On the Hardness of
Strong Metric Dimension

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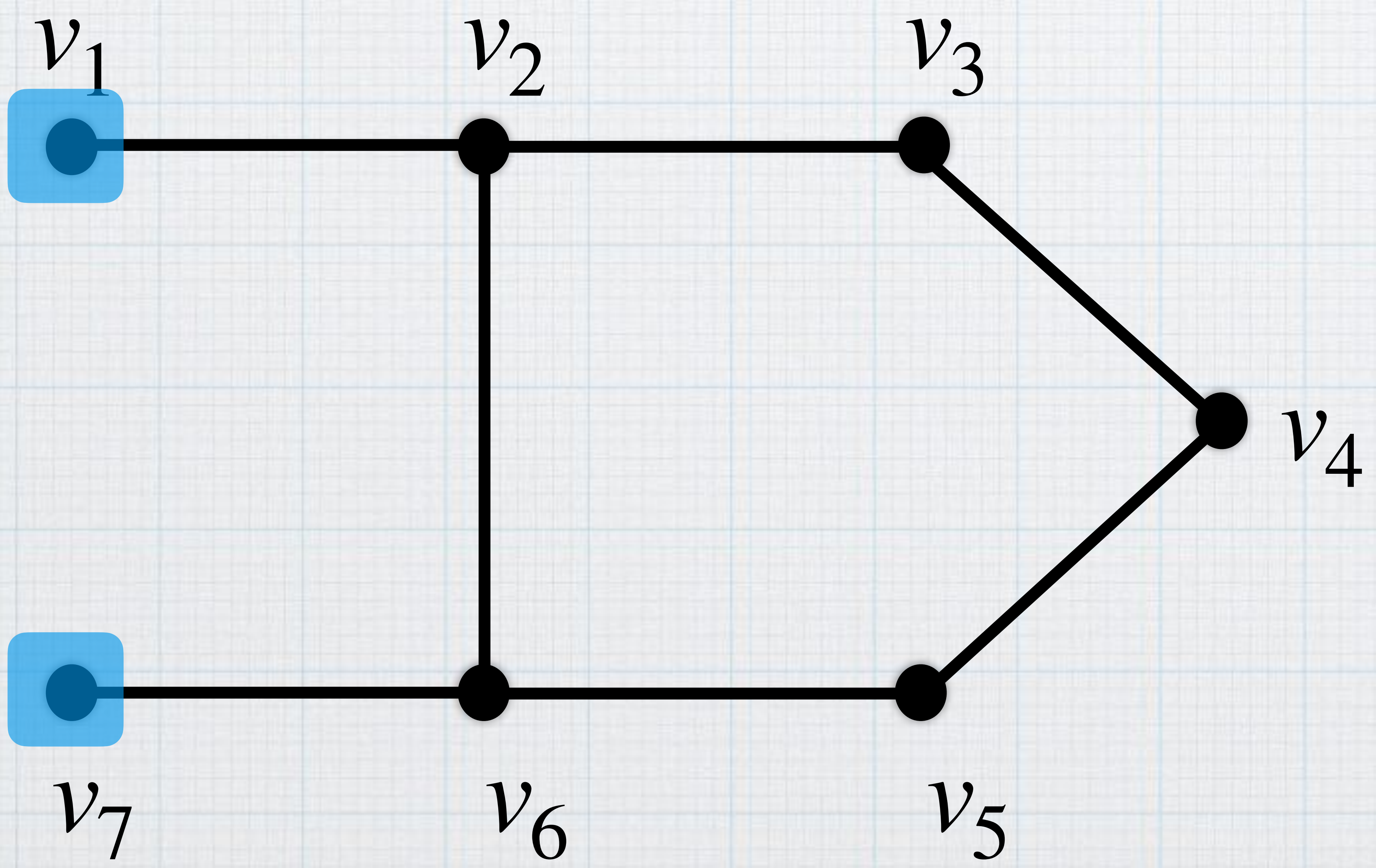
~~Strong~~ Metric Dimension (MD)

Input: Graph G and int k

Output: Does there exist a subset S of size k s.t. for every $u, v \in V(G)$, there is a vertex $s \in S$ such as $d(u, s) \neq d(v, s)$?

Given G, is there a *metric generator* (of specified size) that creates different distance profiles for each vertex?



$v_1 : (0, 3)$

$v_2 : (1, 2)$

$v_3 : (2, 3)$

$v_4 : (3, 3)$

$v_5 : (3, 2)$

$v_6 : (2, 1)$

$v_7 : (3, 0)$

~~Strong~~ Metric Dimension (MD)

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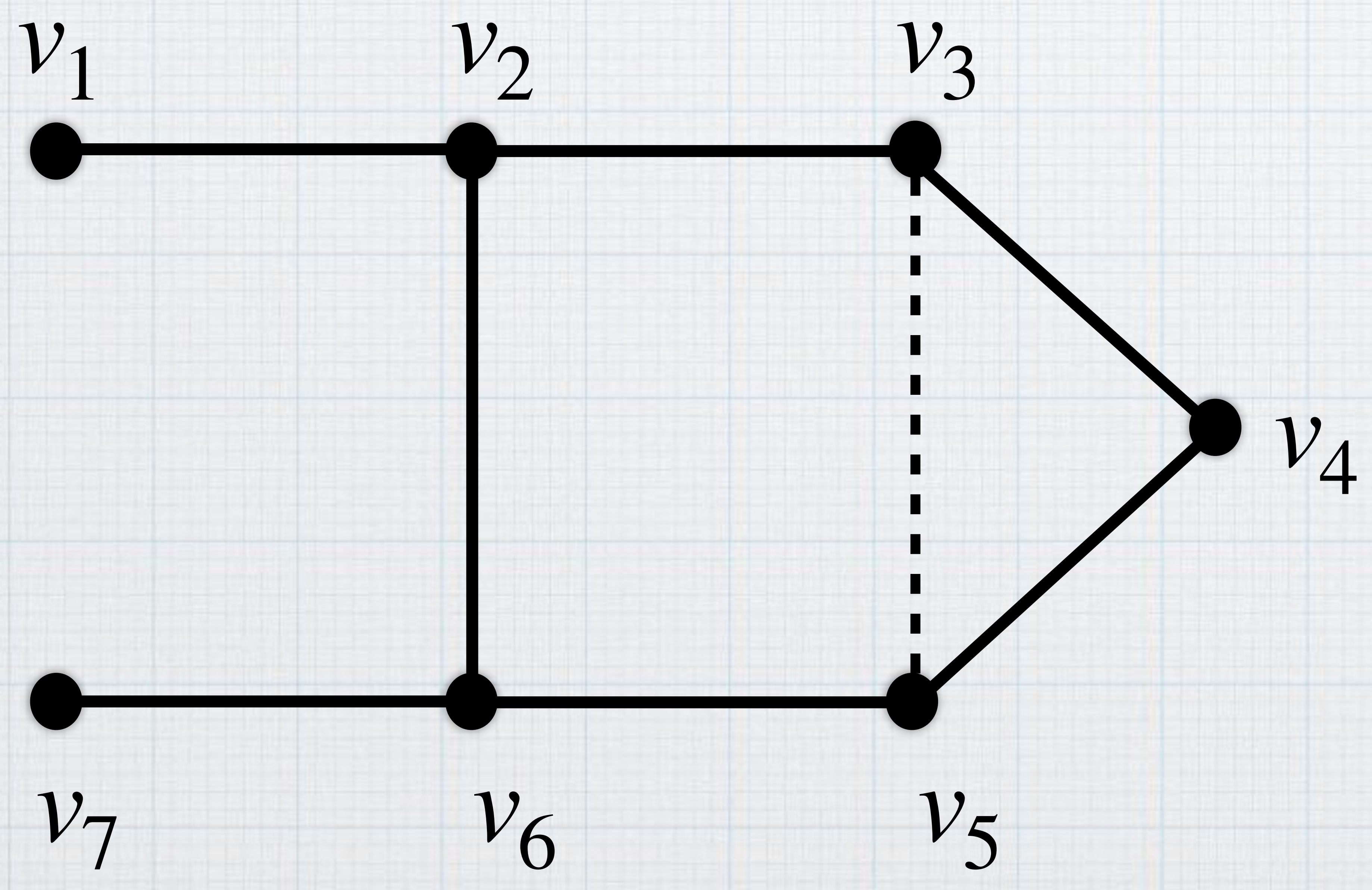
Given a *metric generator* (of specified size) that creates different distance profiles for each vertex, can we reconstruct the graph?



