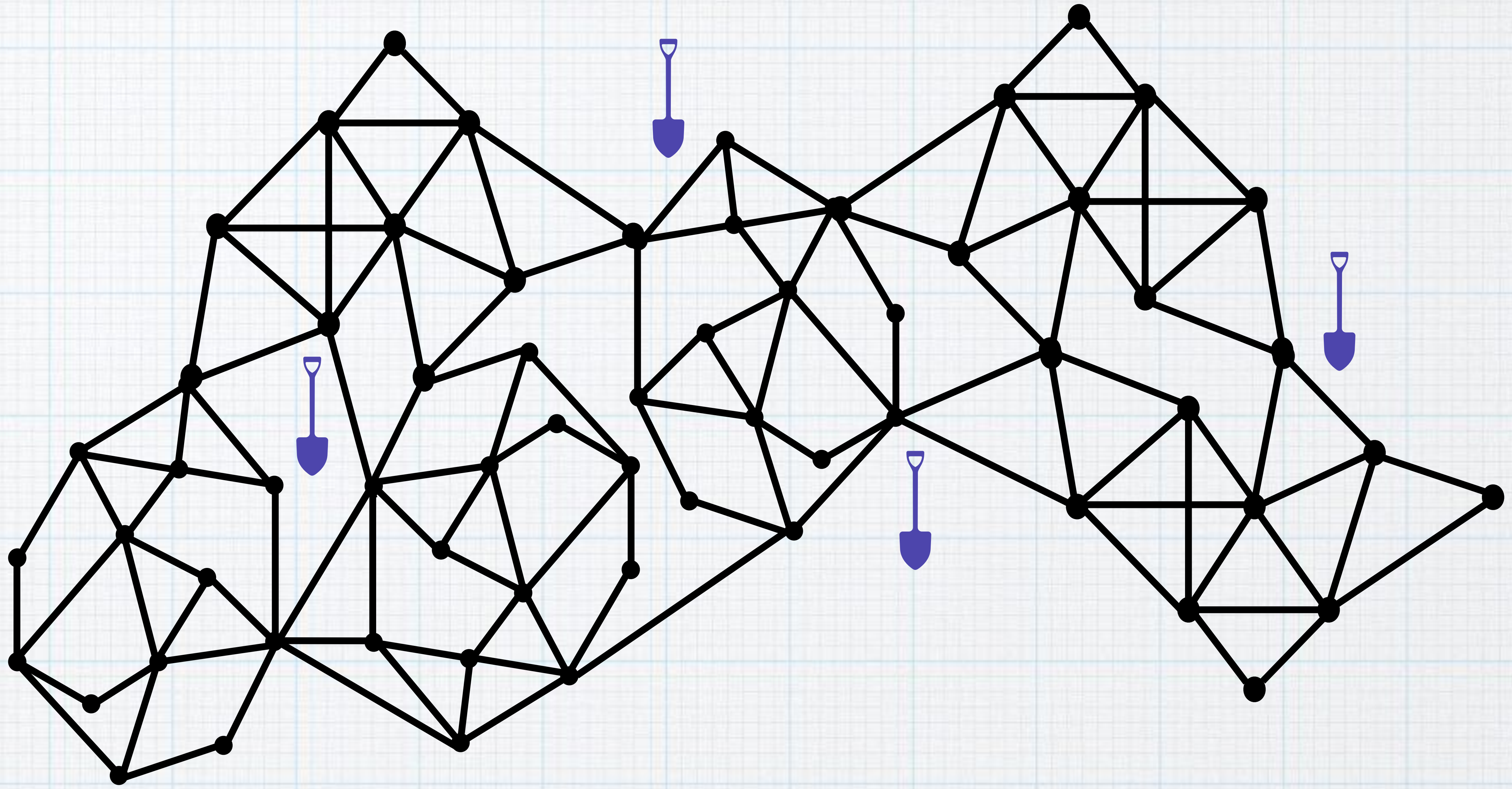

– Effect of a solution vertex is global in `metric graph problems'.

– Effect of a solution vertex is local in `identification problems'.

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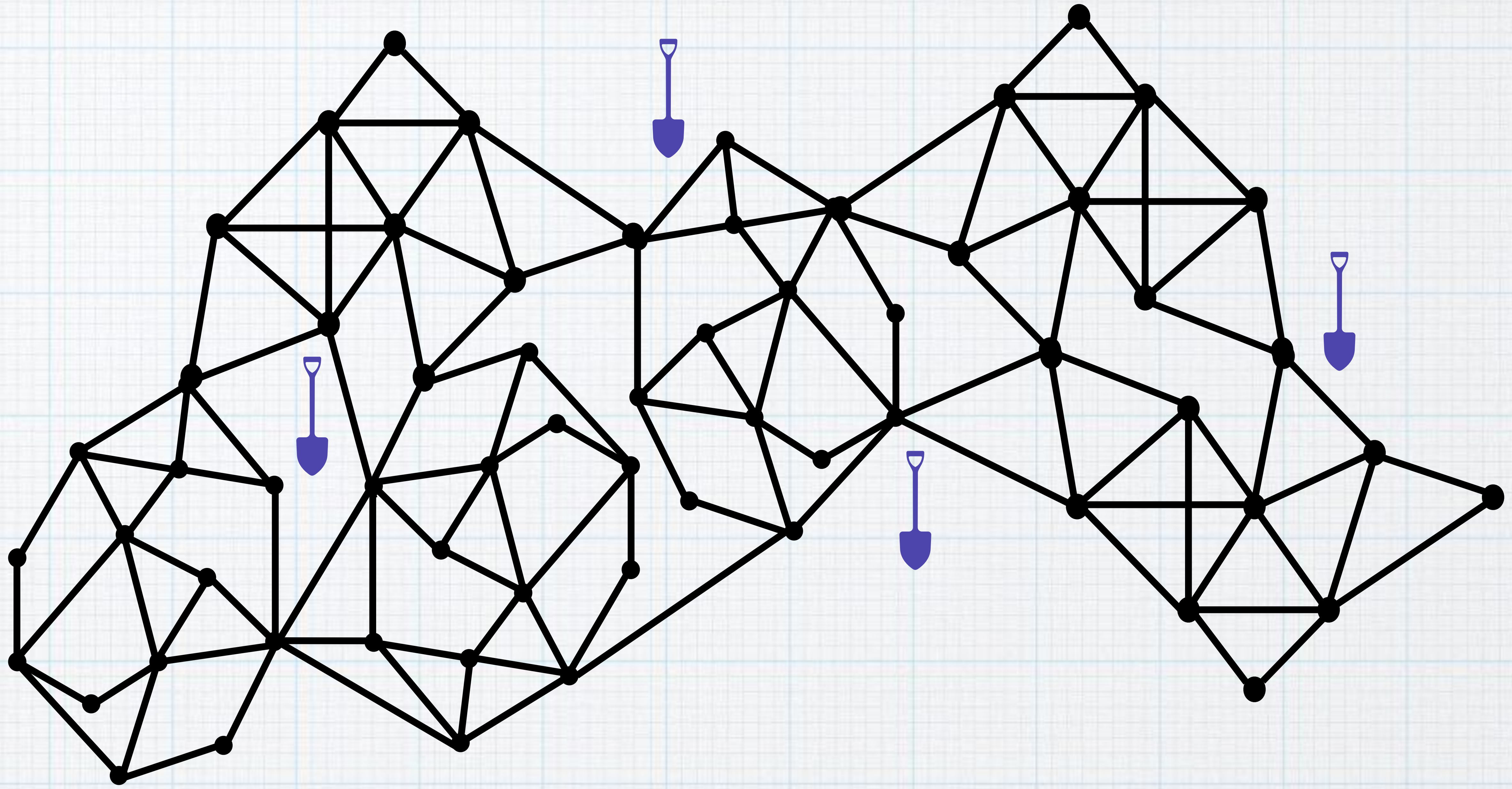
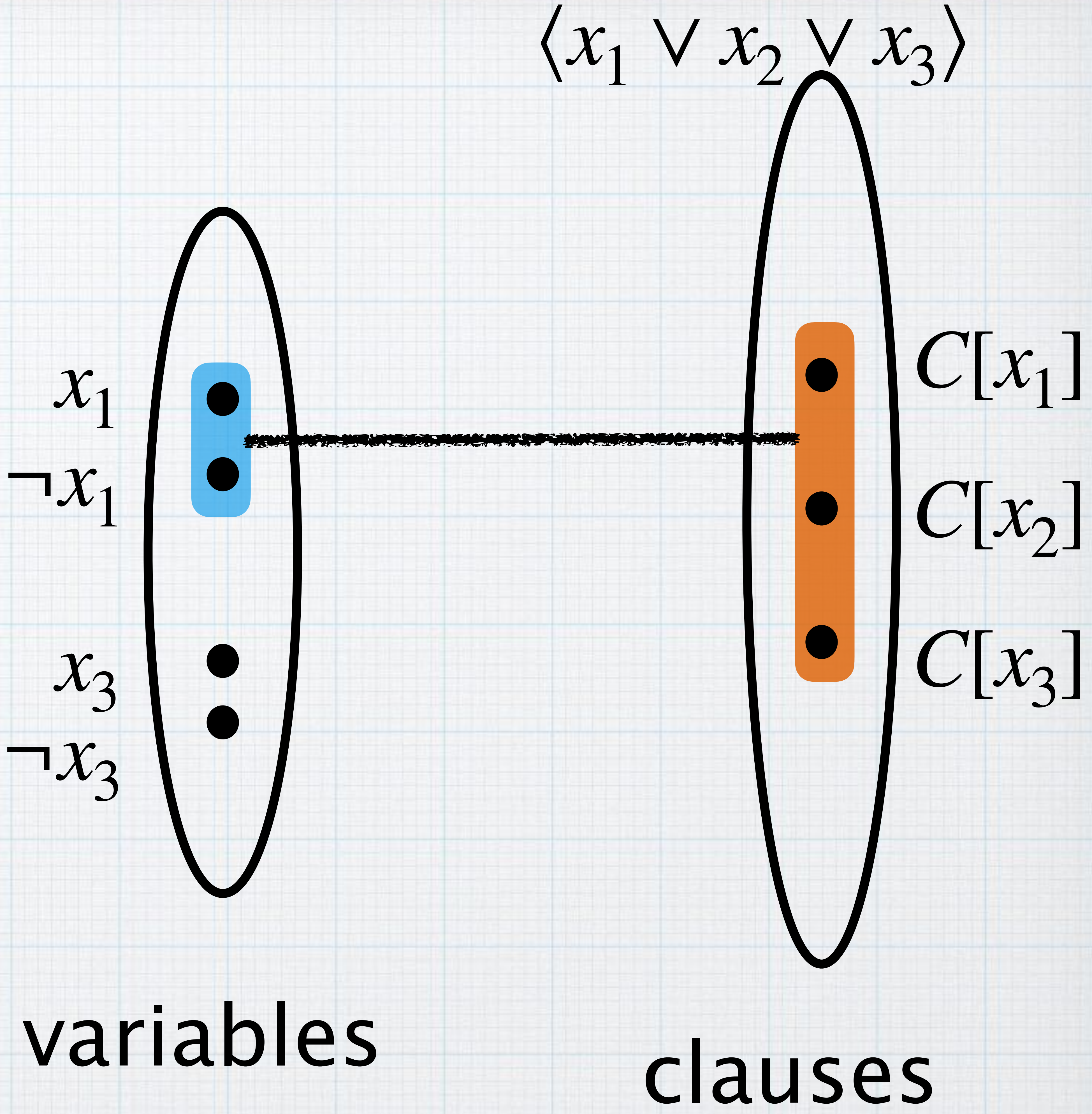


– Effect of a solution vertex is global in `metric graph problems'.

– Effect of a solution vertex is local in `identification problems'.

It is not necessary to use `metric graph problems' to use double exponential lower bound.
We can obtain similar results with `identification problems'.