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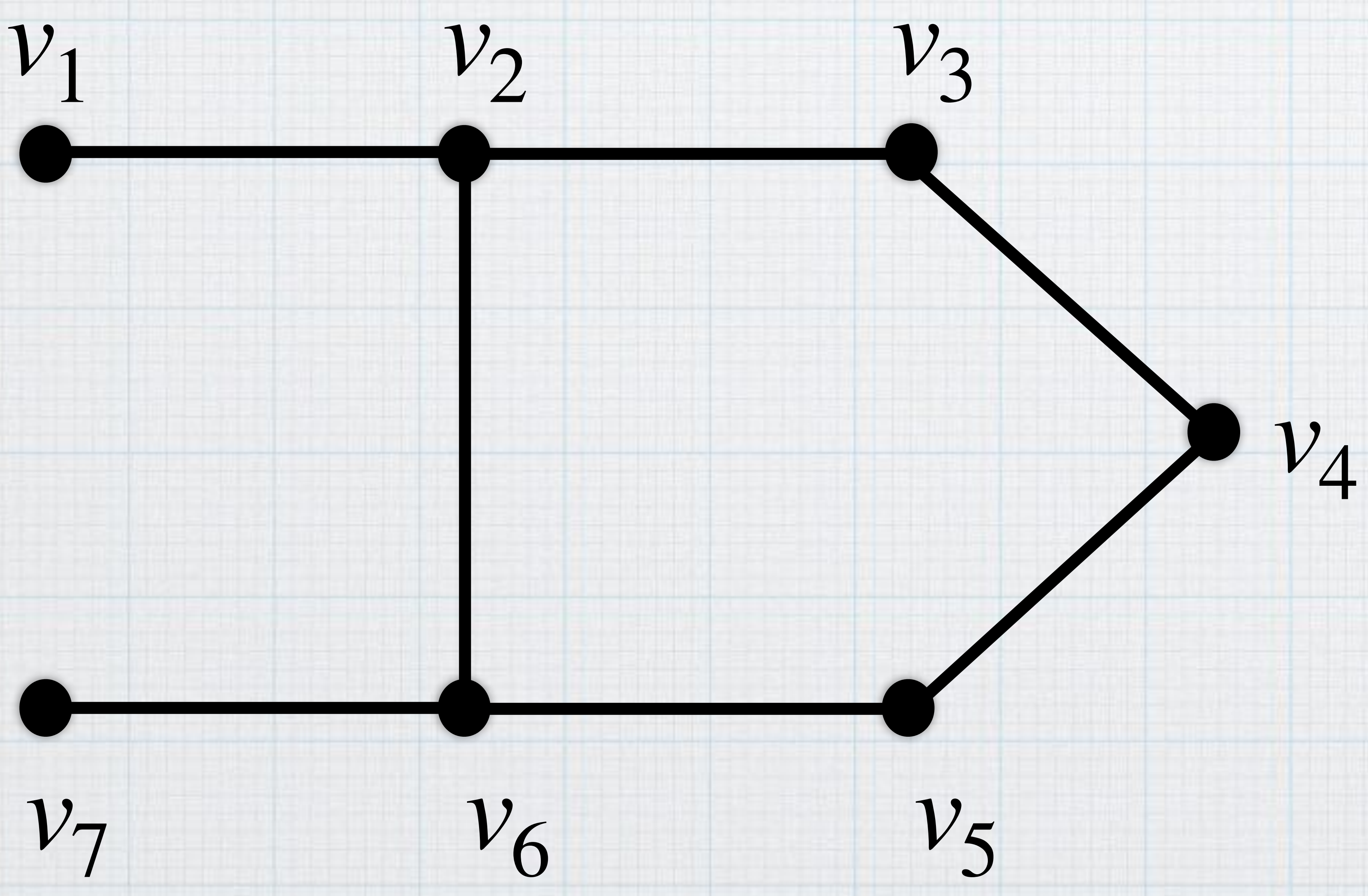
- Sebő and Tannier [Math. of Operation Research (2004)]: Replace ‘metric generator’ by ‘strong metric generator’ to uniquely determine the graph.
- Vertex s strongly resolves pair of vertices u, v if either (i) u is in a v–s path or (ii) v is in u–s path



v_1 resolves (v_2, v_3), (v_2, v_4), $\cdots$

v_1 does not resolve (v_3, v_5)

Strong Metric Dimension (SMD)

Input: Graph G and int k

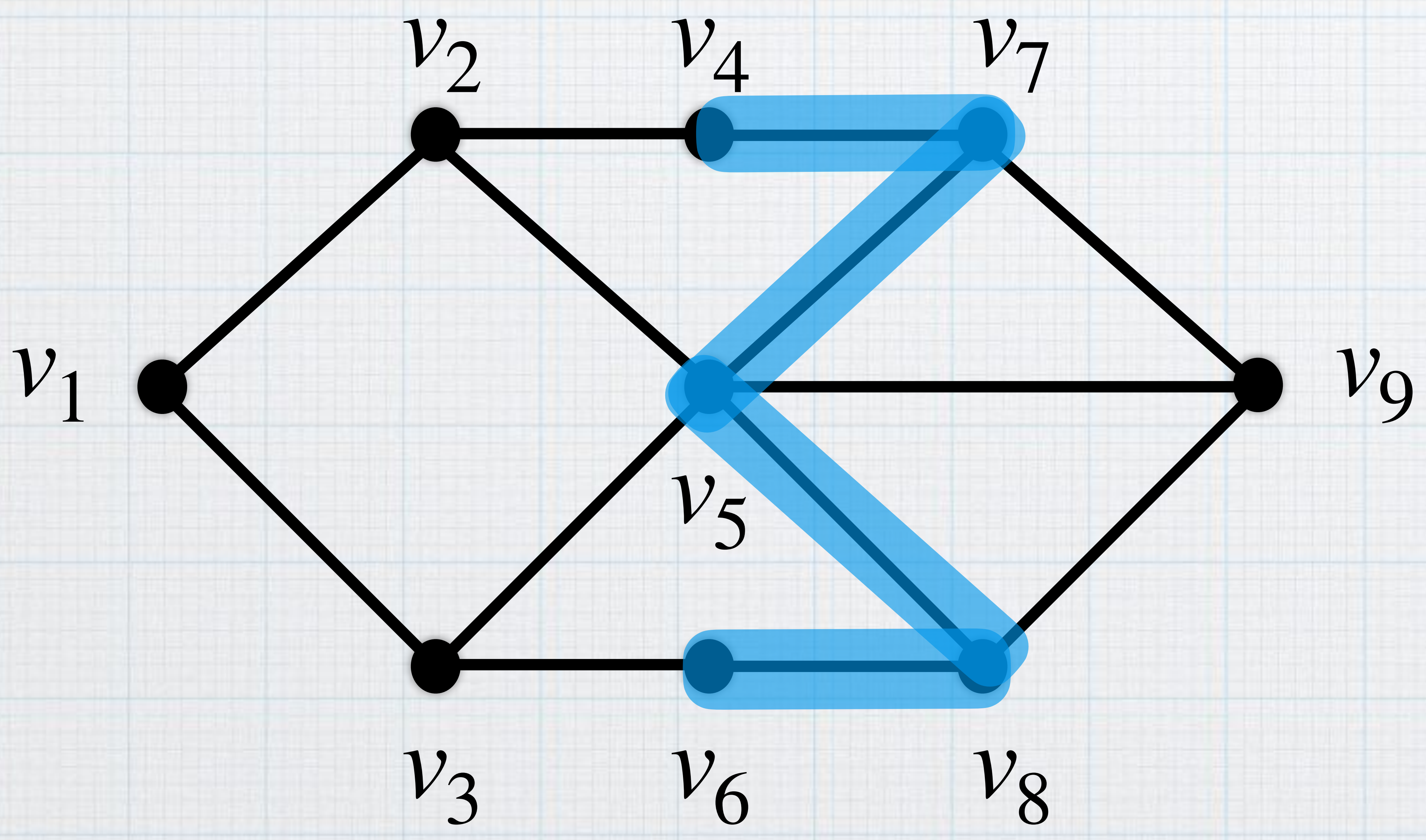
Output: Does there exist a subset S of size k s.t. for every $u, v \in V(G)$, there is a vertex $s \in S$ such as either (i) u is in a v–s path or (ii) v is in u–s path?

- Oellermann and Peters-Fransen [DAM (2007)]: SMD and Vertex Cover are closely related.
- SMD is NP-hard and FPT parameterized by the solution size.
- DasGupta and Mobasheri [DAM (2017)]: Involved conditional lower bounds and hardness for VC also transfer to SMD.
- These reductions are not useful for structural parameters.

Geodetic Set (GS)

Input: Graph G and int k

Output: Does there exist a subset S of size k such that every vertex in G lies on shortest path between two vertices in S?



$S = \{v_4, v_6\}$ covers all vertices.

Metric Graph Problem

Input: Graph G, a metric (ex. shortest distance), and int k

Output: Does there exist a subset S of size k that satisfies some property defined using this metric?

- Metric Graph Problem paramaterized by tree-width?

Q: Can we apply Courcelle's theorem?

A: No MSO formula to encode: Given a subset X and two vertices s, t does X form a shortest s–t path?[#]

- Courcell's theorem results in FPT algorithms parameterized by tree-width plus diameter.

Denis Kuperberg. Public communication on the Theoretical Computer Science Stack Exchange. https://cstheory.stackexchange.com/a/52509, 2023.

	tw+diam			
MD	FPT [1]			
GD	FPT [1]			
SMD	FPT [1]			

- [1] B. Courcelle. The monadic second-order logic of graphs. I. recognizable sets of finite graphs. Inf. Comput. (1990).

	tw+diam	tw		
MD	FPT [1]	W[1]-h [2a] paraNP [2b]		
GD	FPT [1]			
SMD	FPT [1]			

- [2a] É. Bonnet, N. Purohit. Metric Dimension Parameterized by Treewidth. IPEC (2019), Algorithmica (2021)
- [2b] S. Li and M. Pilipczuk. Hardness of metric dimension in graphs of constant treewidth. IPEC (2021), Algorithmica (2022)

	tw+diam	tw		diam
MD	FPT [1]	W[1]-h [2a] paraNP [2b]		paraNP [3]
GD	FPT [1]			
SMD	FPT [1]			

- [3] F. Foucaud, G. B. Mertzios, R. Naserasr, A. Parreau, and P. Valicov. Identification, location-domination and metric dimension on interval and permutation graphs. Algorithmica (2017).

	tw+diam	tw	fvs+pw	diam
MD	FPT [1]	W[1]-h [2a] paraNP [2b]	W[1]-h [4]	paraNP [3]
GD	FPT [1]			
SMD	FPT [1]			

- [4] E.Galby, L.Khazaliya, F.McInerney, R.Sharma, and P.Tale. Metric dimension parameterized by feedback vertex set and other structural parameters. MFCS (2022), SIDMA (2023).

	tw+diam	tw	fvs+pw	diam
MD	FPT [1]	W[1]-h [2a] paraNP [2b]	W[1]-h [4]	paraNP [3]
GD	FPT [1]			paraNP [5]
SMD	FPT [1]			

- [5] D. Chakraborty, F. Foucaud, H. Gahlawat, S. K. Ghosh, and B. Roy. Hardness and approximation for the geodetic set problem in some graph classes. CALDAM (2020).