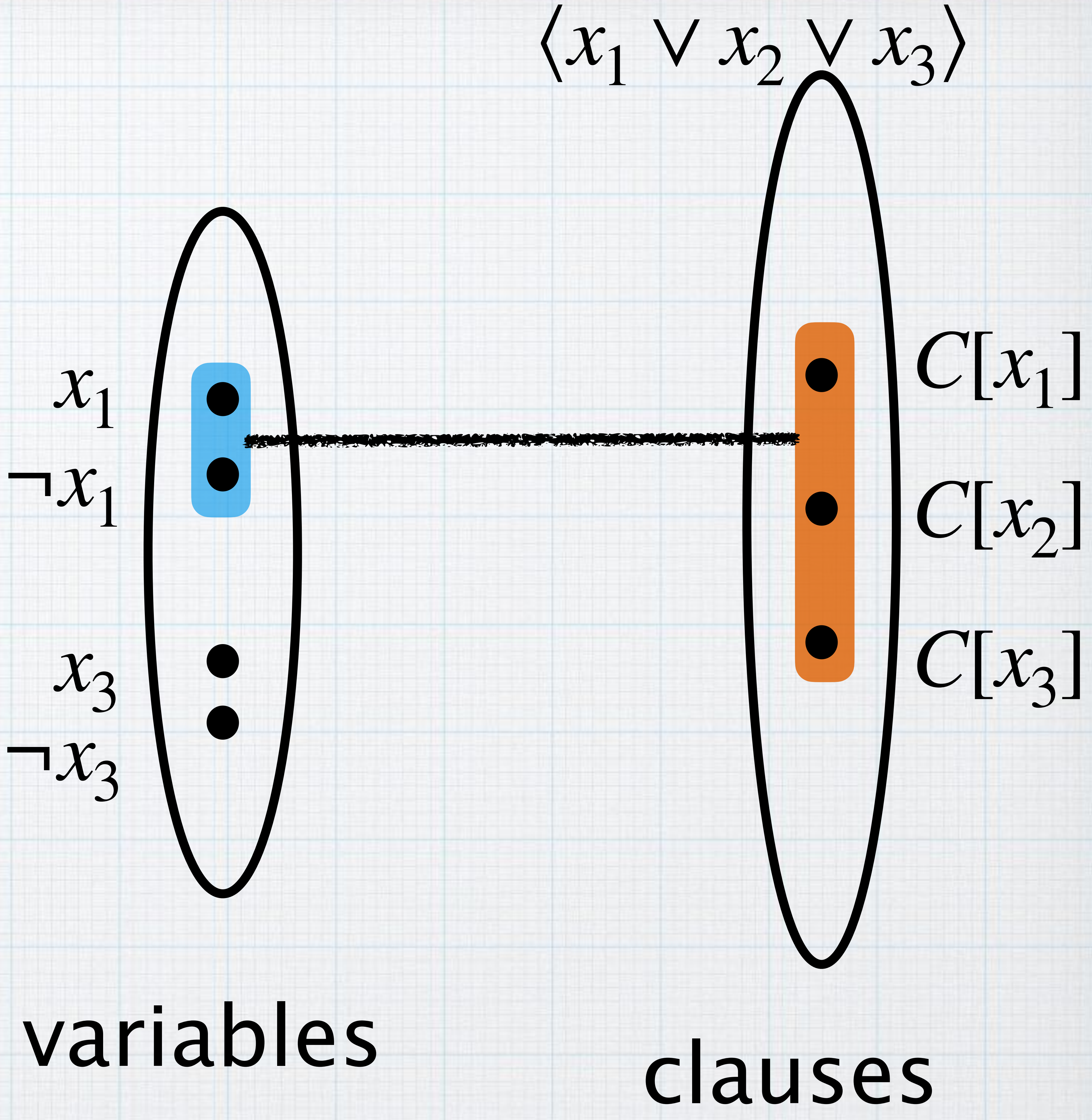
3-SAT to Loc-Dom-Set



$X_1 = \{1,3,5\}$ $X_3 = \{1,3,6\}$

3-SAT to Loc-Dom-Set

- n-variable formula → graph with tw $= \mathcal{O}(\log(n))$



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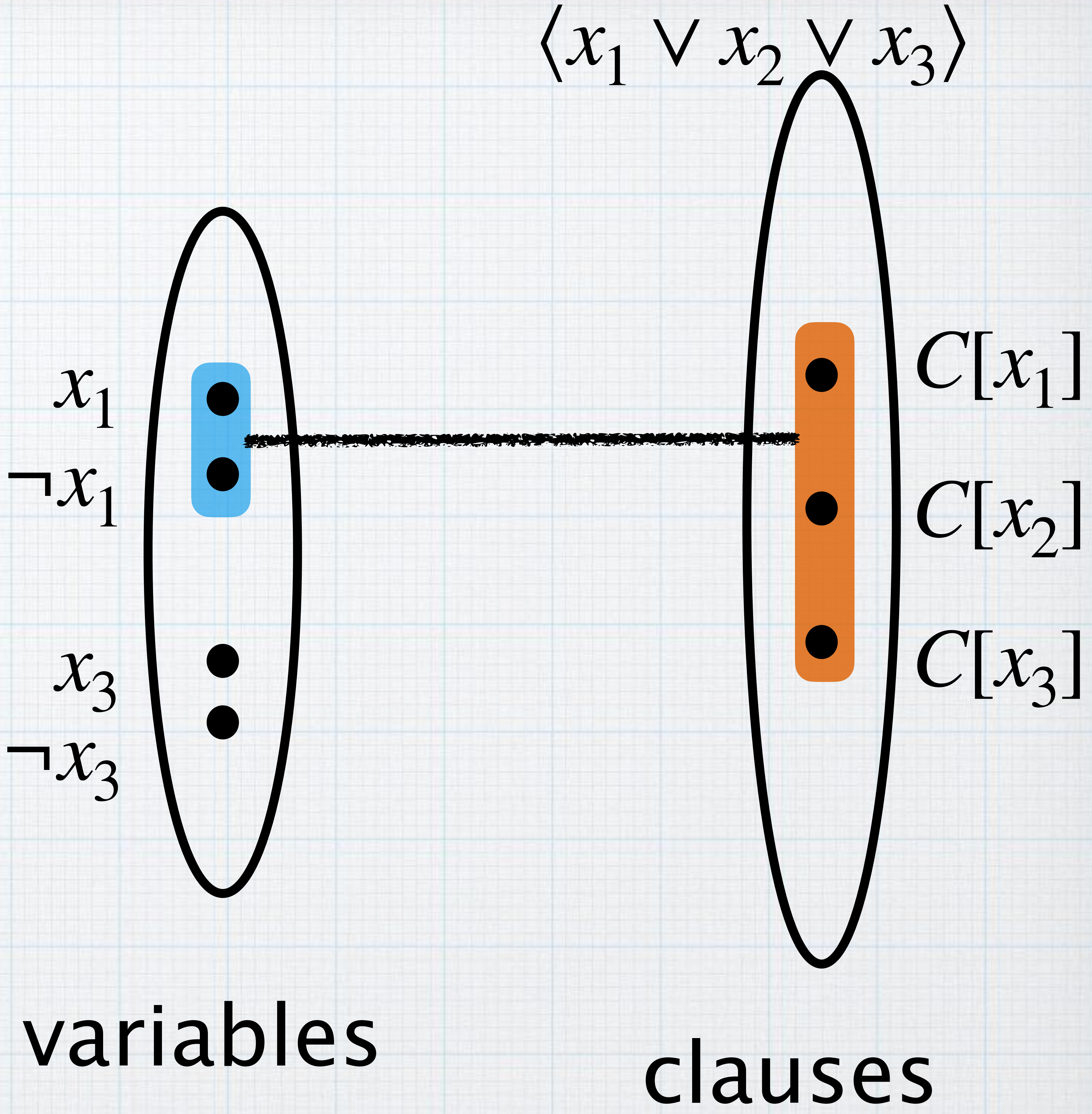

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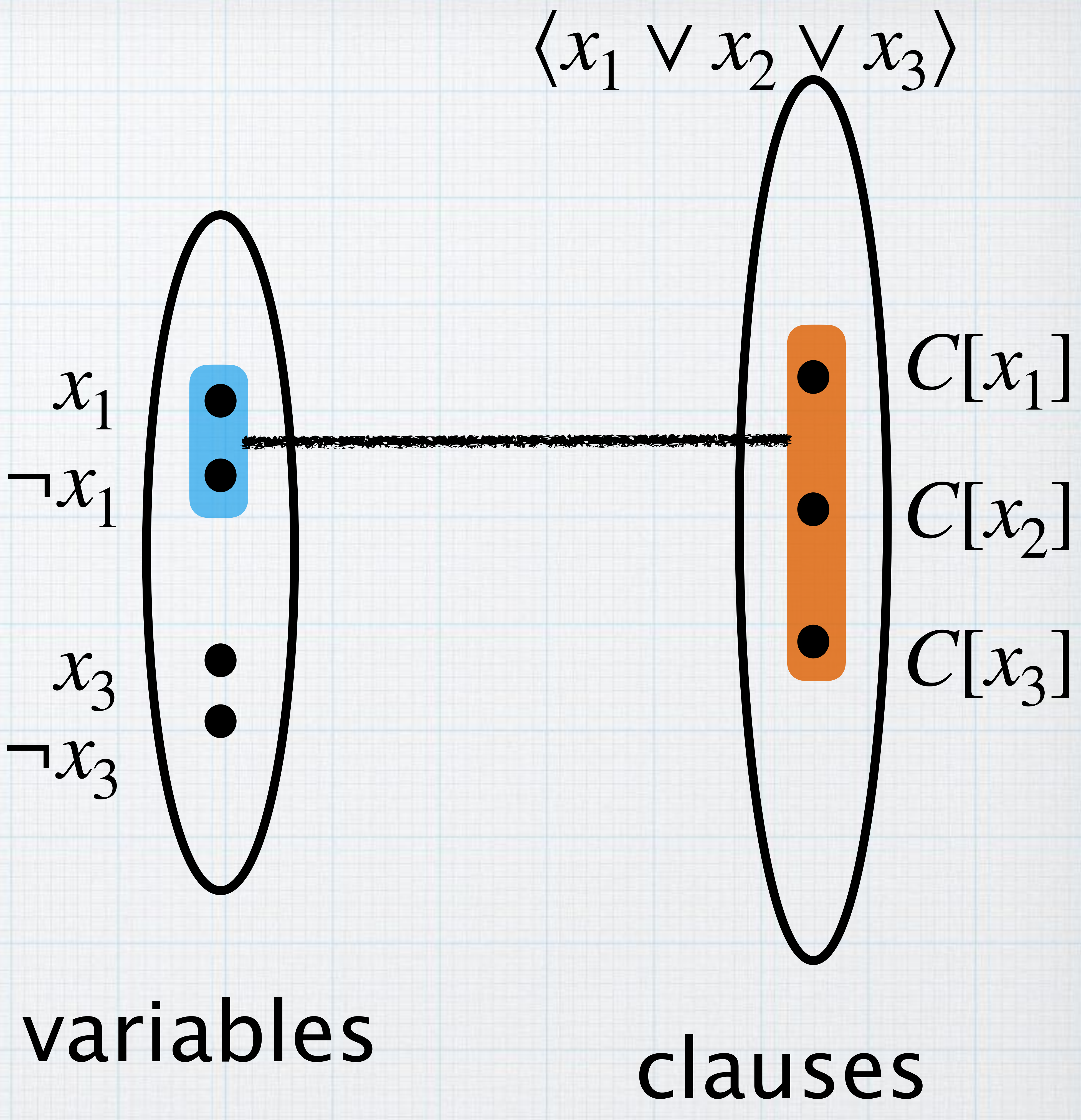
3-SAT to Loc-Dom-Set

- n-variable formula → graph with tw $= \mathcal{O}(\log(n))$
- Add pair of vertices for each variable
- Add 3 vertices for each clause



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3-SAT to Loc-Dom-Set

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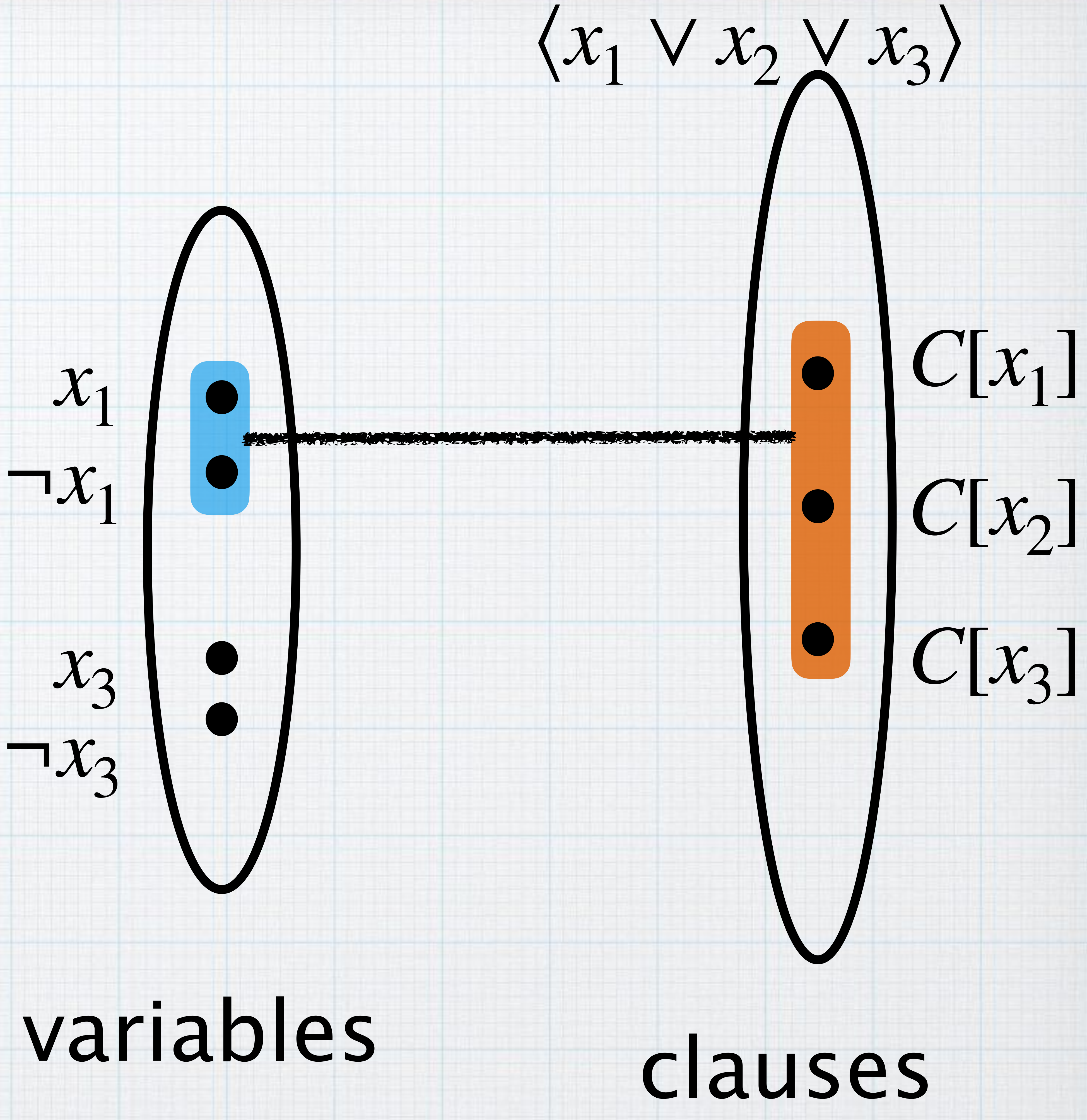