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SMD	FPT [1]			

- [6a] L. Kellerhals and T. Koana. Parameterized complexity of geodetic set. IPEC (2020), Journal of Graph Algorithms and Applications (2022).
- [6b] P. Tale. Geodetic set on graphs of constant pathwidth and feedback vertex set number. IPEC (2025).

	tw+diam	tw	fvs+pw	diam
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GD	FPT [1]	paraNP [6b]	W[1]-h [6a] paraNP [6b]	paraNP [5]
SMD	FPT [1]	paraNP*	paraNP*	paraNP*#

- * This paper.
- # Implied by F. Foucaud, E. Galby, L. Khazaliya, S. Li, F. Mc Inerney, R. Sharma, and P. Tale. Problems in NP can admit double-exponential lower bounds when parameterized by treewidth or vertex cover. ICALP (2024) if you really really trust the authors.

- F. Foucaud, E. Galby, L. Khazaliya, S. Li, F. Mc Inerney, R. Sharma, and P. Tale. Problems in NP can admit double-exponential lower bounds when parameterized by treewidth or vertex cover. ICALP (2024)

Thm: MD and GS admit $2^{\text{diam}^{\mathcal{O}(\text{tw})}} \cdot n^{\mathcal{O}(1)}$-time algo, but do not admit $2^{\text{diam}^{o(\text{tw})}} \cdot n^{\mathcal{O}(1)}$-time algo unless the ETH fails.

Thm: SMD admits $2^{2^{\mathcal{O}(\text{vc})}} \cdot n^{\mathcal{O}(1)}$-time algo, but does not admit $2^{2^{o(\text{vc})}} \cdot n^{\mathcal{O}(1)}$-time algo unless the ETH fails.

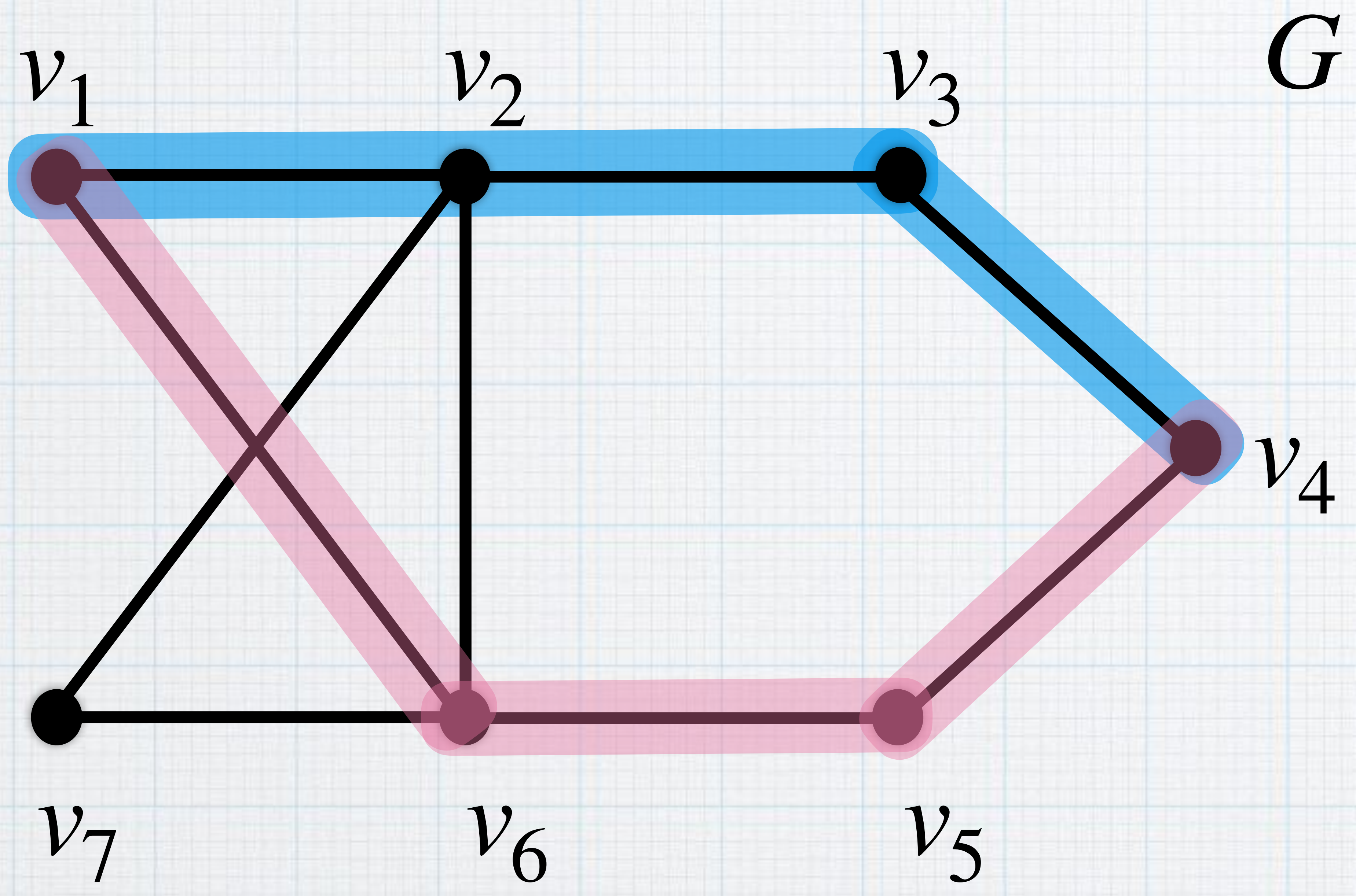
Thm (Implied): The SMD problem remains NP-hard even when diameter is a constant (≥ 3).

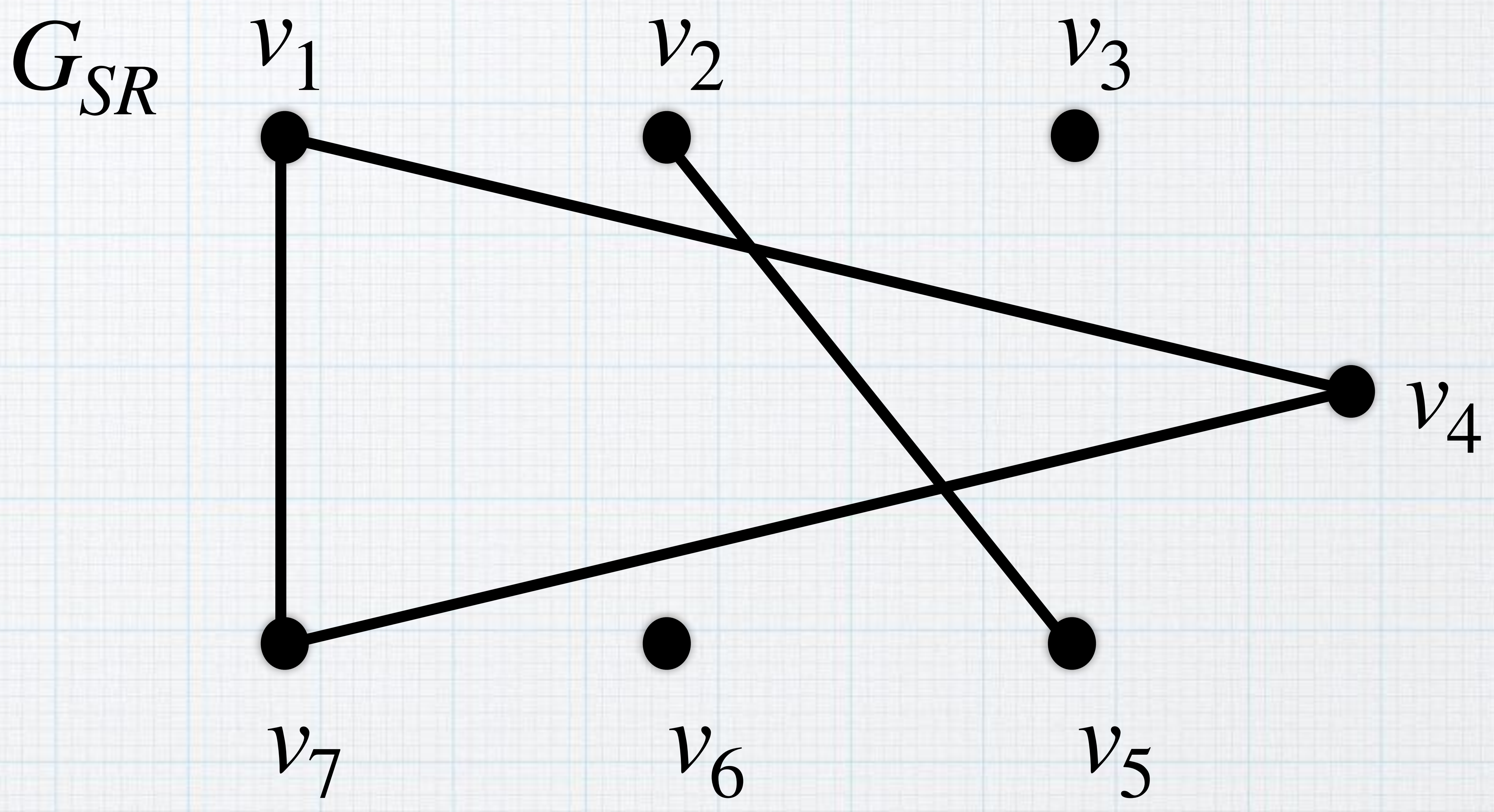
Our Results

Thm: Strong Metric Dimension remains NP-hard even on graphs of diameter two.

Thm: Strong Metric Dimension remains NP-hard even on graphs of the feedback vertex set number 25 and pathwidth 27.

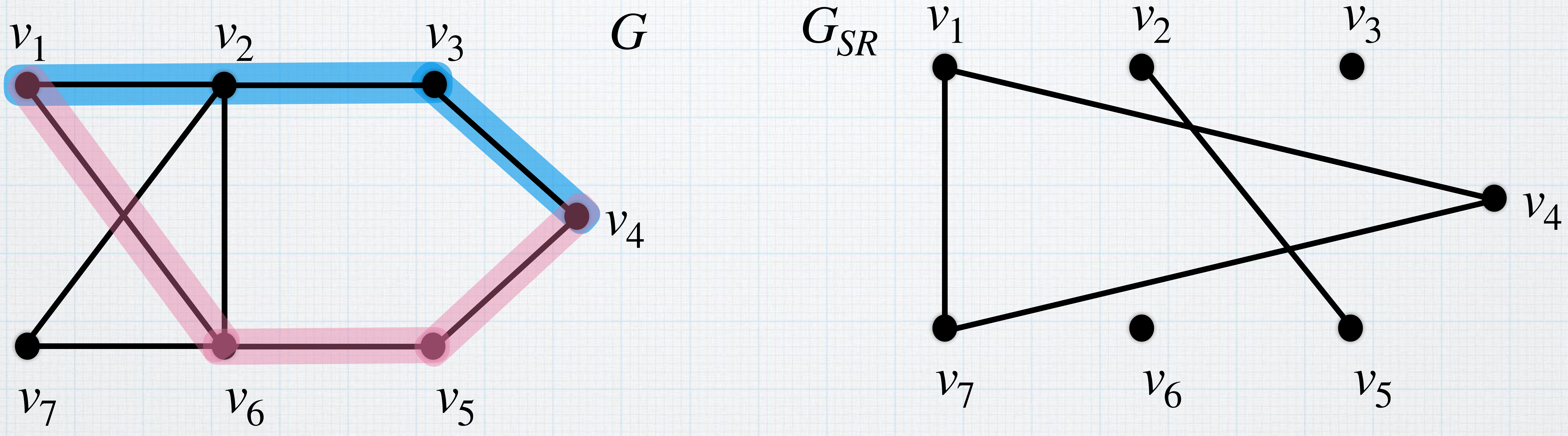
Strong Metric Dimension and Vertex Cover





- Vertex u is maximally distant from v if for every $y \in N(u)$: $\mathrm{dist}(y, v) \leq \mathrm{dist}(u, v)$.
- The strong resolving graph (G_{SR}) of G, has same vertex set and u, v are adjacent in G_{SR} iff they mutually maximally distant in G.

Strong Metric Dimension and Vertex Cover



- Oellermann and Peters-Fransen [DAM (2007)]: (G, k) is a yes-instance of SMD iff (G_{SR}, k) is a yes-instance of Vertex Cover.
- This reduction is not useful for structural parameters.

Strong Metric Dimension para by tw.

Exact-3-Partitioned-3-SAT

Input: A 3-CNF formula ψ with partition $\langle X, Y, Z \rangle$ of variables s.t. every clause of ψ contains exactly one variable from each part.

Output: Does there a satisfying assignment?

- 3-Partitioned-3-SAT: M. Lampis, N. Melissinos, and M. Vasilakis. Parameterized max min feedback vertex set. MFCS (2024), SIDMA (2025).
- Exact-3-Partitioned-3-SAT: F. Foucaud, E. Galby, L. Khazaliya, S. Li, F. Mc Inerney, R. Sharma, and P. Tale. Problems in NP can admit double-exponential lower bounds when parameterized by treewidth or vertex cover. ICALP (2024)

Strong Metric Dimension para by tw.