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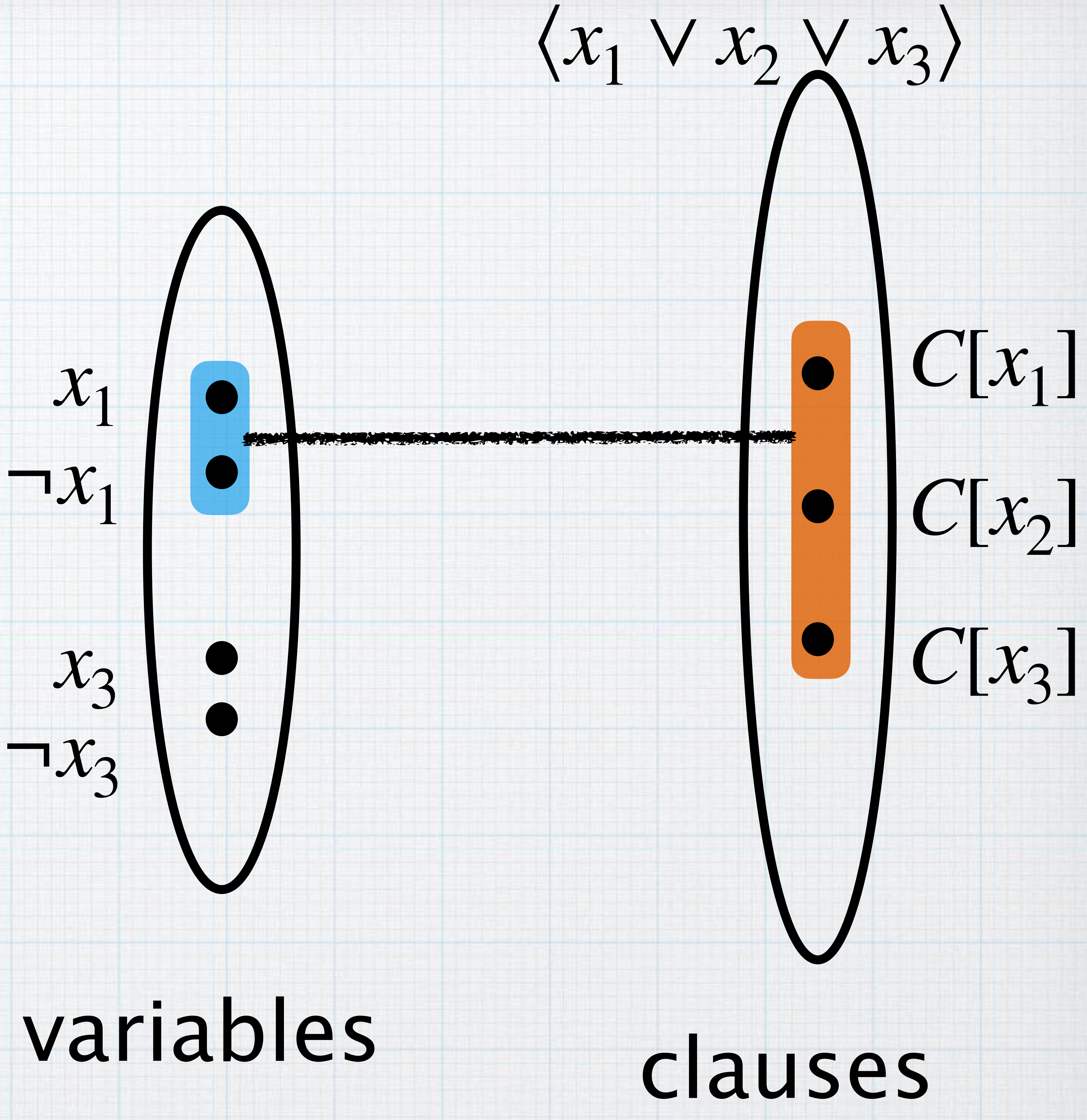

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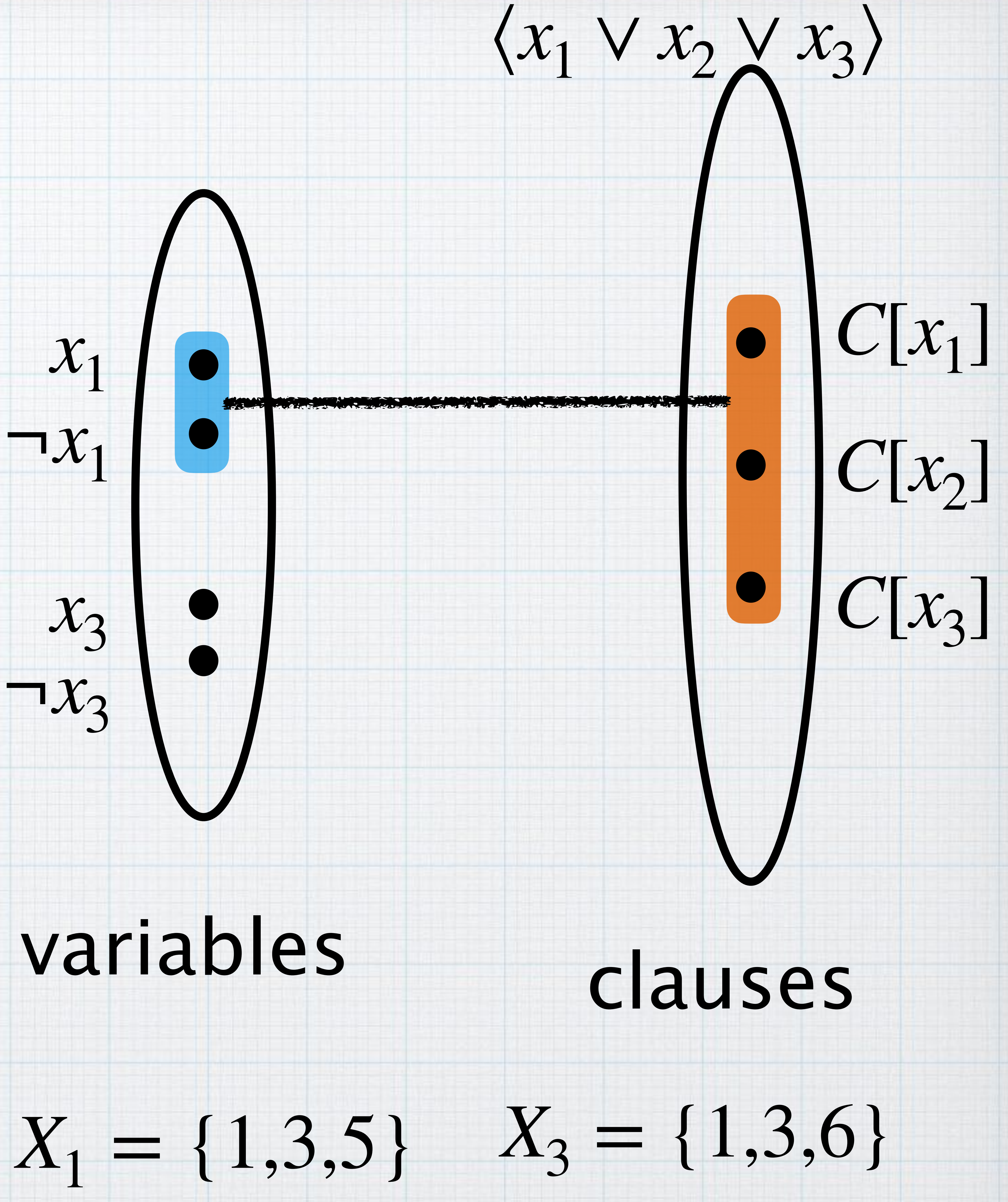

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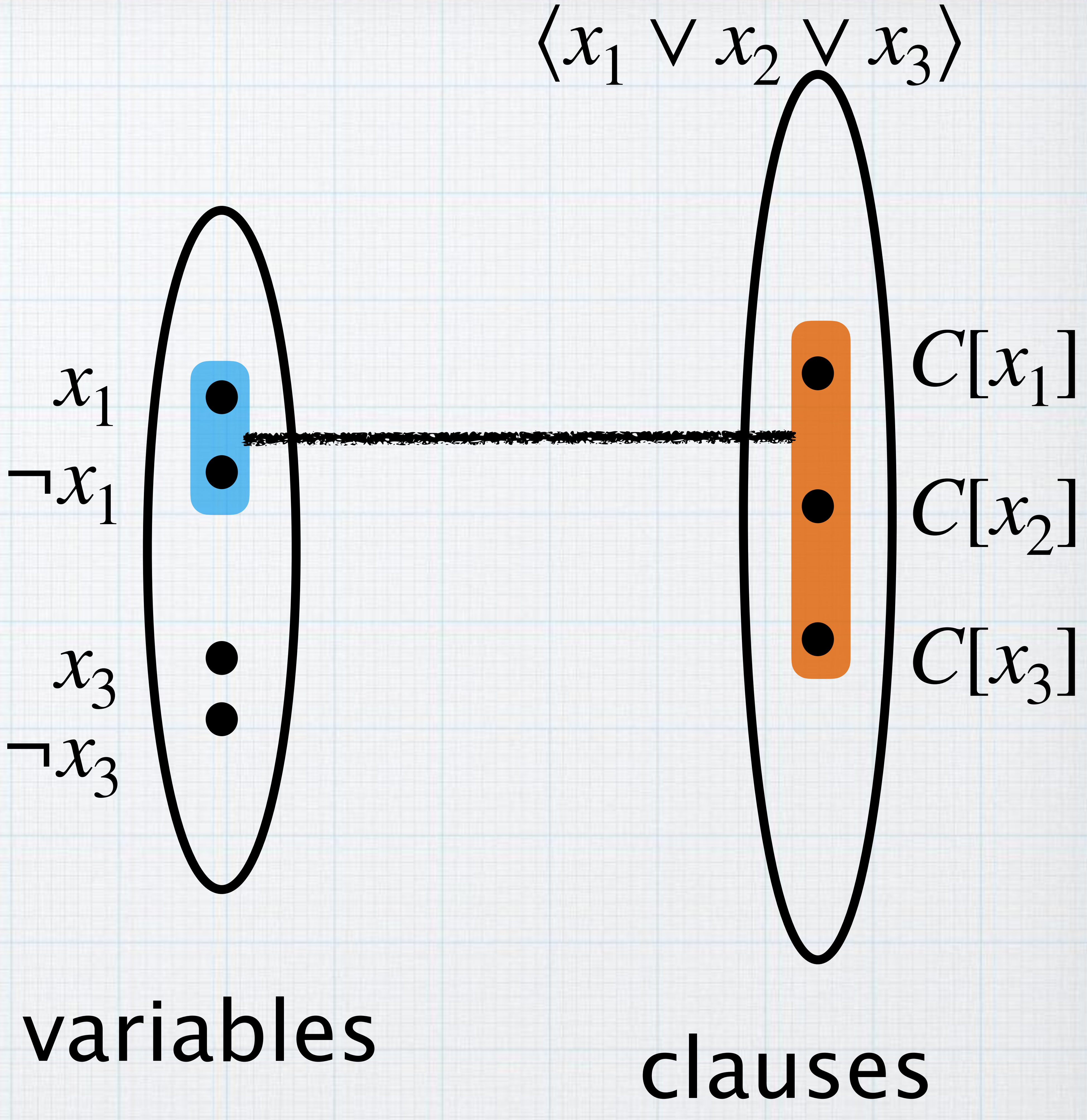