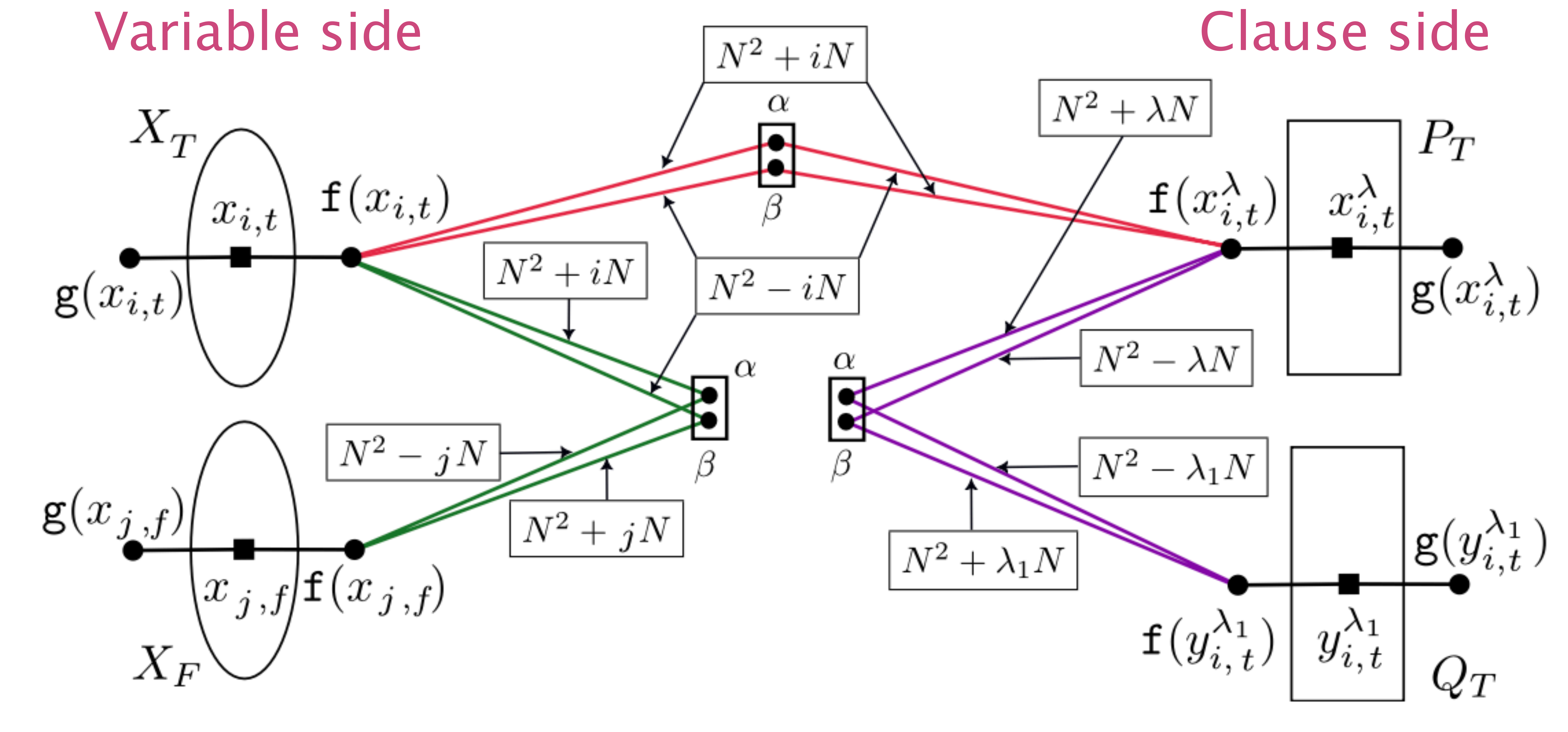
Instance $(\psi, \langle X, Y, Z \rangle)$
of Exact-3-Part-3-SAT

Instance (G, k)
of SMD

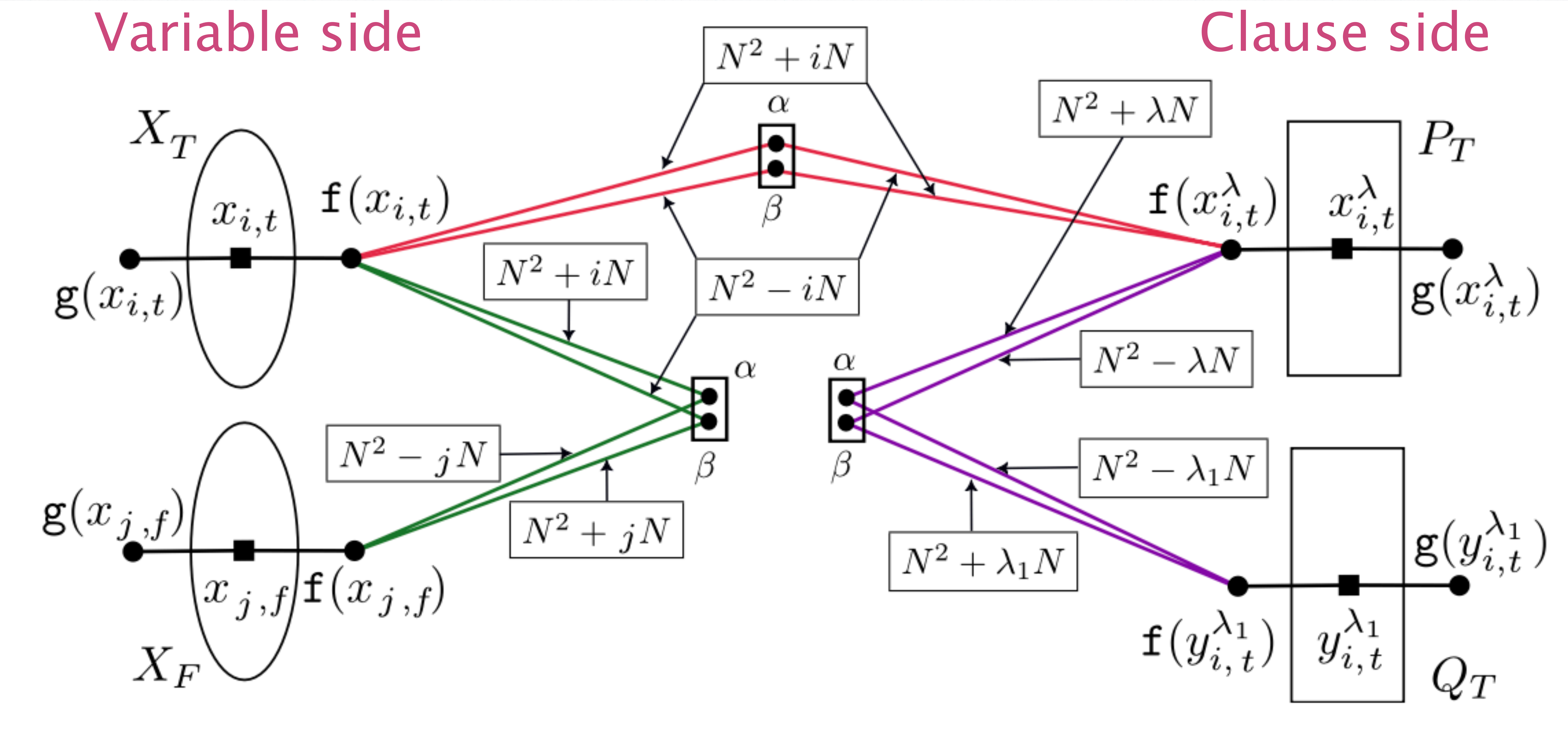
Instance $(H,\ 3n + 2m)$
of Vertex Cover

Instance $(G_{SR},\ k)$
of Vertex Cover

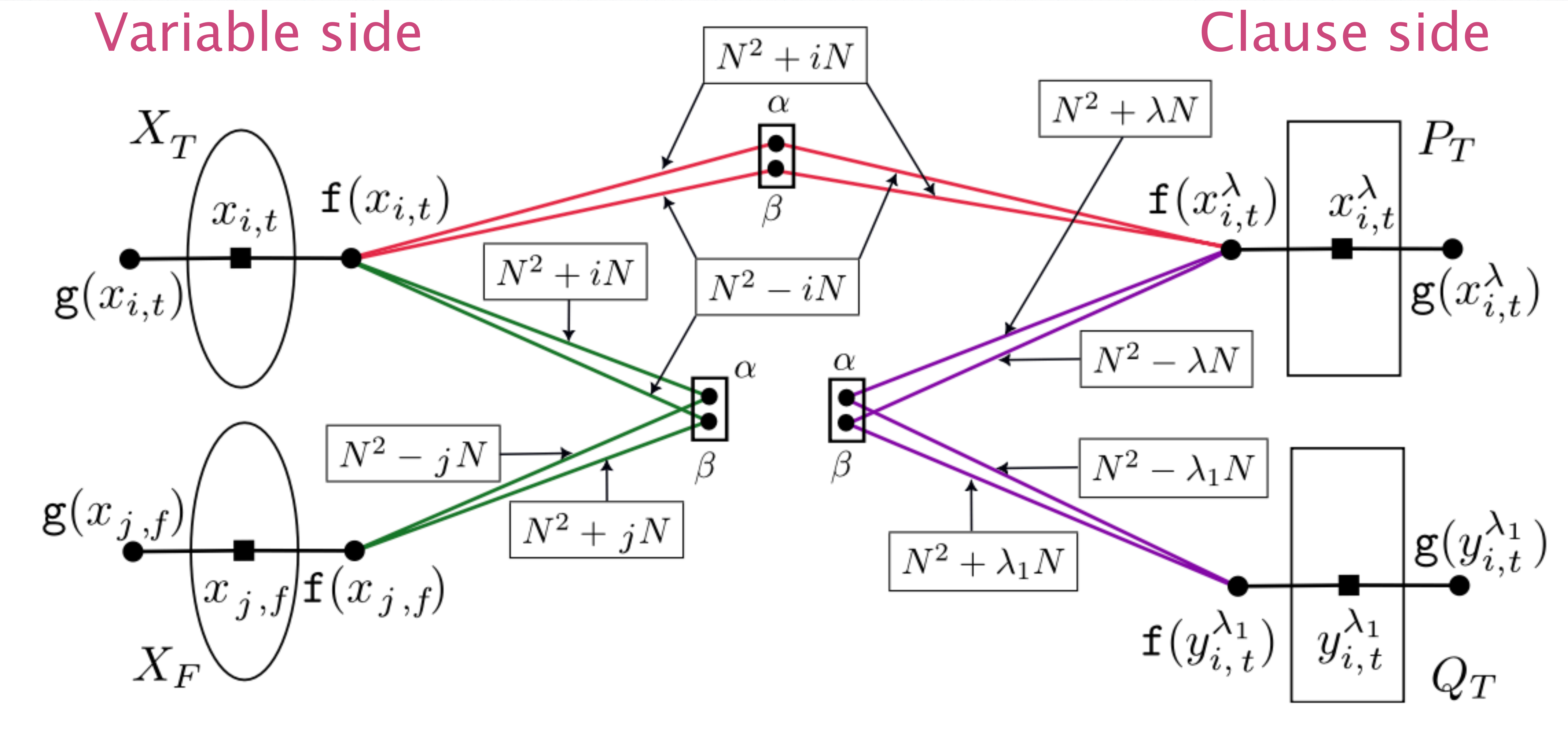
Lemma: Set C is a vertex cover of G_{SR} iff $C \backslash X$ is a vertex cover of H (for set X that can be determined).



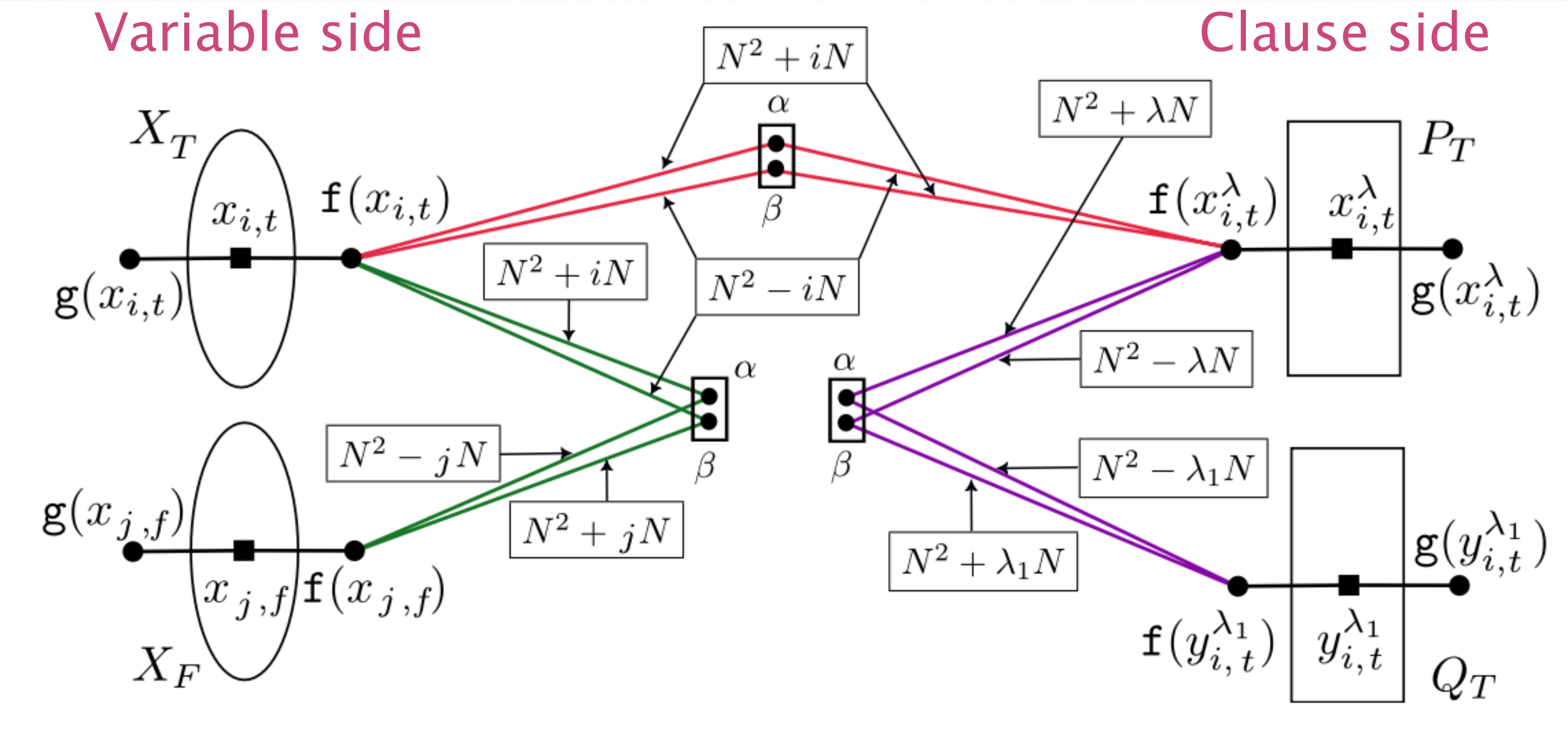
- All vertices of type $g(\,\cdot\,)$ are connected to semi-global vertex g by path of length $N^2 + 1$.