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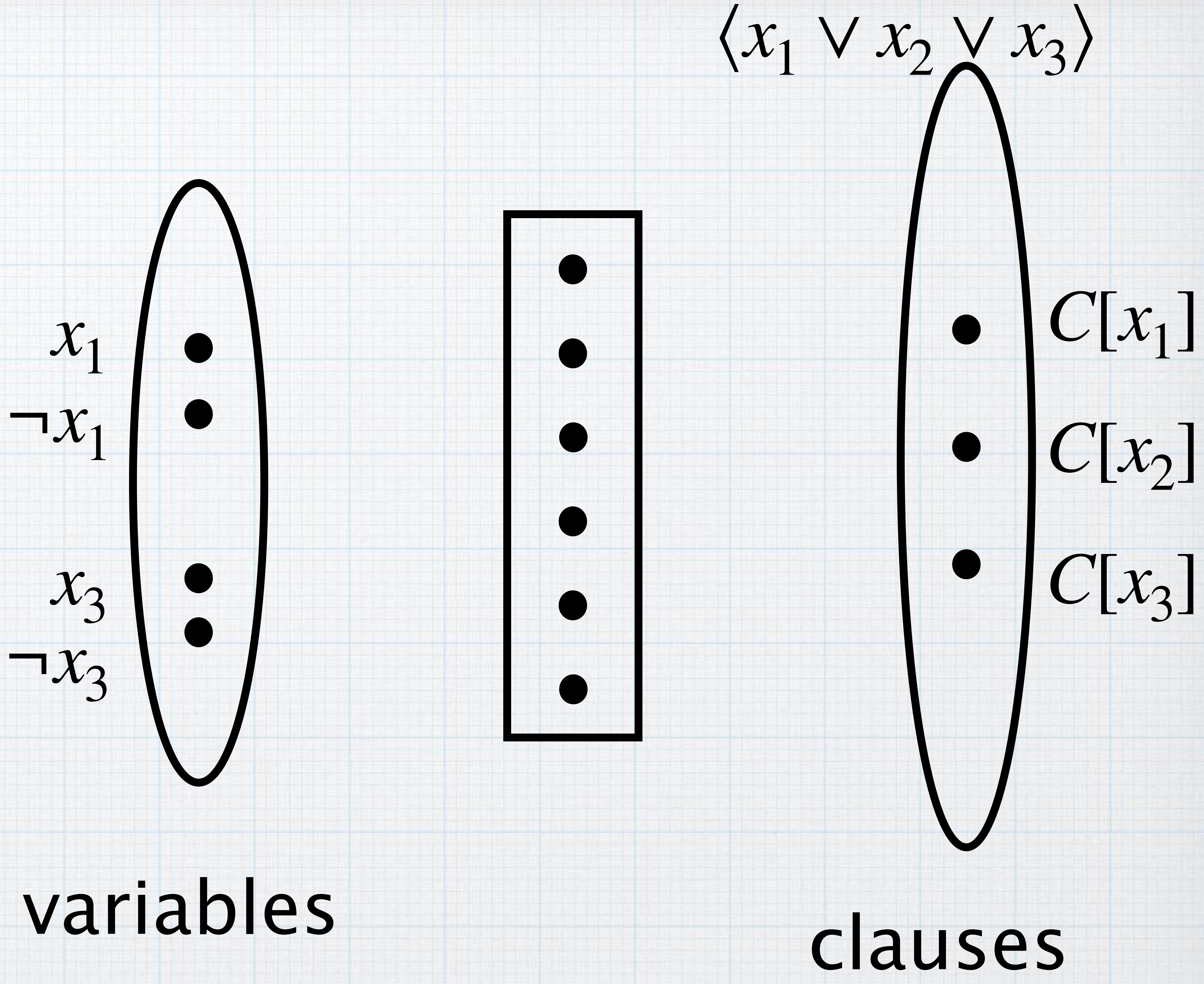

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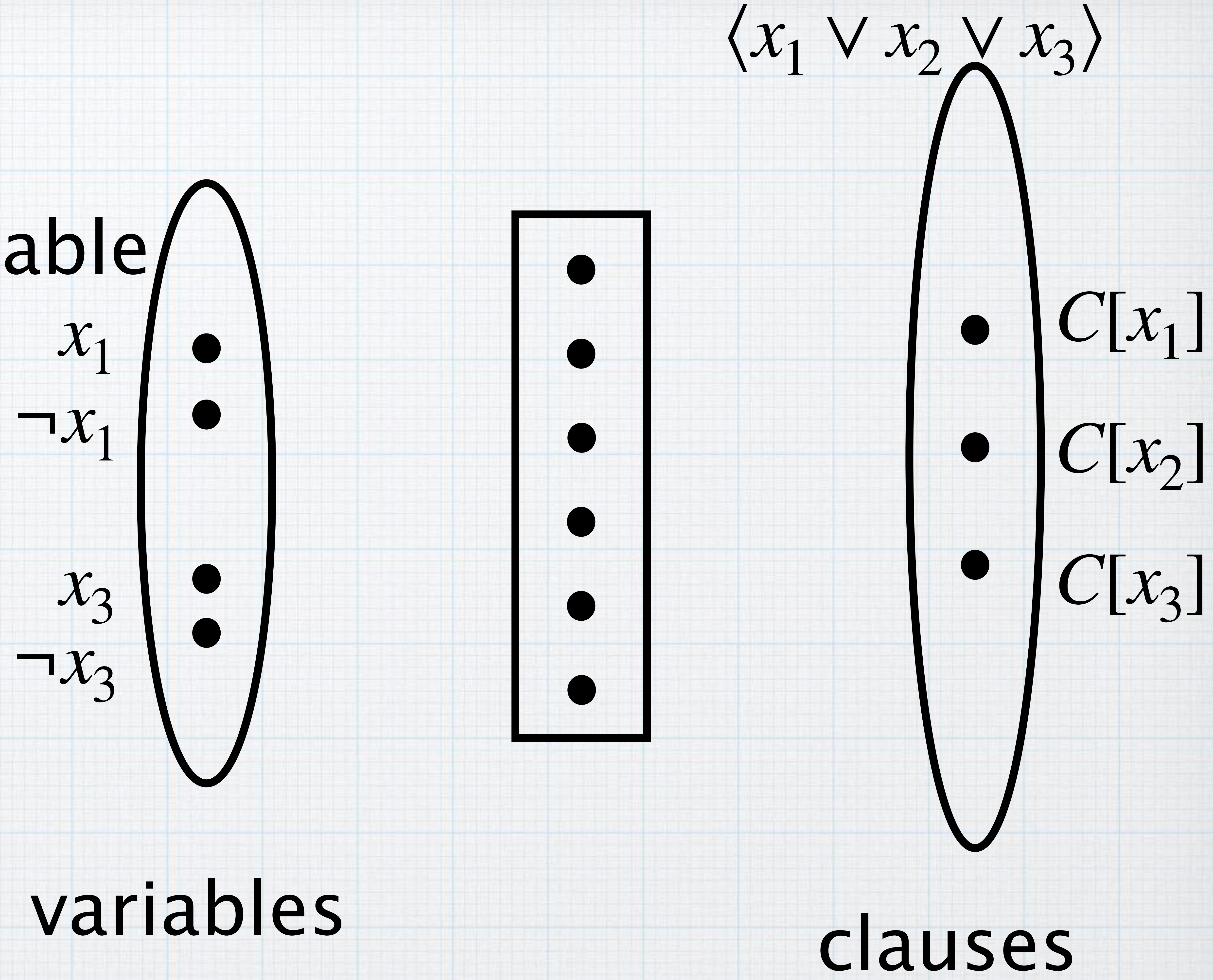

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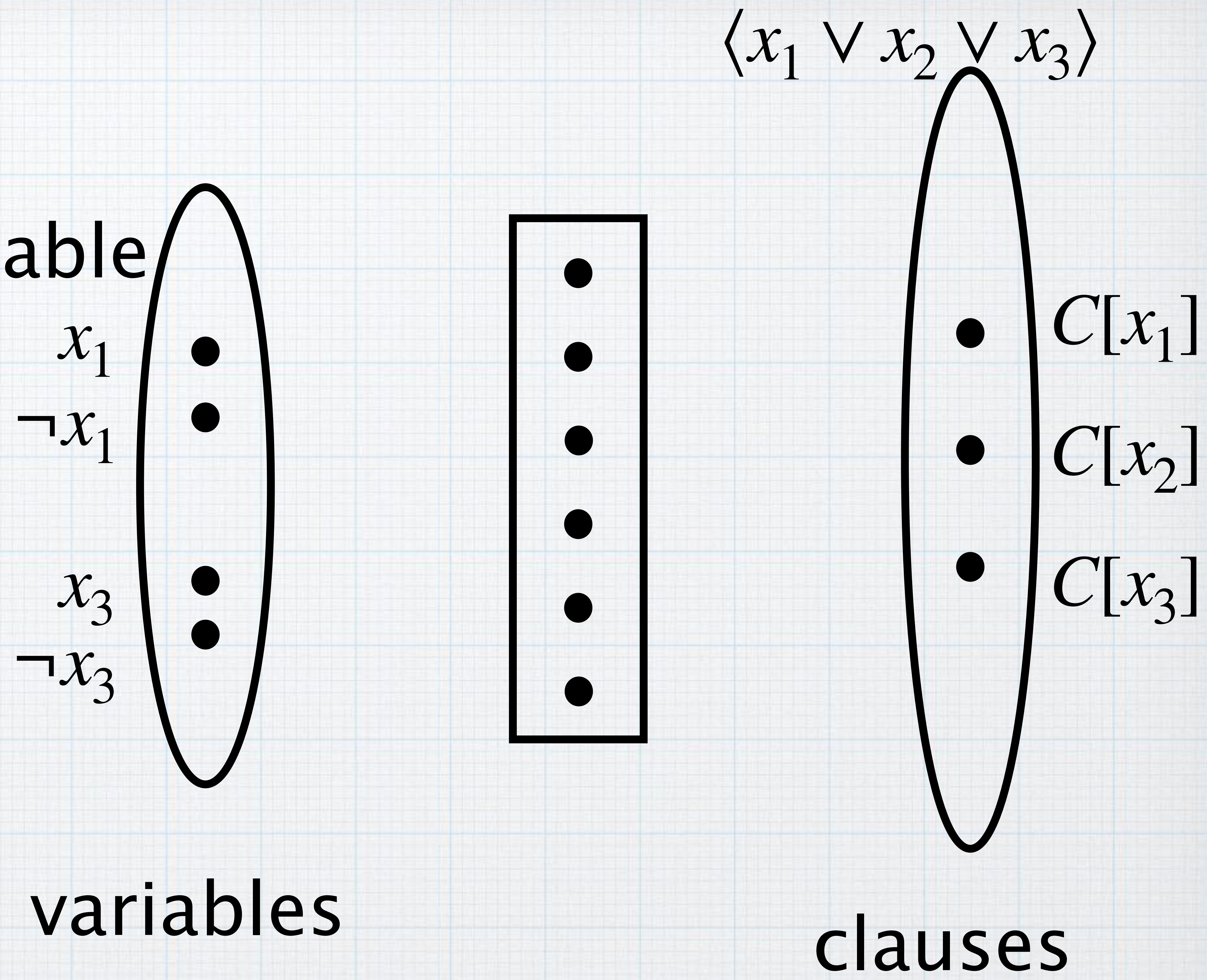

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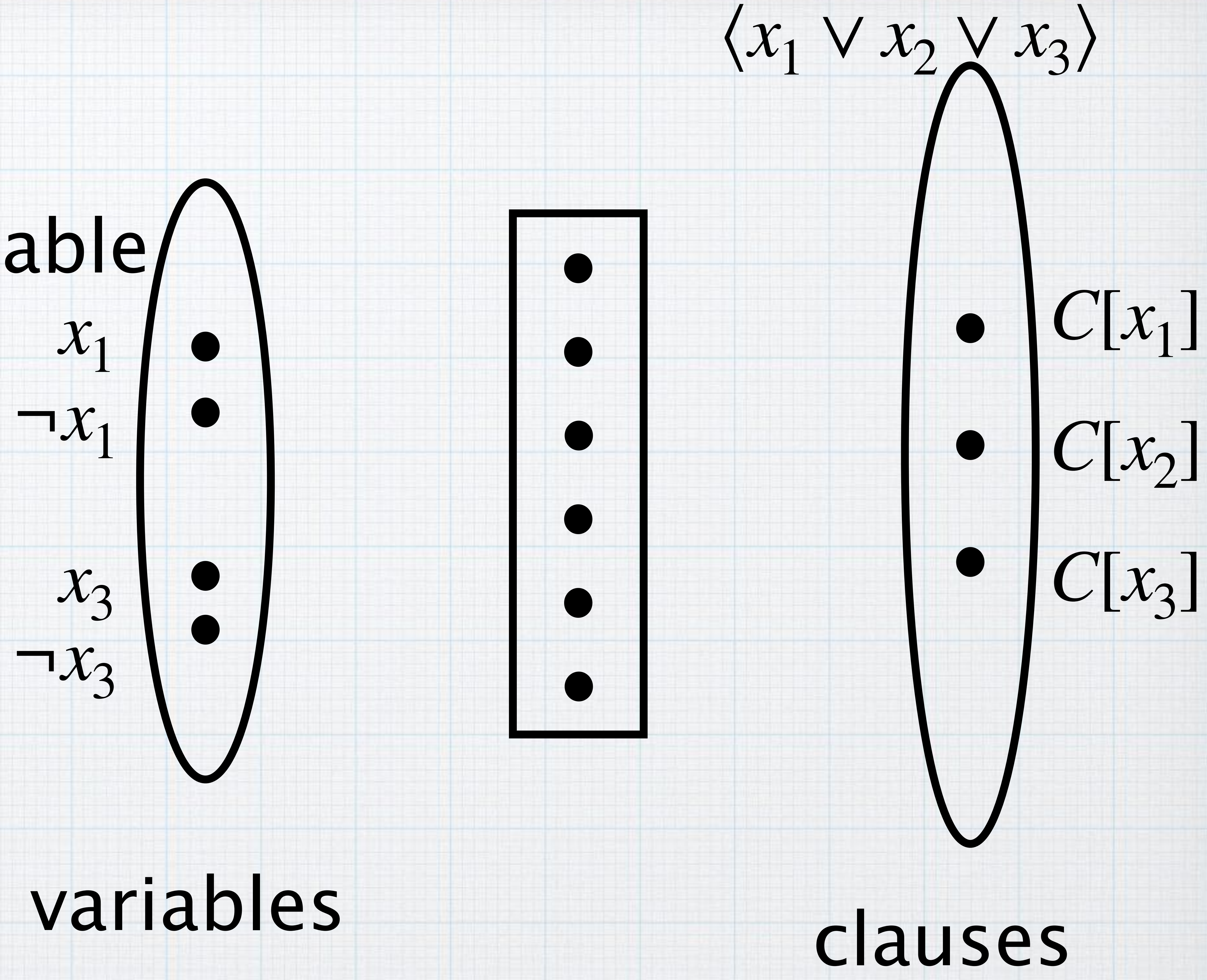