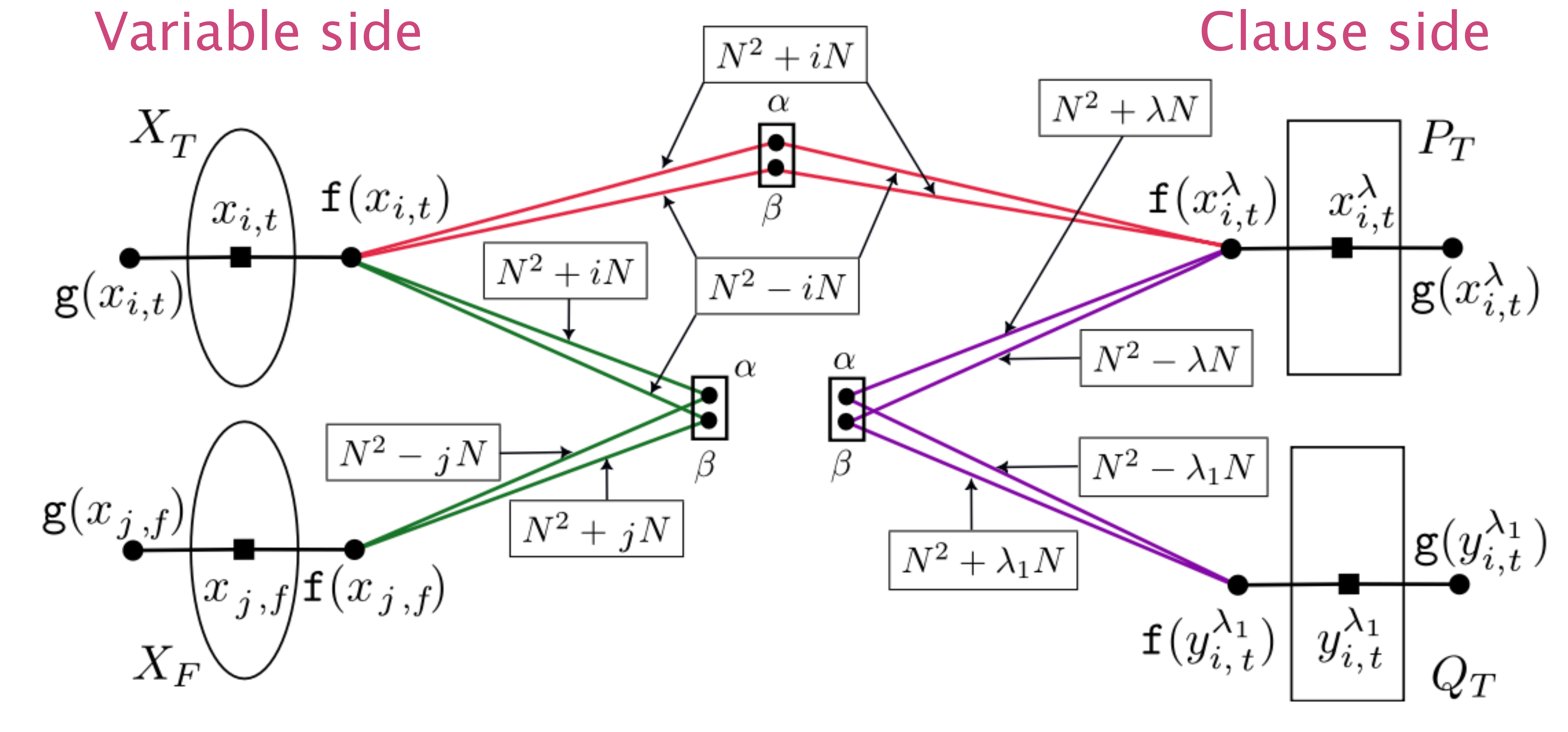
- Green portals: Matching edge across x_i and $\neg x_i$



- Red portals: Edge across x_i and C_j if variable x_i appears in C_j.



- Purple portals: Triangle across three vertices of C_j.



- Remove portal vertices and semi-global vertex results in collection of subdivided starts centered at ■-vertices.

Strong Metric Dimension para by tw.

Instance $(\psi, \langle X, Y, Z \rangle)$
of Exact-3-Part-3-SAT

Instance (G, k)
of SMD

Instance $(H,\ 3n + 2m)$
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Instance $(G_{SR},\ k)$
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Thm: Strong Metric Dimension remains NP-hard even on graphs of the feedback vertex set number 25 and pathwidth 27.

Conclusion and Open Question

	tw+diam	tw	fvs+pw	diam
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GD	FPT [1]	paraNP [6b]	W[1]-h [6a] paraNP [6b]	paraNP [5]
SMD	FPT [1]	paraNP*	paraNP*	paraNP*

- Is Metric Dimension paraNP-hard when para by fvs + pw?

Thank you! Grazie!