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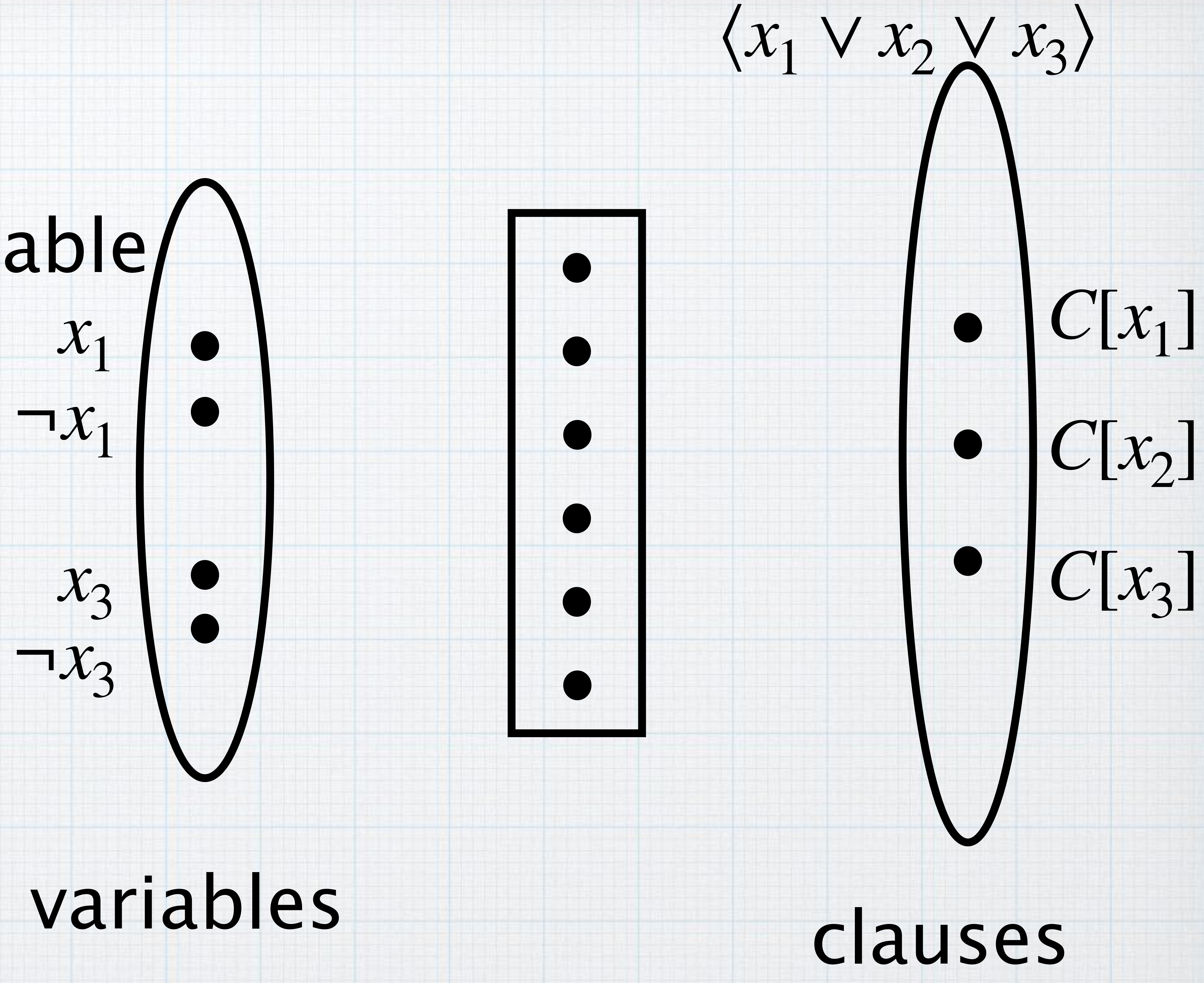

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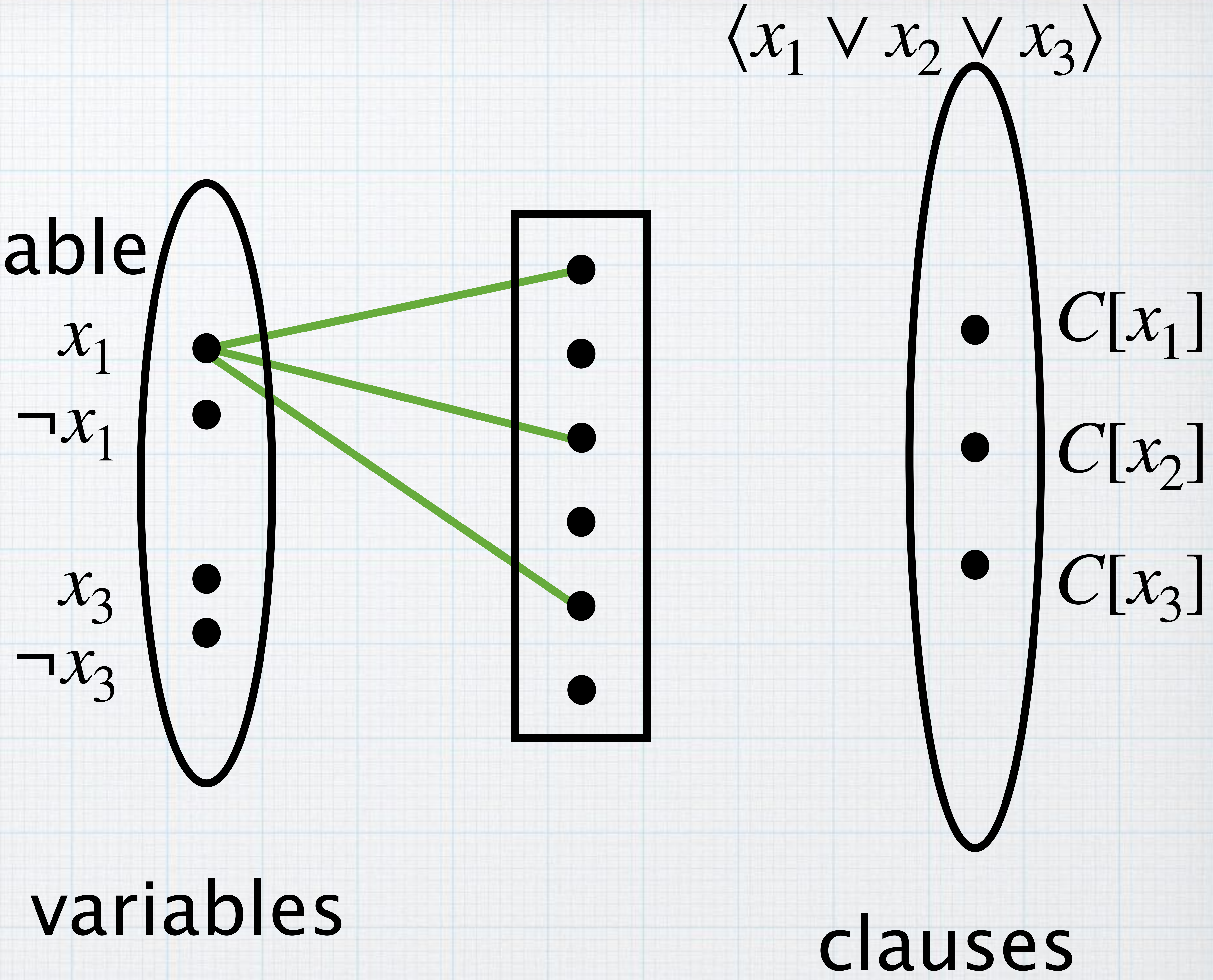

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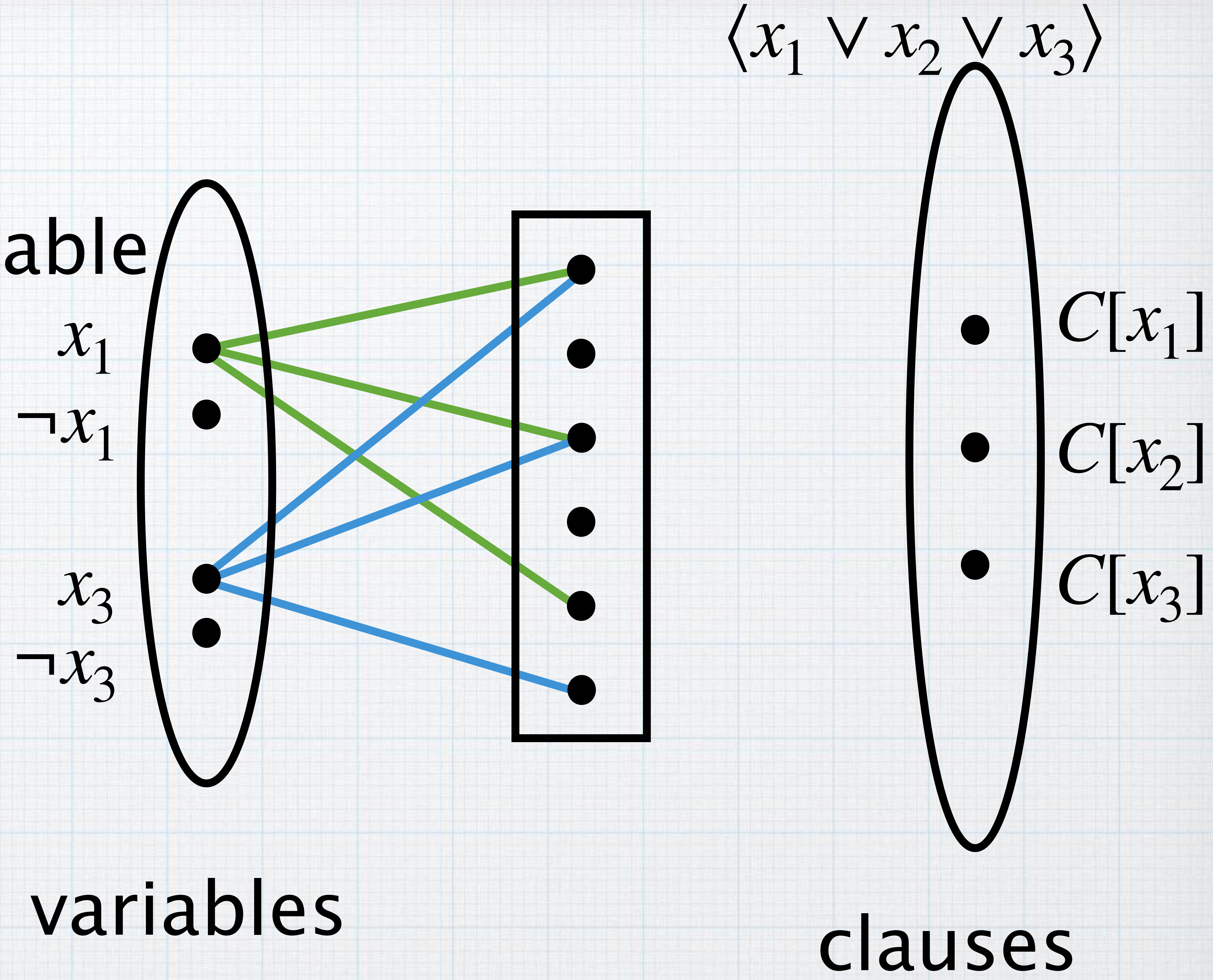

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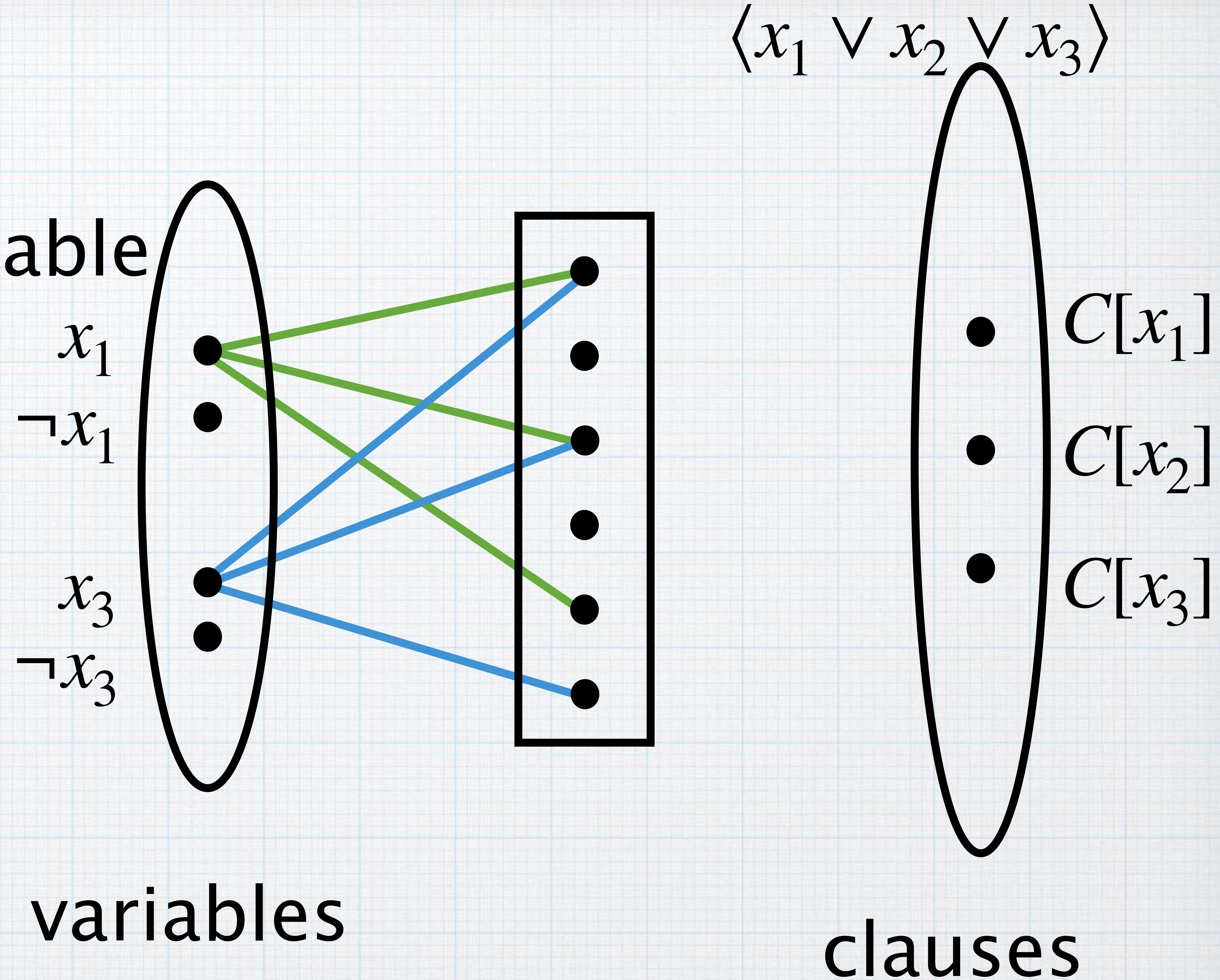

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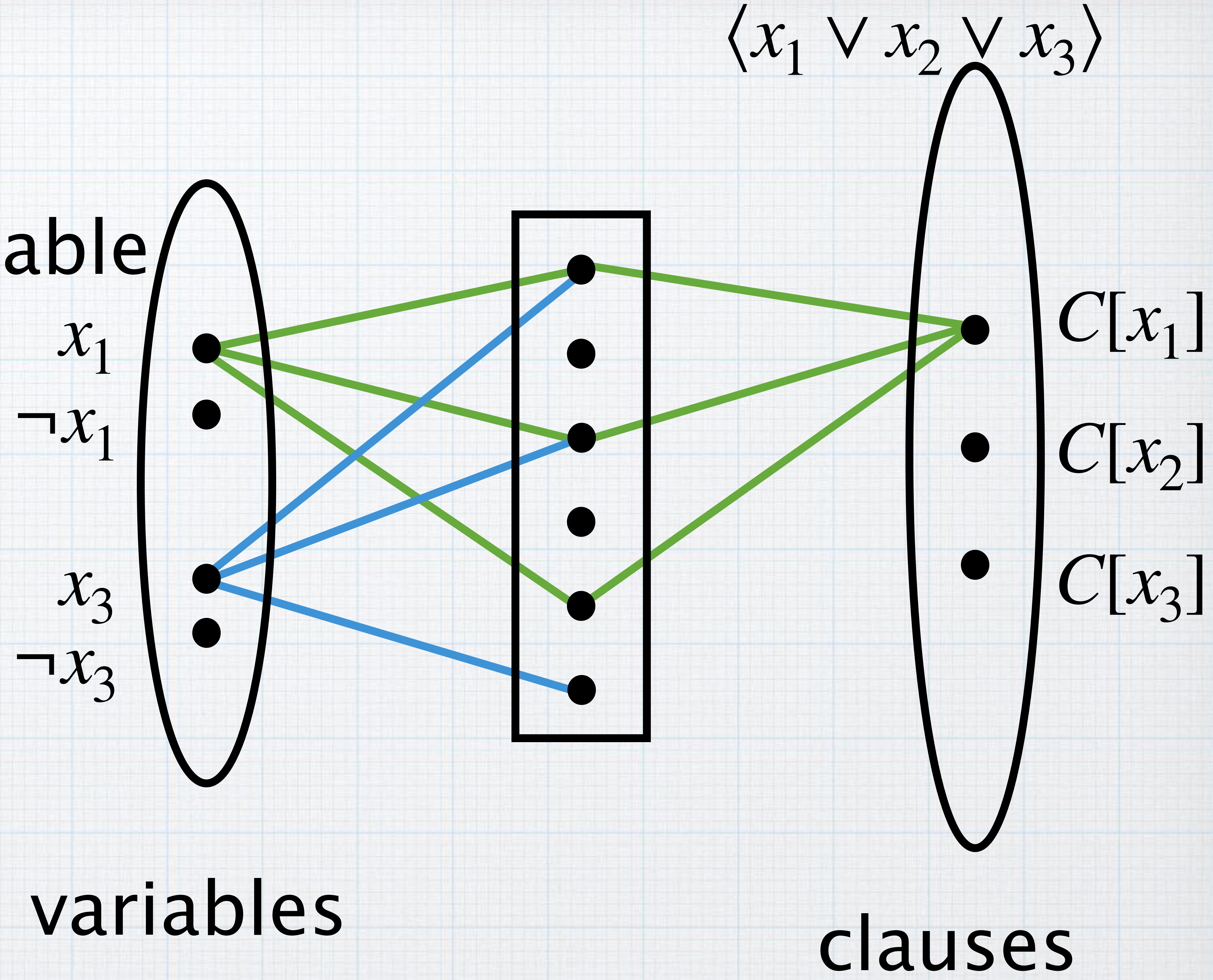

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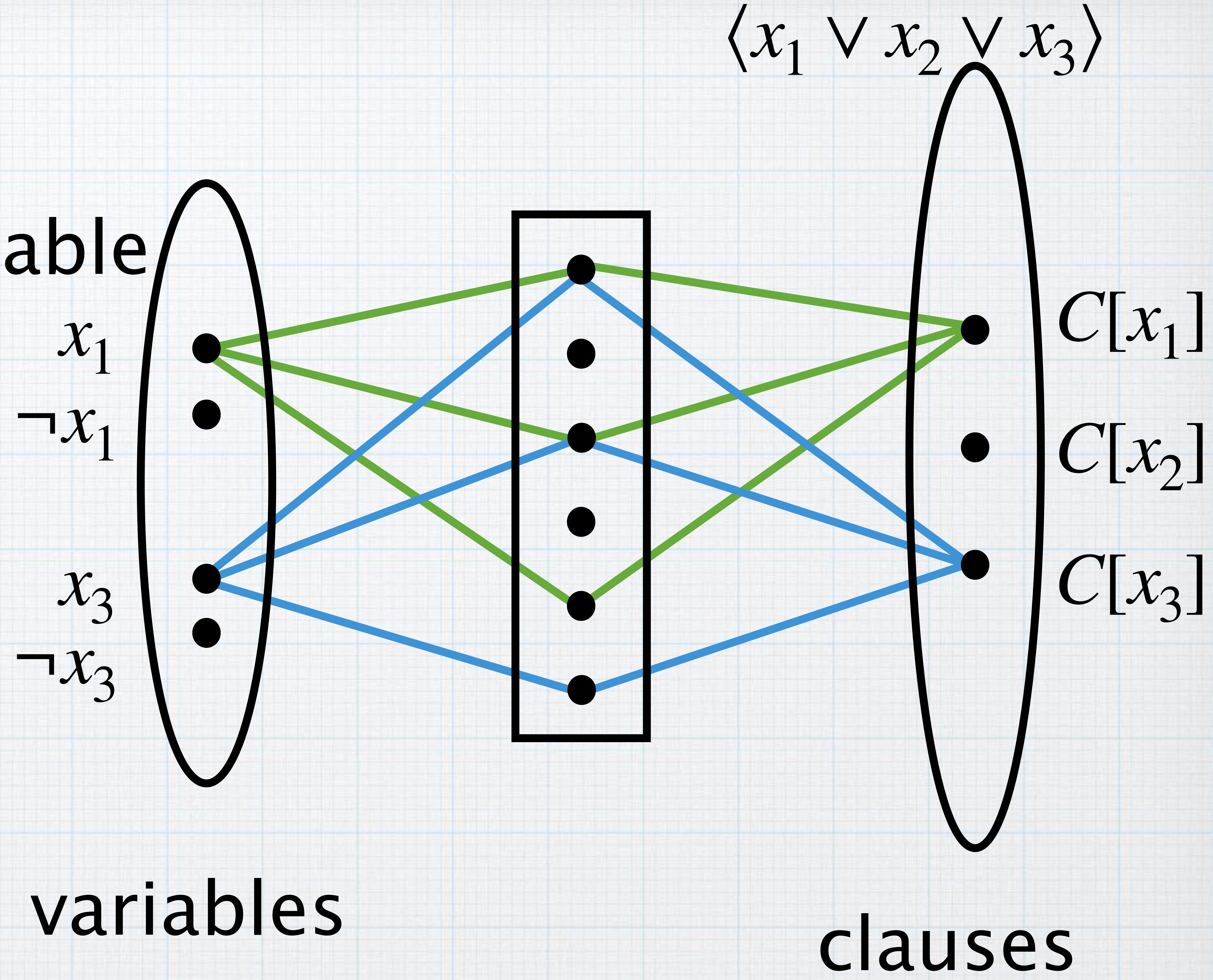

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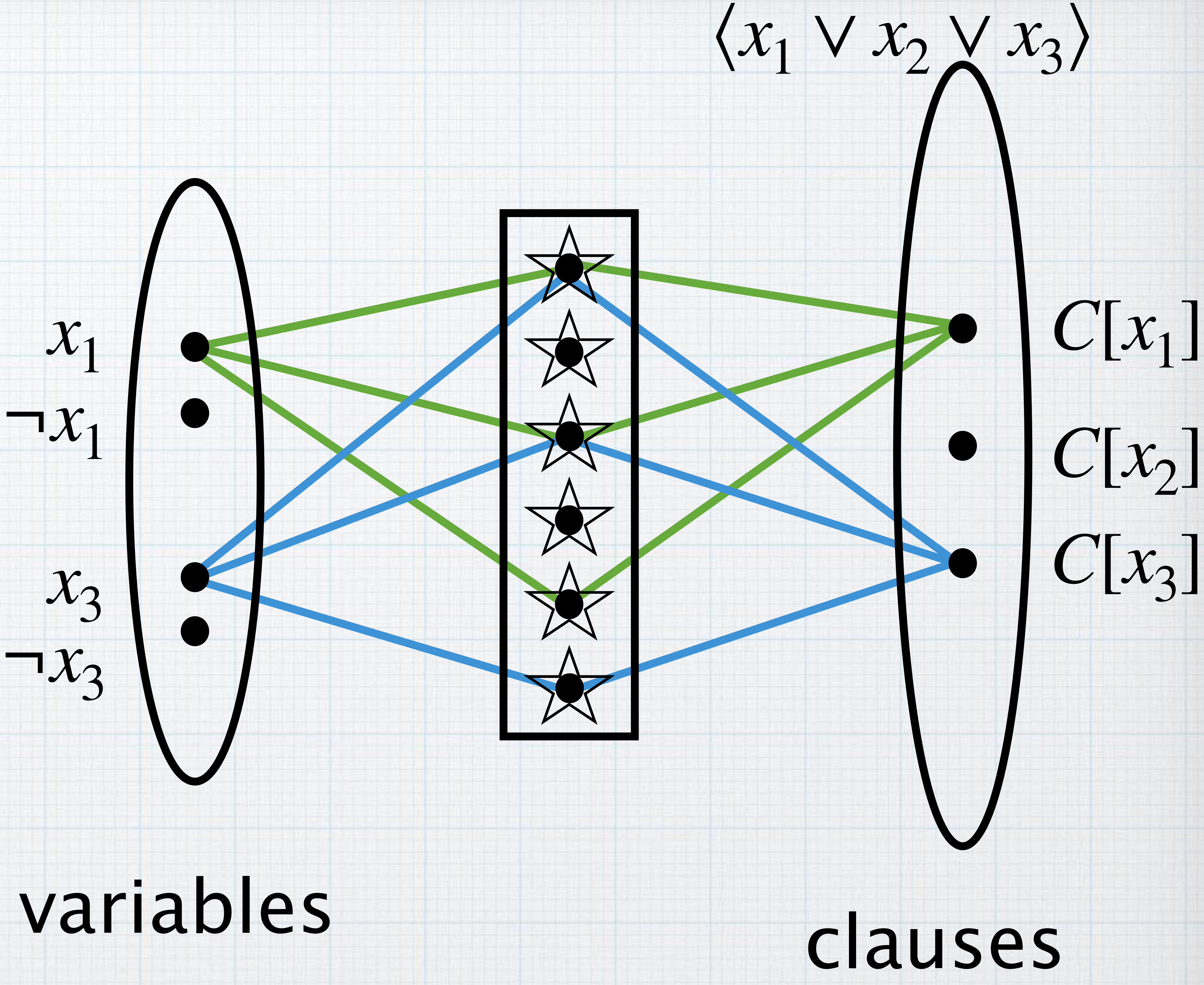

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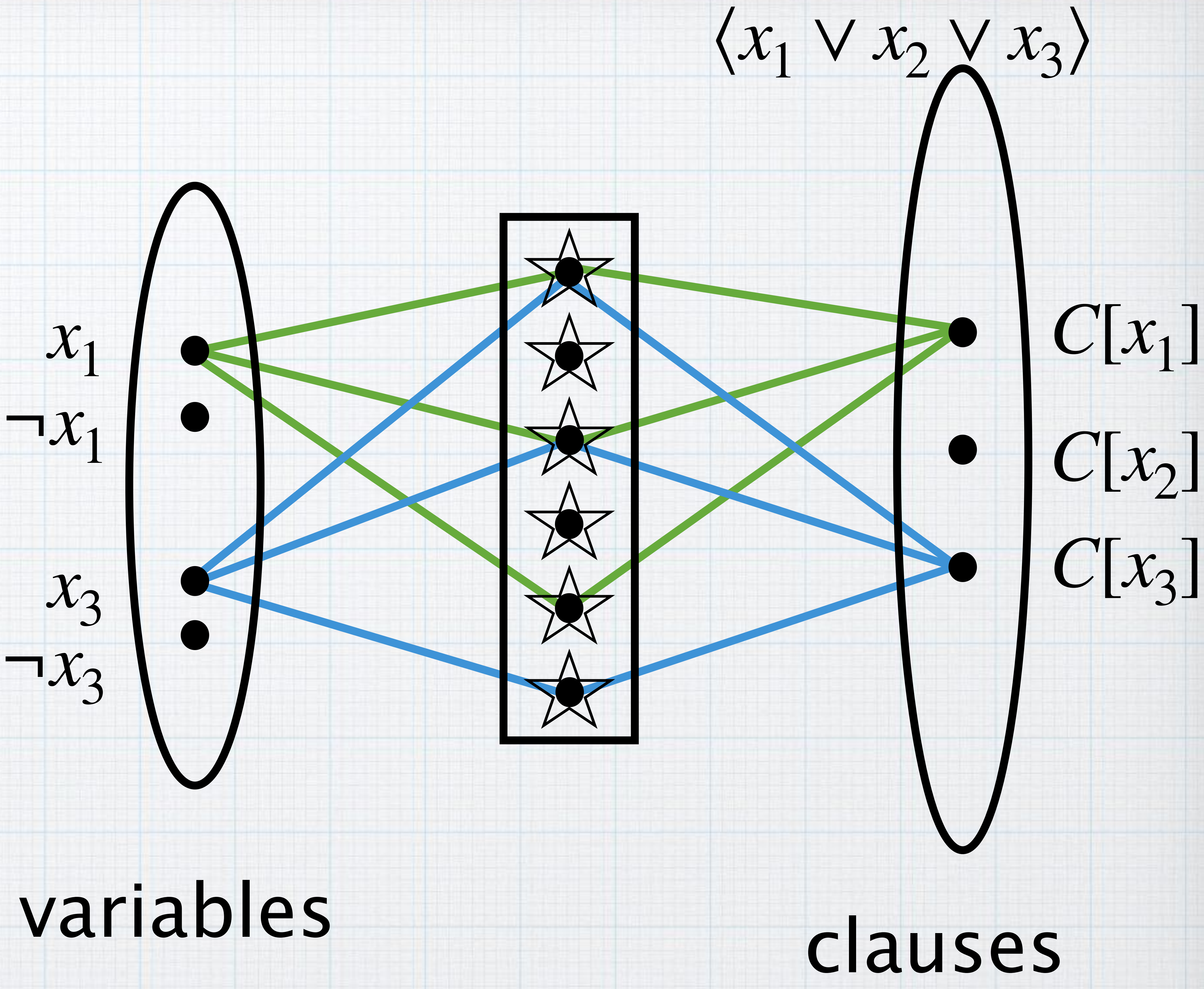

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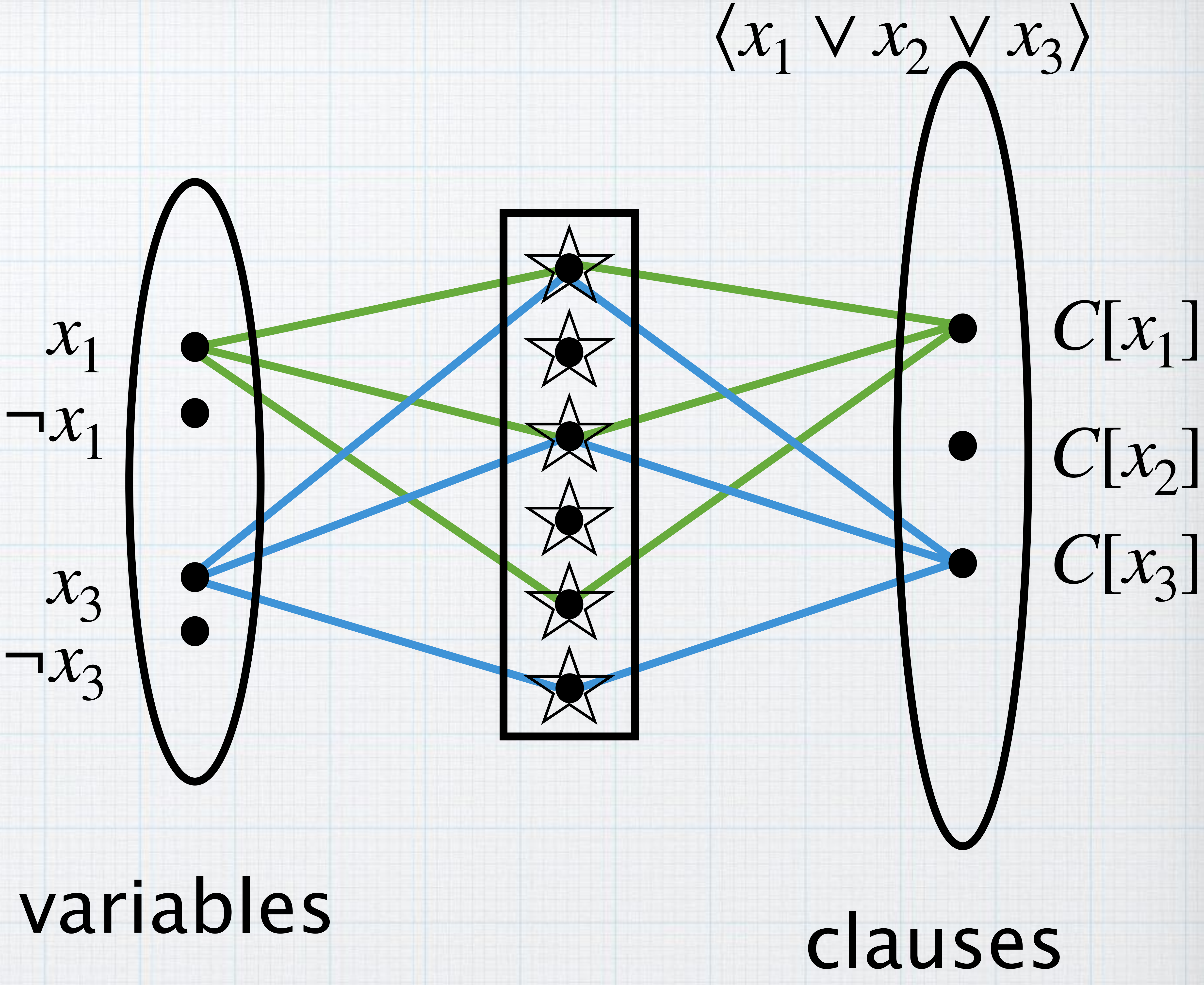

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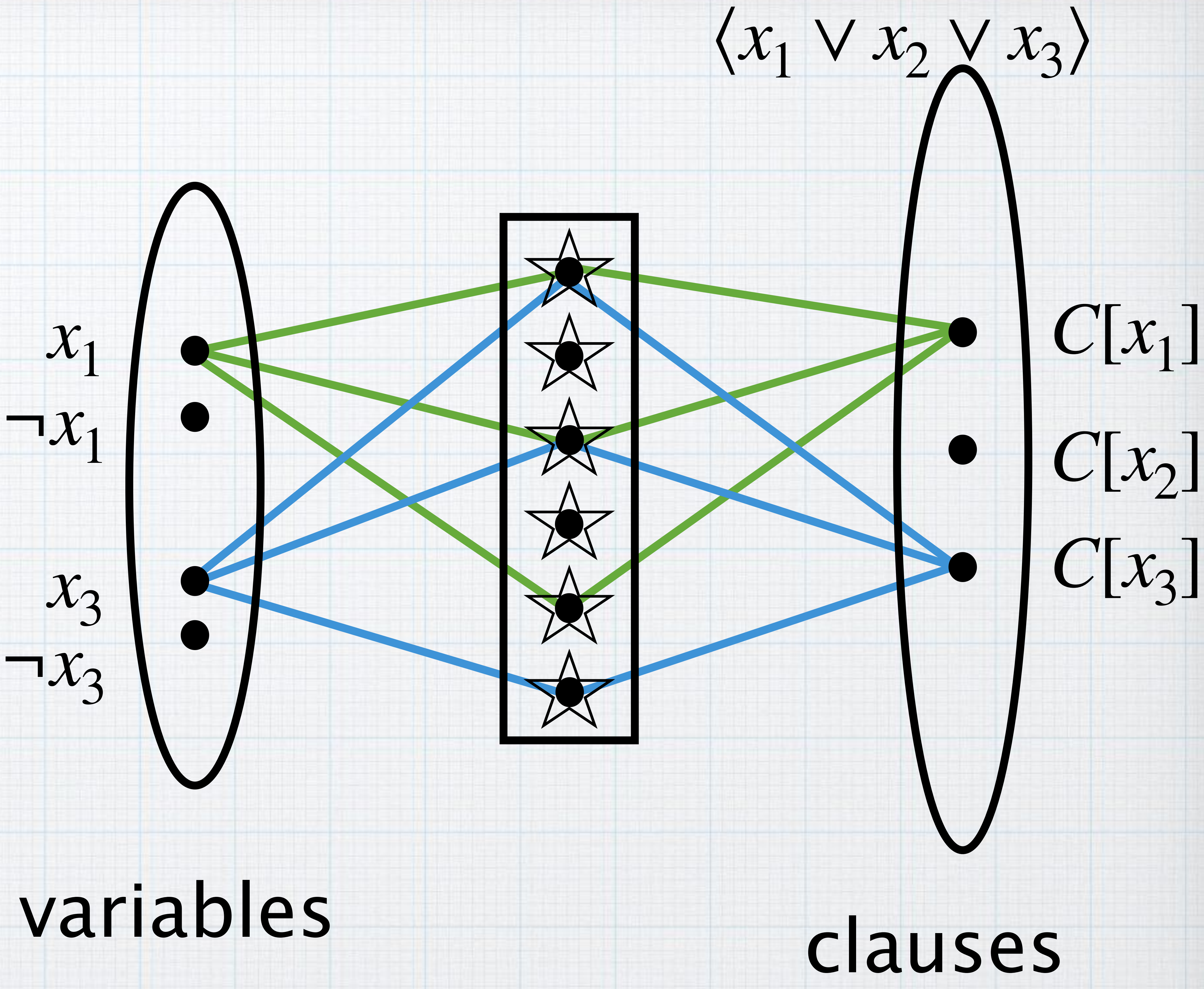

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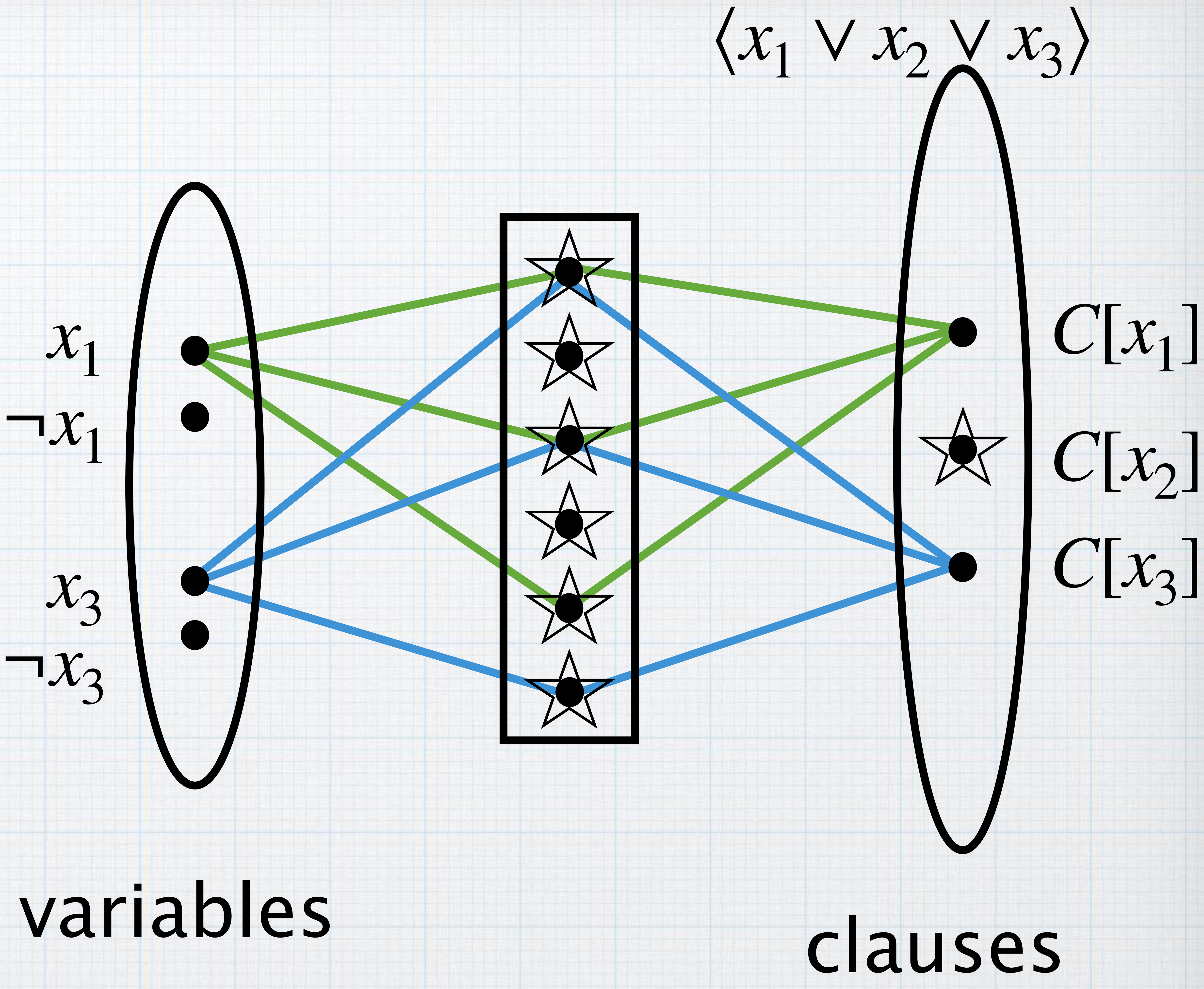

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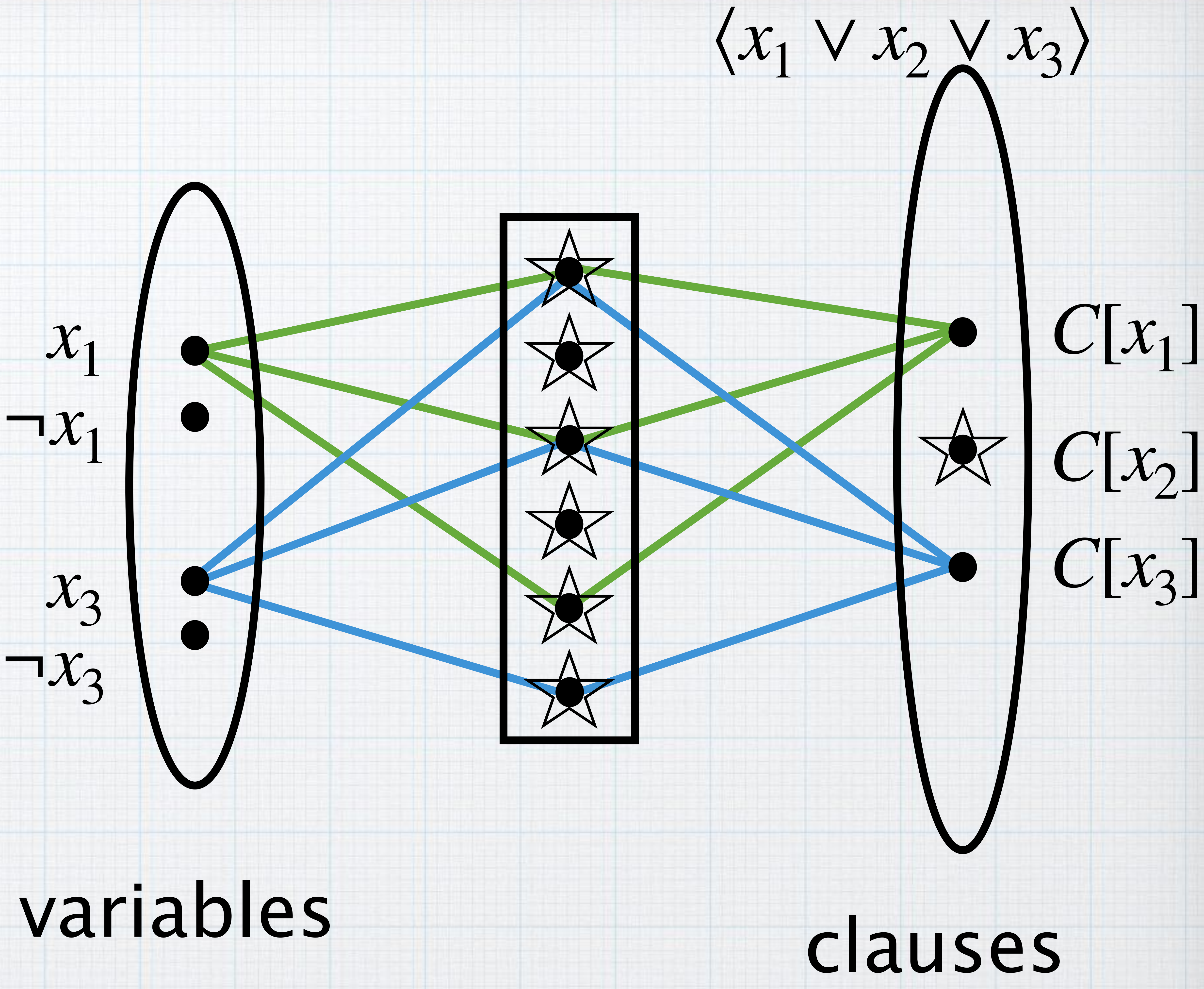

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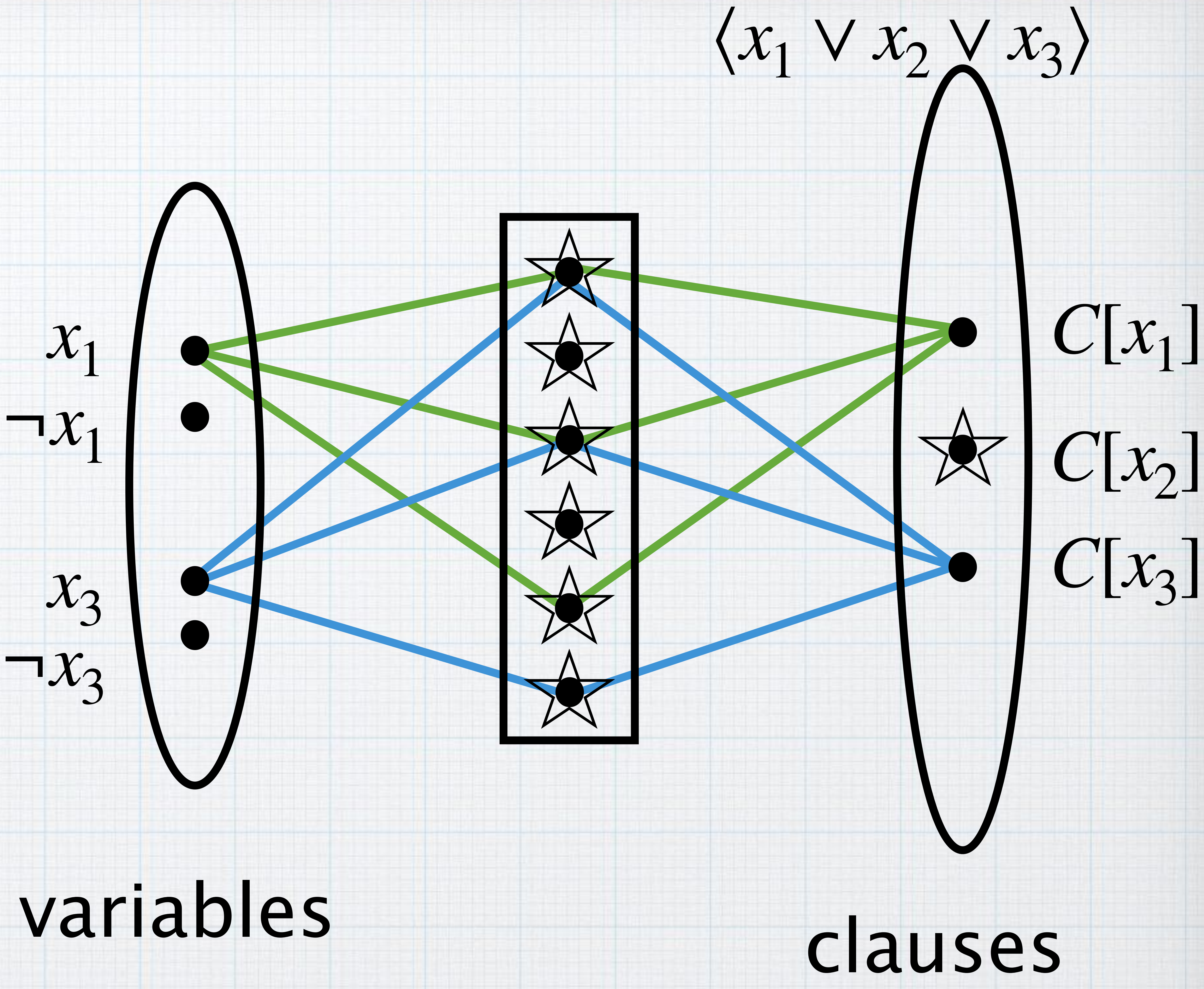

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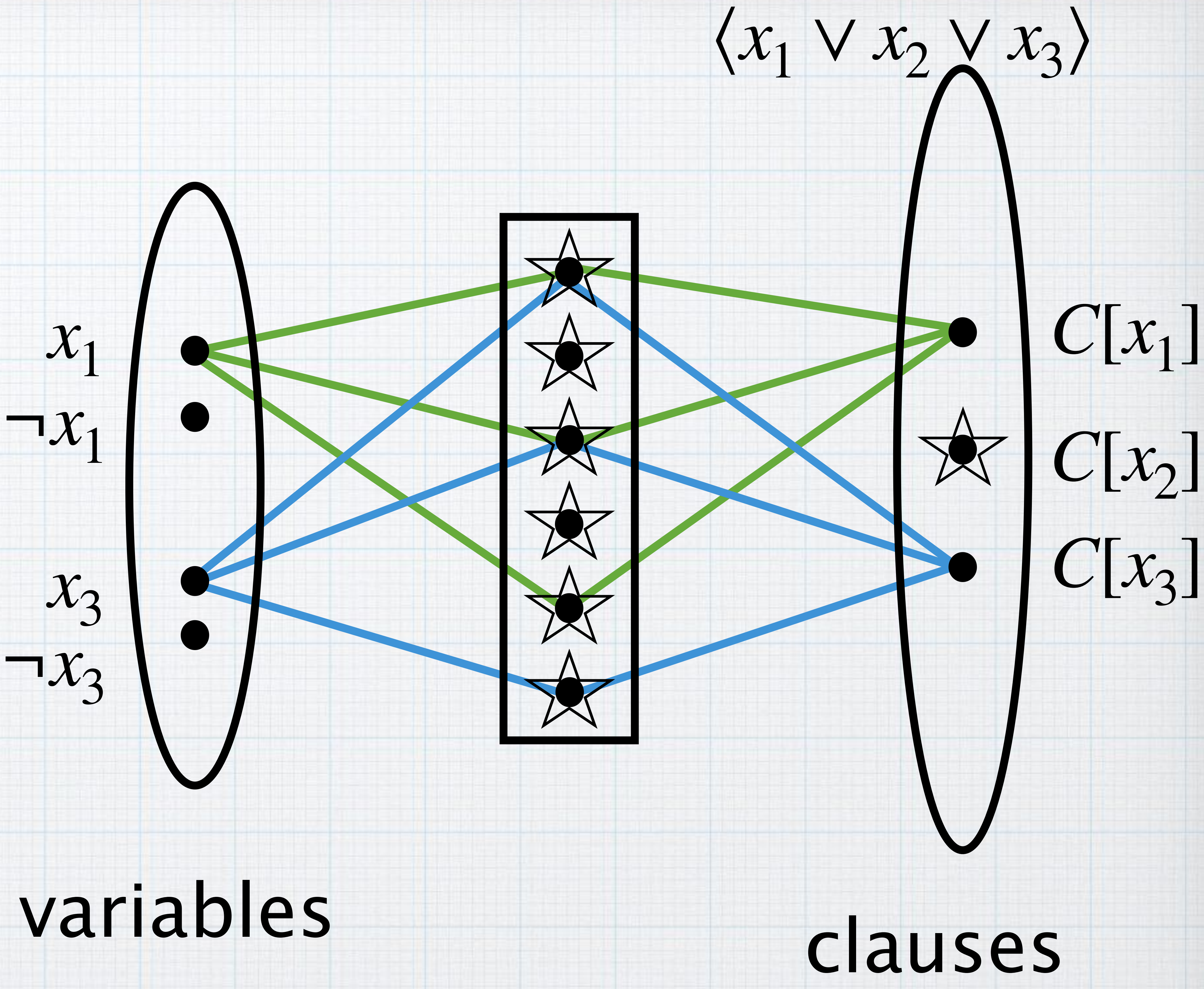

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Useful to encode 3-SAT instance.

Future Directions

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- Can our tight double-exponential lower bound for Locating-Dominating Set parameterized by treewidth be applied to the feedback vertex set number (a larger parameter)?

Thank you

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Thm [Marx, Mitsou (ICALP'16)]. The λ–Choosability problem

— admits $2^{2^{\mathcal{O}(\mathrm{tw})}} \cdot n^{\mathcal{O}(1)}$–time algo, but

— does not admit $2^{2^{o(\mathrm{tw})}} \cdot n^{\mathcal{O}(1)}$ algo unless the ETH fails.